Koherentní stavy

mějme zvolenou část. repr.

koher. stav

$$|coh:\phi\rangle = \exp\left(\tfrac{1}{2}\langle\phi,\phi\rangle\right)\hat{W}[\phi]\,|vac\rangle$$

shift operátor

$$\hat{W}[\phi] = \exp(i\,\phi\cdot\partial f\cdot\hat{\Phi}) = \exp(i\hat{L}_\phi)$$

$$\hat{W}[\phi]^\dagger\,\hat{\Phi}\,W[\phi] = \hat{\Phi} + \phi\hat{\mathbb{1}}$$

důkaz:

$$[\hat{\Phi}, i\phi\cdot\partial f\cdot\hat{\Phi}] = -i[\hat{\Phi},\hat{\Phi}]\cdot\partial f\cdot\phi = \phi \quad\Rightarrow\quad [\hat{\Phi}, F(i\phi\cdot\partial f\cdot\hat{\Phi})] = \phi F'(i\phi\cdot\partial f\cdot\hat{\Phi})$$

$$\Downarrow\; \hat{W}[\phi]^\dagger\hat{\Phi}\hat{W}[\phi] = \hat{\Phi} + \hat{W}[\phi]^\dagger[\hat{\Phi}, \exp(i\phi\cdot\partial f\cdot\hat{\Phi})] = \hat{\Phi} + \phi\,\hat{W}[\phi]^\dagger\hat{W}[\phi]$$

$$= \hat{\Phi} + \phi\hat{\mathbb{1}}$$

koherentní stav je tak "posunuté" vakuum

ale! neortonormální, nenormalizované!

pomocné lemma

vynechat

$$\hat{W}[\phi_1]\hat{W}[\phi_2] = \exp(i\phi_1\cdot\partial f\cdot\phi_2)\,\hat{W}[\phi_2]\hat{W}[\phi_1]$$

$$\exp(\hat{a}[\phi_1])\exp(\hat{a}[\phi_2]^\dagger) = \exp(\langle\phi_1,\phi_2\rangle)\exp(\hat{a}[\phi_2]^\dagger)\exp(\hat{a}[\phi_1])$$

$$= \exp(\tfrac{1}{2}\langle\phi_1,\phi_2\rangle)\exp(\hat{a}[\phi_1] + \hat{a}[\phi_2]^\dagger)$$

důkaz:

důsledek $\exp(\hat{A}+\hat{B}) = \exp(-\tfrac{1}{2}[\hat{A},\hat{B}])\exp\hat{A}\,\exp\hat{B}$

pro $[\hat{A},[\hat{A},\hat{B}]] = [\hat{B},[\hat{A},\hat{B}]] = 0$

normální uspořádání shift. oper.

$$\hat{W}[\phi] = \exp(\hat{a}[\phi]^\dagger - \hat{a}[\phi]) =$$

$$= \exp(-\tfrac{1}{2}\langle\phi,\phi\rangle)\exp(\hat{a}[\phi]^\dagger)\exp(-\hat{a}[\phi])$$

vztah koherentních a částicových stavů

$$|coh\,\phi\rangle = \exp(\hat{a}[\phi]^\dagger)\,|vac\rangle$$

$$= \sum_n \frac{1}{\sqrt{n!}}\,|\phi : n\rangle$$

- superpozice částic v módu ϕ
- všechny počty částic (včetně vakua)

neortonormalita a nenormovanost

$$\langle coh\,\phi_1|coh\,\phi_2\rangle = \exp(\langle\phi_1|\phi_2\rangle)$$

$$\Uparrow\ \exp(\hat{a}[\phi_1])\exp(\hat{a}[\phi_2]^\dagger) = \exp(\langle\phi_1,\phi_2\rangle)\exp(\hat{a}[\phi_2]^\dagger)\exp(\hat{a}[\phi_1])$$

- nejsou kolmé i pro kolmé módy $\langle\phi_1,\phi_2\rangle = 0$ (obsahuje vakuum)
- nejsou normované i pro norm. módy $\langle coh\,\phi|coh\,\phi\rangle = \exp(\langle\phi|\phi\rangle)$

relace úplnosti

$$\hat{\mathbb{1}} = \int_{\phi\in\mathcal{G}} d\mu(\phi)\,|coh\,\phi\rangle\langle coh\,\phi|$$

$$d\mu(\phi) = \exp(-\langle\phi,\phi\rangle)\,d\Gamma$$

$$d\Gamma = \left(\mathrm{Det}\,\frac{\partial f}{2\pi}\right)^{\frac{1}{2}} = \left(\mathrm{Det}\,\frac{\partial f}{2\pi}\right)^{\frac{1}{2}} = \prod_x \frac{d\varphi^x\,d\pi_x}{2\pi}$$

