Greenovy funkce

klasické Gr. fce

G_{ret} retardovaná $\quad G_{ret}(x|y) = \Theta(x|y)\, G_{ret}(x|y)$

G_{adv} advancovaná $\quad G_{adv}(x|y) = \Theta(y|x)\, G_{adv}(x|y)$

$G_{sym} = \frac{1}{2}(G_{ret} + G_{adv})$ symetrické

$G_c = G_{ret} - G_{adv}$ kauzální

$$G_{ret}^T = G_{adv} \qquad G_{sym}^T = G_{sym} \qquad G_c^T = -G_c$$
$$G_{ret} = \Theta\, G_c \qquad G_{adv} = -\Theta^T G_c$$

kvantové Gr. fce

mějme zvolenou část. interpretaci s vakuem $|vac\rangle$

$$G^+(x|y) = \langle vac|\, \hat{\Phi}(x)\, \hat{\Phi}(y)\, |vac\rangle$$
$$G^-(x|y) = \langle vac|\, \hat{\Phi}(y)\, \hat{\Phi}(x)\, |vac\rangle$$

Wightmanovy Gr. fce

$$G^H(x|y) = \langle vac|\big(\hat{\Phi}(x)\hat{\Phi}(y) + \hat{\Phi}(y)\hat{\Phi}(x)\big)|vac\rangle$$

Hadamardova Gr. fce

$$G^H = G^+ + G^- \qquad G^+ = \frac{1}{2}(G^H - i G_c) \qquad G^{+T} = G^- \qquad G^{H} = G^H$$
$$-iG_c = G^+ - G^- \qquad G^- = \frac{1}{2}(G^H + iG_c) \qquad G^{+*} = G^- \qquad G^{H*} = G^H$$

Feynmanova Gr. fce

$$G^F(x|y) = \langle vac|\, T\, \hat{\Phi}(x)\, \hat{\Phi}(y)\, |vac\rangle =$$

$$= \begin{cases} x > y & \langle vac|\hat{\Phi}(x)\hat{\Phi}(y)|vac\rangle = G^+ \\ x \sim y & \langle vac|\hat{\Phi}(x)\hat{\Phi}(y)|vac\rangle = \frac{1}{2}G^H \\ x < y & \langle vac|\hat{\Phi}(y)\hat{\Phi}(x)|vac\rangle = G^- \end{cases}$$

$$G^F = \frac{1}{2}G^H - \frac{i}{2}\Theta\, G_c + \frac{i}{2}\Theta^T G_c = \frac{1}{2}G^H - i\, G_{sym}$$
$$= G^+ + i\,\Theta^T G_c = G^+ - i\, G_{adv}$$
$$= G^- - i\,\Theta\, G_c = G^- - i\, G_{ret}$$

$$G^\pm = G^F + i\, G_{adv/ret} \qquad G^H = 2(G^F + i G_{sym}) \qquad G^{FT} = G^F$$

vztah k pos-neg. frekv. rozštěpení

$$G^+ = \langle vac|\hat{\Phi}\hat{\Phi}|vac\rangle = \langle vac|\hat{\Phi}^+\hat{\Phi}^-|vac\rangle$$

$G^+(x|y)$ pos. fr. v x
neg. fr. v y

$$G^+ = P^+ \cdot G^+ \cdot P^-$$

$$G^- = P^- \cdot G^- \cdot P^+$$

platí (kompaktabilita J a ∂f)

$$-iG_c = \underbrace{\langle vac|[\hat{\Phi},\hat{\Phi}]|vac\rangle}_{G^+ - G^-} = \underbrace{-iP^+ \cdot G_c \cdot P^-}_{G^+} \underbrace{-iP^- \cdot G_c \cdot P^+}_{G^-}$$