$\det J = 1$

viz dále

koherentní stav v částicové bázi

$\mathbf{u} = \{u_k\}$ norm. báze v $\mathfrak{S}_J$

$|\mathbf{u}:\mathbf{n}\rangle$ částicová báze

$$|\mathrm{coh}\,\phi\rangle = \sum_{\mathbf{n}} \frac{1}{\sqrt{\mathbf{n}}} \underbrace{\langle \mathbf{u}, \phi\rangle^{\mathbf{n}}}_{\prod_k \langle u_k, \phi\rangle^{n_k}} |\mathbf{u}:\mathbf{n}\rangle$$

důkaz:

$$|\mathrm{coh}\,\phi\rangle = \exp(\underbrace{\langle \hat{\Phi}, \phi\rangle}_{\hat{a}[\phi]^\dagger})|\mathrm{vac}\rangle =$$

$$= \exp\Big(\sum_k \langle \hat{\Phi}, u_k\rangle\langle u_k, \phi\rangle\Big)|\mathrm{vac}\rangle$$

$$= \prod_k \exp(\langle u_k, \phi\rangle\, \hat{a}_k^\dagger)|\mathrm{vac}\rangle$$

$$= \prod_k \sum_{n_k} \frac{1}{n_k!} \langle u_k, \phi\rangle^{n_k}\, \hat{a}_k^{\dagger n_k} |\mathrm{vac}\rangle$$

$$= \sum_{\mathbf{n}} \underbrace{\frac{1}{\sqrt{\mathbf{n}!}} \langle \mathbf{u}, \phi\rangle^{\mathbf{n}}}_{\prod_k \frac{1}{\sqrt{n_k!}}\langle u_k, \phi\rangle^{n_k}} \underbrace{|\mathbf{u}:\mathbf{n}\rangle}_{\frac{1}{\sqrt{\mathbf{n}!}}\hat{a}[\mathbf{u}]^{\dagger\mathbf{n}}|\mathrm{vac}\rangle = \prod_k \frac{1}{\sqrt{n_k!}}\hat{a}_k^{\dagger n_k}|\mathrm{vac}\rangle}$$

vlastní vektory anihil. operátorů

$$\hat{a}[\phi_0]\,|\mathrm{coh}:\phi\rangle = \langle\phi_0,\phi\rangle\,|\mathrm{coh}\,\phi\rangle$$

důkaz

$$\langle\phi_0,\hat{\Phi}\rangle\exp(\tfrac{1}{2}\langle\phi,\phi\rangle)\hat{W}[\phi]|\mathrm{vac}\rangle =$$
$$= \exp(\tfrac{1}{2}\langle\phi,\phi\rangle)\hat{W}[\phi]\langle\phi_0,\hat{\Phi}+\phi\hat{\mathbb{1}}\rangle|\mathrm{vac}\rangle$$
$$= \langle\phi_0,\phi\rangle\exp(\tfrac{1}{2}\langle\phi,\phi\rangle)\hat{W}[\phi]|\mathrm{vac}\rangle$$
$$= \langle\phi_0,\phi\rangle|\mathrm{coh}\,\phi\rangle$$

neboli $|\mathrm{coh}\,\phi\rangle$ je vl. vekt. neg. fr. části $\hat{\Phi}^-$

$$\hat{\Phi}^-|\mathrm{coh}\,\phi\rangle = \phi^-|\mathrm{coh}\,\phi\rangle$$

akce kreačních oper.

$$\hat{a}[\phi_0]^\dagger\,|\mathrm{coh}\,\phi\rangle = \phi_0\cdot\frac{\delta}{\delta\phi}|\mathrm{coh}\,\phi\rangle$$

důkaz:

$$\hat{a}[\phi_0]^\dagger|\mathrm{coh}\,\phi\rangle = \hat{a}[\phi_0]^\dagger\exp(\hat{a}[\phi]^\dagger)|\mathrm{vac}\rangle =$$
$$= -i\,\hat{\Phi}^-\cdot\partial\mathcal{F}\cdot\phi_0\,\exp(-i\,\hat{\Phi}^-\cdot\partial\mathcal{F}\cdot\phi)|\mathrm{vac}\rangle =$$
$$= \phi_0\cdot\frac{\delta}{\delta\phi}\exp(-i\,\hat{\Phi}^-\cdot\partial\mathcal{F}\cdot\phi)|\mathrm{vac}\rangle = \phi_0\cdot\frac{\delta}{\delta\phi}|\mathrm{coh}:\phi\rangle$$

střední počet částic

$$\frac{\langle\mathrm{coh}\,\phi|\,\hat{n}[\phi_0]\,|\mathrm{coh}\,\phi\rangle}{\langle\mathrm{coh}\,\phi|\mathrm{coh}\,\phi\rangle} = \frac{\langle\phi,\phi_0\rangle\langle\phi_0,\phi\rangle}{\langle\phi_0,\phi_0\rangle}$$ – ortogonální módy nepřispívají!