⇓

$$G^\pm = \mp i\, P^\pm \cdot G_c \cdot P^\mp = \mp i\, P^\pm \cdot G_c$$

$$G^\pm \cdot \partial f = \pm i P^\pm \qquad \text{tj} \quad \pm i P^\pm \cdot \phi = G^\pm \cdot \partial f \cdot \phi$$

$$G^H \cdot \partial f = J \qquad \text{tj} \quad J \cdot \phi = G^H \cdot \partial f \cdot \phi$$

$$G^H = -J \cdot G_c = G_c \cdot J$$

rovnice pro Gr.-fce

G_{ret}, G_{adv}, G_{sym}, iG^F splňuje

$$\tilde{F} \cdot G = \delta \qquad G \text{ je inverze } F$$

G_c, $G^\pm$, G^H řeší homogenní rovnici

$$\tilde{F} \cdot G = 0$$

navzájem se liší okrajovými podmínkami

rozdíl dvou Gr. fcí typu F^{-1} je homog. Gr.-fce

$$G_c \cdot \partial f \cdot G_c = -G_c$$

$$G_H \cdot \partial f \cdot G_H = G_c \qquad \Leftarrow \quad J \cdot \partial f \cdot J = \partial f$$

$$G^\pm \cdot \partial f \cdot G^\pm = \pm i G^\pm \qquad \Leftarrow \quad P^{\pm 2} = P^\pm$$

rozklad do báze

$$G^+ = \sum_{\mathfrak{r}} u^+_{\mathfrak{r}} u^-_{\mathfrak{r}} \qquad G^- = \sum_{\mathfrak{r}} u^-_{\mathfrak{r}} u^+_{\mathfrak{r}}$$

$$G^H = \sum_{\mathfrak{r}} \left(u^+_{\mathfrak{r}} u^-_{\mathfrak{r}} + u^-_{\mathfrak{r}} u^+_{\mathfrak{r}} \right)$$

$$G_c = i \sum_{\mathfrak{r}} \left(u^+_{\mathfrak{r}} u^-_{\mathfrak{r}} - u^-_{\mathfrak{r}} u^+_{\mathfrak{r}} \right)$$

Greenovy fce ve statickém případě

obecné řešení – Cauchyho propagátor

$$\varphi(t) = \cos(\Omega t)\cdot\varphi_0 + \Omega^{-1}\cdot\sin(\Omega t)\cdot\tilde{q}^{-1}\cdot\pi_0$$

lze odečíst kauz. gr. fci

kauzální gr. fce a zl. gr. fce

$$G_c(t_2|t_1) = \Omega^{-1}\cdot\sin(\Omega(t_1-t_2))\cdot\tilde{q}^{-1}$$

$$G_{ret}(t_2|t_1) = \Theta(t_2-t_1)\; G_c(t_2|t_1) = G_{adv}(t_1|t_2)$$

$$G_{adv}(t_2|t_1) = -\Theta(t_1-t_2)\; G_c(t_2|t_1) = G_{ret}(t_1|t_2)$$

$$G_{avg}(t_2|t_1) = \frac{1}{2}\,\mathrm{sign}(t_2-t_1)\; G_c(t_2|t_1)$$

Wightmanovy gr. fce

$$G^+ = \langle vac|\hat{\Phi}\,\hat{\Phi}|vac\rangle$$

$$G^+(t_2|t_1) = \langle vac|\hat{\varphi}^+(t_2)\,\hat{\varphi}^-(t_1)|vac\rangle$$

$$= \sum_{k,l}\langle vac|\hat{a}_k\,\varphi_k^+(t_2)\,\varphi_l^-(t_1)\,\hat{a}_l^+|vac\rangle$$

$$= \sum_k \varphi_k^+(t_2)\,\varphi_k^-(t_1) = \sum_k \frac{1}{2\omega_k}\exp(-i\omega_k(t_2-t_1))\,\vartheta_k\vartheta_k$$

$$= \frac{1}{2}\Omega^{-1}\cdot\exp(-i\Omega(t_2-t_1))\cdot\tilde{q}^{-1}$$

$$G^-(t_2|t_1) = \frac{1}{2}\Omega^{-1}\cdot\exp(+i\Omega(t_2-t_1))\cdot\tilde{q}^{-1}$$

Hadamard

$$G^H(t_2|t_1) = \Omega^{-1}\cdot\cos(\Omega(t_2-t_1))\cdot\tilde{q}^{-1}$$

Feynman

$$G^F(t_2|t_1) = \frac{1}{2}G^H(t_2|t_1) - i\,G_{avg}(t_2|t_1)$$

$$= \frac{1}{2}\Omega^{-1}\cdot\cos(\Omega(t_2-t_1))\cdot\tilde{q}^{-1} - i\,\frac{1}{2}\Omega^{-1}\cdot\sin(\Omega|t_2-t_1|)\cdot\tilde{q}^{-1}$$

$$= \frac{1}{2}\Omega^{-1}\cdot\exp(-i\Omega|t_2-t_1|)\cdot\tilde{q}^{-1}$$

Termální Gr. fce

$$G^+_\beta = \langle \hat{\Phi} \hat{\Phi} \rangle_\beta \qquad G^-_\beta = G^{+\,T}_\beta \qquad \text{a t. d.}$$

$$\hat{\Phi} = \sum_k \frac{1}{\sqrt{2\omega_k}} \left(\exp(-i\omega_k t)\, \hat{a}_k + \exp(i\omega_k t)\, \hat{a}^+_k \right) \vartheta_k$$

$\Downarrow$

$$G^+_\beta = \sum_k \frac{1}{2\omega_k} \left(\exp(-i\omega_k \Delta t) \langle \hat{a}_k \hat{a}^+_k \rangle + \exp(i\omega_k \Delta t) \langle \hat{a}^+_k \hat{a}_k \rangle \right) \vartheta_k \vartheta_k$$

$\Downarrow$

$$G^\pm_\beta = \sum_k \frac{1}{2\omega_k} \frac{1}{1-\exp(-\beta\omega_k)} \, q^\pm_k(\Delta t) \; \vartheta_k \vartheta_k$$

$$q^\pm_k(\Delta t) = \exp(\mp i\omega_k \Delta t) + \exp(\pm i\omega_k (\Delta t \pm i\beta))$$

Integrální reprezentace Greenových funkcí

$$[\tilde{\partial}_t^2 + \Omega^2]\cdot G = \tilde{q}^{-1}\,\delta(\Delta t)$$

↖ normováno k dt

$$\tilde{f} = \tilde{q}\cdot[\tilde{\partial}_t^2 + \Omega^2]$$

Four. tr. + rozklad do módů v 1. argum.

$$G(t,x|t',x') = \sum_k \frac{1}{\sqrt{2\pi}}\int d\omega\, \exp(-i\omega t)\, \vartheta_k(x)\, g_k(\omega|t'x')$$

$$[\tilde{\partial}_t^2 + \Omega^2]\cdot G = \sum_k \frac{1}{\sqrt{2\pi}}\int d\omega\, (-\omega^2+\omega_k^2)\, g_k(\omega|t'x')\exp(-i\omega t)\,\vartheta_k(x)$$

$$\tilde{q}^{-1}(x|x')\,\delta(t|t') = \sum_k \frac{1}{2\pi}\int d\omega\, \exp(-i\omega(t-t'))\,\vartheta_k(x)\,\vartheta_k(x')$$

⇓

$$g_k(\omega|t'x') = \frac{1}{\sqrt{2\pi}}\,\frac{1}{-\omega^2+\omega_k^2}\,\exp(i\omega t')\,\vartheta_k(x')$$

⇓

$$G(tx|t'x') = \sum_k \frac{1}{2\pi}\int d\omega\, \frac{\exp(-i\omega(t-t'))}{-\omega^2+\omega_k^2}\,\vartheta_k(x)\,\vartheta_k(x')$$

problém s póly na $\omega^2 = \omega_k^2$

různé volby integrační cesty vede na různé Gr. fce

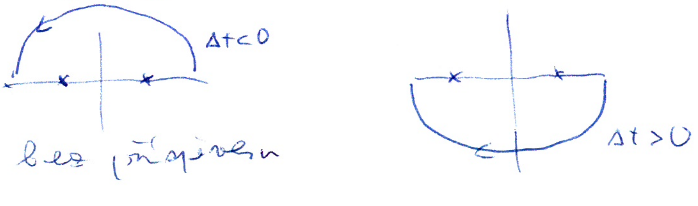
obecně

$$\int_{\omega\in\mathbb{R}} d\omega\, \frac{\exp(-i\omega\Delta t)}{-\omega^2+\omega_k^2} = \int d\omega\, \frac{1}{2\omega_k}\left[\frac{\exp(-i\omega\Delta t)}{\omega+\omega_k} - \frac{\exp(-i\omega\Delta t)}{\omega-\omega_k}\right]$$

póly: $\omega = -\omega_k$, $\omega = \omega_k$

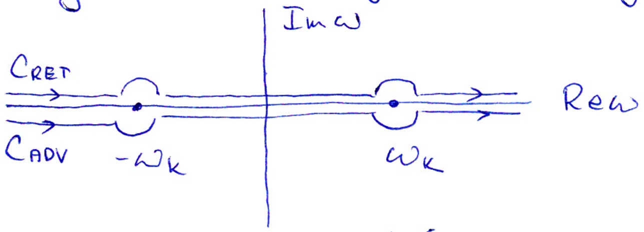
pro $\Delta t < 0$ lze oblouk uzavřít v horní ω-polorovině