tj. $$\frac{\langle\mathrm{coh}\,\phi|\,\hat{n}[\phi]\,|\mathrm{coh}\,\phi\rangle}{\langle\mathrm{coh}\,\phi|\mathrm{coh}\,\phi\rangle} = \langle\phi,\phi\rangle$$

důkaz

$$\hat{n}[\phi_0] = \frac{\hat{a}[\phi_0]^\dagger\hat{a}[\phi_0]}{\langle\phi_0,\phi_0\rangle} \qquad \hat{a}[\phi_0]|\mathrm{coh}\,\phi\rangle = \langle\phi_0,\phi\rangle|\mathrm{coh}\,\phi\rangle$$

celkový počet částice

$$\frac{\langle\mathrm{coh}\,\phi|\,\hat{N}\,|\mathrm{coh}\,\phi\rangle}{\langle\mathrm{coh}\,\phi|\mathrm{coh}\,\phi\rangle} = \langle\phi,\phi\rangle$$ – všechny částice v módu ϕ

střední hodnota pole

$$\frac{\langle coh\,\phi|\hat{\Phi}|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \phi$$

důkaz:

$$\frac{\langle coh\,\phi|\hat{\Phi}|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \frac{\langle coh|\hat{\Phi}^- + \hat{\Phi}^+|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \phi^- + \phi^+ = \phi$$

díky $\hat{\Phi}^+|coh\,\phi\rangle = \phi|coh\,\phi\rangle \qquad \langle coh\,\phi|\hat{\Phi}^+ = \phi^+\langle coh\,\phi|$

komplementarita s část. popisem

$$\frac{\langle\phi:n|\hat{\Phi}|\phi:n\rangle}{\langle\phi:n|\phi:n\rangle} = 0$$

či

$$\langle u:m|\hat{\Phi}|u:m\rangle = 0$$

asi vynechat

jeden mód ϕ_0 $\qquad \langle \phi_0, \phi_0 \rangle = 1$

$\hat{a} = \hat{a}[\phi_0] \qquad [\hat{a}, \hat{a}^+] = \hat{\mathbb{1}} \qquad \hat{n} = \hat{a}^+ \hat{a}$

část. stavy

$$|n\rangle = |\phi_0 : n\rangle = \frac{1}{\sqrt{n!}} \hat{a}^{+n} |vac\rangle \qquad \langle n | n \rangle = 1$$

koh. stav $\quad \phi = \varkappa \phi_0 \qquad \langle \phi, \phi \rangle = |\varkappa|^2$

$$|koh\, \phi\rangle_{\varkappa \phi_0} = \exp\left(\tfrac{1}{2}|\varkappa|^2\right) \hat{W}[\phi] |vac\rangle$$

$$= \exp\left(\hat{a}[\phi]^+\right) |vac\rangle =$$

$$= \exp\left(\varkappa \hat{a}^+\right) |vac\rangle$$

$$= \sum_n \frac{1}{\sqrt{n!}} \varkappa^n |n\rangle$$

$$\langle koh\, \phi | koh\, \phi \rangle = \exp\left(|\varkappa|^2\right)$$

$$|\widetilde{koh}\, \phi\rangle = \hat{W}[\phi] |vac\rangle =$$

$$= \exp\left(-\tfrac{1}{2}|\varkappa|^2\right) \sum_n \frac{1}{\sqrt{n!}} \varkappa^n |n\rangle$$

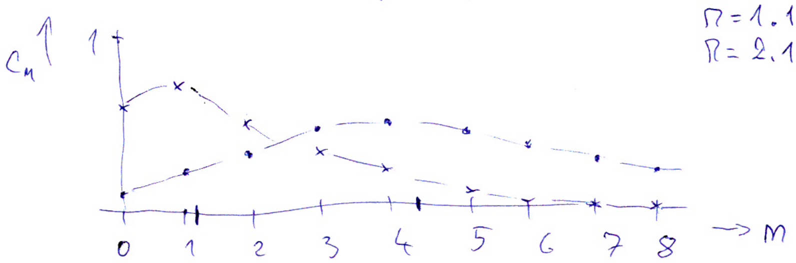
$$\langle \widetilde{koh}\, \phi | \widetilde{koh}\, \phi \rangle = 1$$

$$\langle \widetilde{koh}\, \phi | \hat{n} | \widetilde{koh}\, \phi \rangle = \frac{\langle koh\, \phi | \hat{n} | koh\, \phi \rangle}{\langle koh\, \phi | koh\, \phi \rangle} = \langle \phi, \phi \rangle = |\varkappa|^2$$

obsazovací koefic pro $|\widetilde{koh}\, \varkappa \phi_0\rangle = \sum c_n |n\rangle$

$\varkappa = 1.1$ × × ×

$\varkappa = 2.1$ • • •



$\langle \hat{n} \rangle = 1.1^2 \qquad \langle \hat{n} \rangle = 2.1^2$