$\Delta t > 0$ lze oblouk uzavřít v dolní ω-polorovině



Retardovaná a advancovaná Gr. fce

uvažujme následující integr. cesty:

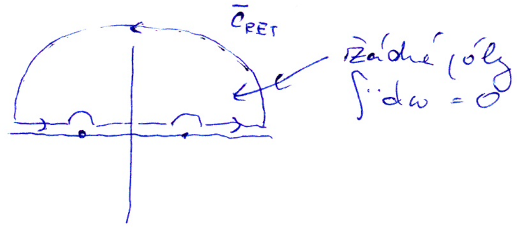


pro cestu C_{RET} platí:

$\Delta t < 0 \quad \mathrm{Im}\,\omega > 0 \Rightarrow \mathrm{Re}(-i\omega\Delta t) < 0$

$\Rightarrow C_{RET}$ lze uzavřít v polorovině $\mathrm{Im}\,\omega > 0$ kde

$$\int_{C_{RET}} \frac{\exp(-i\omega\Delta t)}{-\omega^2+\omega_k^2}\, d\omega = 0$$

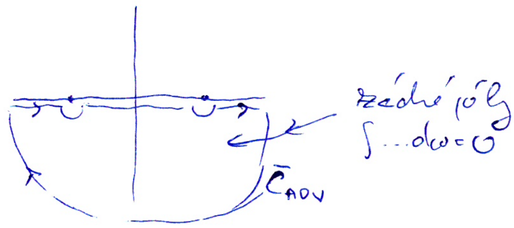


$\Rightarrow G_{C_{RET}}(tx|t'x') = 0$ pro $t<t'$ tj $G = G_{RET}$

pro cestu C_{ADV} obdobně platí

$G_{C_{ADV}}(tx|t'x') = 0$ pro $t>t'$

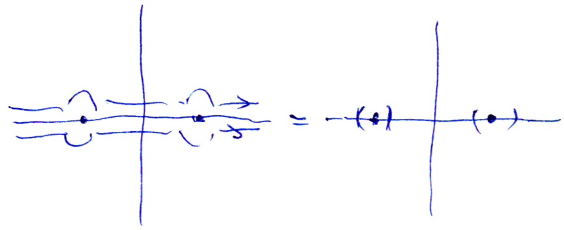
$\Rightarrow G = C_{ADV}$



symetrická Gr. fce

$$G_{sym} = \frac{1}{2}\left(G_{ret} + G_{adv}\right)$$

integr. cesta je průměr nad a pod → "principal value"



explicitní výpočet

$$G_{ret}(tx|t'x') = \sum_k \frac{1}{2\pi}\int_{C_{RET}} d\omega\, \frac{\exp(-i\omega\Delta t)}{-\omega^2+\omega_k^2}\, \vartheta_k(x)\vartheta_k(x')$$

$$\frac{\exp(-i\omega)}{2\omega_k}\left(\frac{1}{-\omega+\omega_k} + \frac{1}{\omega+\omega_k}\right) \sim \frac{i}{2\omega_k}\left(\mathrm{res}_{\omega_k}\frac{\exp(\ )}{-\omega+\omega_k} + \mathrm{res}_{-\omega_k}\frac{\exp(\ )}{\omega+\omega_k}\right)$$ pro $\Delta t > 0$

$$= \Theta(\Delta t)\sum_k \frac{i}{2\omega_k}\left(\exp(-i\omega_k\Delta t) - \exp(i\omega_k\Delta t)\right)\vartheta_k(x)\vartheta_k(x')$$

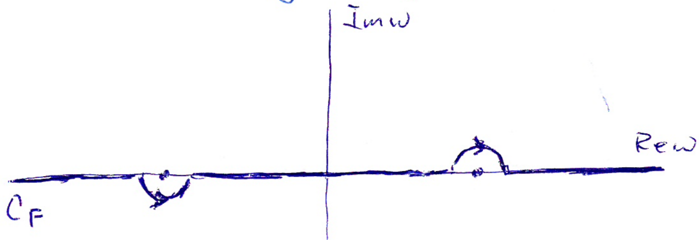
$$= \Theta(\Delta t)\sum_k \frac{\sin(\omega_k\Delta t)}{\omega_k}\vartheta_k(x)\vartheta_k(x')$$

$$= \Theta(\Delta t)\frac{\sin(\Omega\Delta t)}{\Omega}\cdot\tilde{q}^{-1}$$

$$G_{adv}(tx|t'x') = -\Theta(-\Delta t)\frac{\sin(\Omega\Delta t)}{\Omega}\cdot\tilde{q}^{-1}$$

$$G_{sym}(tx|t'x') = \frac{\sin(\Omega|\Delta t|)}{2\Omega}\cdot\tilde{q}^{-1}$$

Feynmanova gr. fce

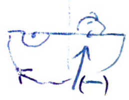


$$\underbrace{\frac{1}{2\omega_k}\left(\frac{\exp(-i\omega\Delta t)}{\omega+\omega_k} - \frac{\exp(-i\omega\Delta t)}{\omega-\omega_k}\right)}$$

$$iG_F(tx|t'x') = \sum_k \frac{1}{2\pi}\int_{C_F} d\omega \, \frac{\exp(-i\omega\Delta t)}{-\omega^2+\omega_k^2} \, \vartheta_k(x)\, \vartheta_k(x')$$

$$= \Theta(-\Delta t)\sum_k \frac{i}{2\omega_k}\exp(i\omega_k\Delta t)\; \vartheta_k(x)\,\vartheta_k(x')$$

$$+\,\Theta(\Delta t)\sum_k \frac{i}{2\omega_k}\exp(-i\omega_k\Delta t)\; \vartheta_k(x)\,\vartheta_k(x')$$



$$= \sum_k \frac{i}{2\omega_k}\exp(-i\omega_k|\Delta t|)\; \vartheta_k(x)\,\vartheta_k(x')$$

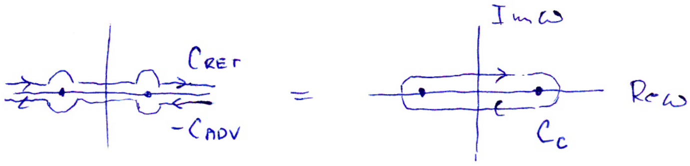
$$= i\,\frac{\exp(-i\Omega|\Delta t|)}{2\Omega}\cdot \hat{g}^{-1}$$

$$= i\,\frac{\cos(\Omega\Delta t)}{2\Omega}\cdot\tilde{g}^{-1} + \frac{\sin(\Omega|\Delta t|)}{2\Omega}\cdot\tilde{g}^{-1}$$

Homogenní Gr. fce

Kauzální Gr. fce

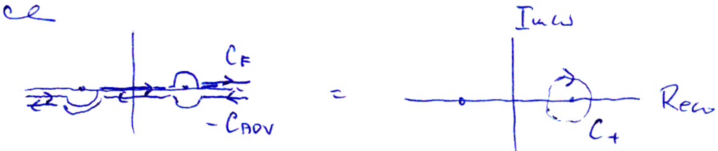
$G_c = G_{ret} - G_{adv}$



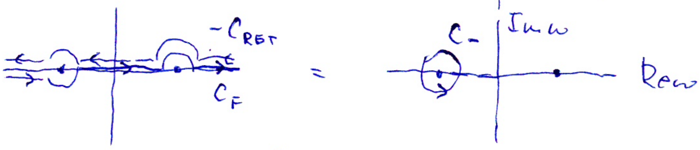
$$G_c = \frac{\sin(\Omega t)}{\Omega} \cdot \tilde{g}^{-1}$$

Wightmanovy Gr. fce

$iG^+ = iG^F - G_{adv}$



$iG^- = iG^F - G_{ret}$



$$G^+ = -i\left(i\,\frac{\exp(-i\Omega|\Delta t|)}{2\Omega} + \Theta(-\Delta t)\,\frac{\sin(\Omega \Delta t)}{\Omega} \right) \cdot \tilde{g}^{-1}$$

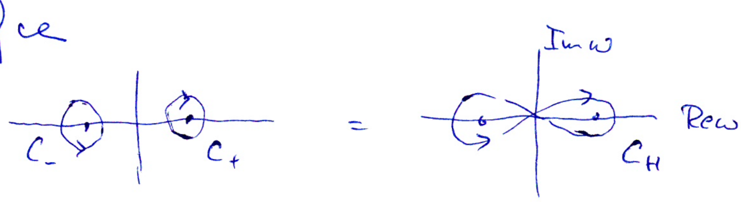
$$= \frac{1}{2\Omega}\Big(\Theta(\Delta t)\exp(-i\Omega\Delta t) + \Theta(-\Delta t)\exp(i\Omega\Delta t) - \Theta(-\Delta t)\exp(i\Omega\Delta t) + \Theta(-\Delta t)\exp(-i\Omega\Delta t)\Big) \cdot \tilde{g}^{-1}$$

$$= \frac{1}{2\Omega}\exp(-i\Omega\Delta t) \cdot \tilde{g}^{-1}$$

$$G^- = \frac{1}{2\Omega}\exp(i\Omega\Delta t) \cdot \tilde{g}^{-1}$$

Hadamardova Gr. fce

$iG^H = iG^+ + iG^-$



$$G^H = \frac{\cos(\Omega \Delta t)}{\Omega} \cdot \tilde{g}^{-1}$$

Analytické vlastnosti Gr. fcí ve stat. př.

ve stat. př. závisí pouze na rozdílu čas. souř.

$$G(t_1 x_1 | t_2 x_2) = G(\Delta t, x_1 | x_2) = G(\Delta t)^{x_1 x_2}$$

$$\Delta t = t_1 - t_2$$

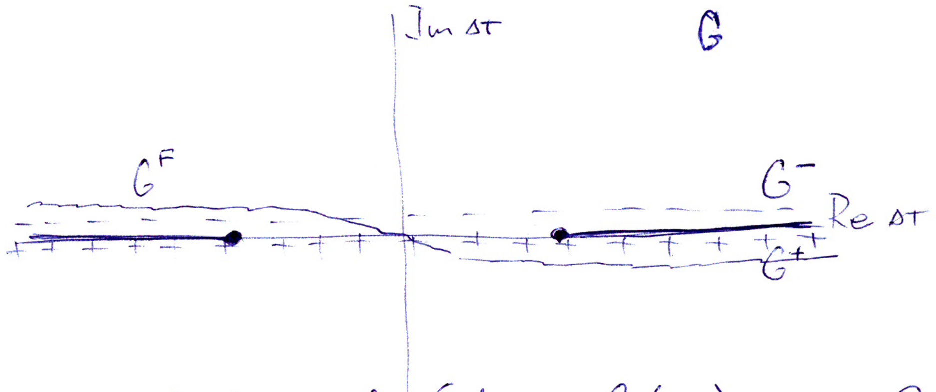
$$G^{\pm}(\Delta t) = \frac{\exp(\mp i \Omega \Delta t)}{2\Omega} \cdot \tilde{g}^{-1}$$

$$= \sum_k \frac{\exp(\mp i \omega_k \Delta t)}{2\omega_k} v_k v_k$$

$G^+(\Delta t)$ celá fce pro $\mathrm{Im}\, \Delta t < 0$

$G^-(\Delta t)$ celé fce pro $\mathrm{Im}\, \Delta t > 0$

$G^+(\Delta t) = G^-(\Delta t)$ pro $\Delta t < s(x_1 | x_2)$



⇓ existuje holomorfní fce $G(\Delta t)$ $\Delta t \in \mathbb{C}$ tak, že

$$G^+(\Delta t) = G(\Delta t - i0)$$

$$G^-(\Delta t) = G(\Delta t + i0)$$

které má póly a řezy na reálné ose

$$|\Delta t| \geq s(x_1 | x_2)$$

$$G^F(\Delta t) = G(\Delta t(1 - i0)) = \frac{\exp(-i\Omega |\Delta t|)}{2\Omega} \cdot \tilde{g}$$

$$= \Theta(-\Delta t)\, G^-(\Delta t) + \Theta(\Delta t)\, G^+(\Delta t)$$

Kauzální Gr. fce

$-iG_c = G^+ - G^-$

G_c charakterizuje skok na reálné ose ΔT

pouze póly ⟺ Huygensův princip

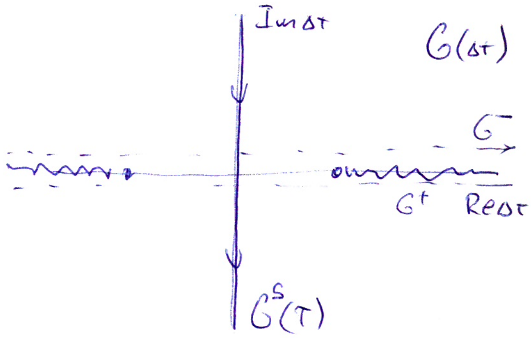
$-iG_{ret} = G^F - G^-$

$-iG_{adv} = G^F - G^+$

Schwingerova Gr. fc.

hodnota analyt. rozš. ~~Gr.~~ fc. na imaginární ose

$$G^S(\tau) = G(-i\tau) \qquad \tau \in \mathbb{R}$$



Víme

$$G(\Delta t) = \begin{cases} \dfrac{\exp(-i\Omega\Delta t)}{2\Omega}\cdot\tilde{q}^{-1} & \mathrm{Im}\,\Delta t < 0 \\[2ex] \dfrac{\exp(i\Omega\Delta t)}{2\Omega}\cdot\tilde{q}^{-1} & \mathrm{Im}\,\Delta t > 0 \end{cases}$$

⇓

$$G^S(\tau) = \begin{cases} \tau < 0 \quad \mathrm{Im}\,\Delta t > 0 & \dfrac{\exp(\Omega\tau)}{2\Omega}\cdot\tilde{q}^{-1} = \\[2ex] \tau > 0 \quad \mathrm{Im}\,\Delta t < 0 & \dfrac{\exp(-\Omega\tau)}{2\Omega}\cdot\tilde{q}^{-1} = \end{cases} \Bigg\} \; \frac{\exp(-\Omega|\tau|)}{2\Omega}\cdot\tilde{q}^{-1}$$

$\Delta t = -i\tau$

G^S řeší riemannovskou úlohu na varietě s metrikou $N^2 d\tau^2 + q$ $\quad \tau \in \mathbb{R}$, i.e.

$$\underbrace{\left[-\partial_\tau^2 + \Omega^2\right]}_{\mathcal{L}} \cdot G^S = \delta(x)\,\tilde{q}^{-1}$$

$\mathcal{L}$ – Laplace-like operator

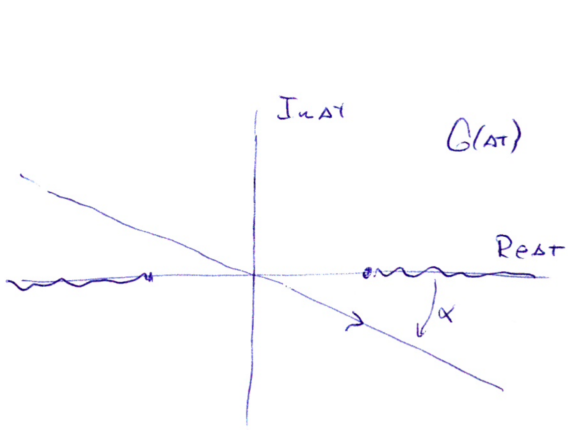
G^S je klesající pro $|\tau| \to \infty$

Wickova rotace

$$\Delta t = e^{-i\alpha}\tau \qquad \alpha \in (0, \tfrac{\pi}{2})$$

$\alpha = 0 \qquad G^F$

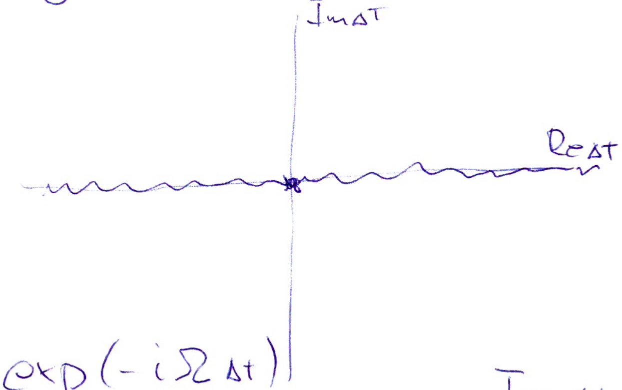
$\alpha = \frac{\pi}{2} \qquad G^S$



Harmonický oscilátor

degenerovaný případ – nemáme prostorupodobnost

$G(\Delta\tau)$

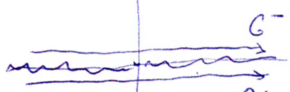


$$G(\Delta t) = \begin{cases} \dfrac{\exp(-i\Omega\Delta t)}{2\Omega} & \mathrm{Im}\,\Delta t < 0 \\ \dfrac{\exp(i\Omega\Delta t)}{2\Omega} & \mathrm{Im}\,\Delta t > 0 \end{cases}$$

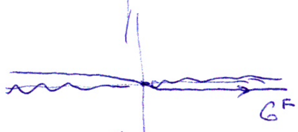
Ω obyčejné číslo

$G(\Delta t)$ fce pouze času

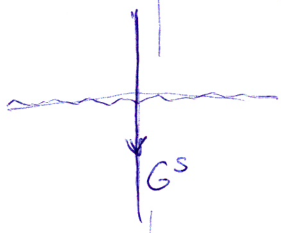
$$G^{\pm}(\Delta t) = \frac{\exp(\mp i\Omega\Delta t)}{2\Omega}$$



$$G^{F}(\Delta t) = \frac{\exp(-i\Omega|\Delta t|)}{2\Omega}$$

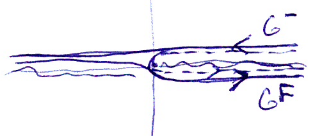


$$G^{S}(\Delta t) = \frac{\exp(-\Omega|\Delta t|)}{2\Omega}$$



$$-i\,G_{ret}(\Delta t) = \begin{cases} 0 & \Delta t < 0 \\ -i\dfrac{\sin(\Omega\Delta t)}{\Omega} & \Delta t > 0 \end{cases}$$

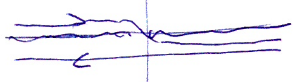
$G_F - G^-$



= skok v řezu pro $\Delta t > 0$

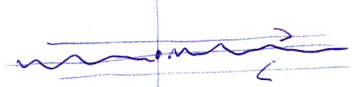
$$-i\,G_{adv}(\Delta t) = \begin{cases} +i\dfrac{\sin(\Omega\Delta t)}{\Omega} & \Delta t < 0 \\ 0 & \Delta t > 0 \end{cases}$$

$G^F - G^+$



= skok v řezu pro $\Delta t < 0$

$$-i\,G_C = G^+ - G^- = -i\frac{\sin(\Omega\Delta t)}{\Omega}$$



= skok v řezu

analytické struktura $G^{\pm}_{\beta}$

G^{+}_{β} holomorfní pro $q^{+}_{k}(z) = \exp(-i\omega_k z) + \exp(i\omega_k(z+i\beta))$

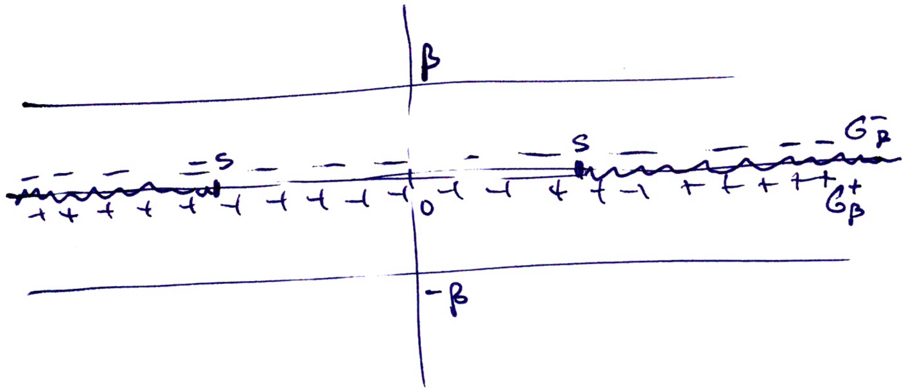
klesající exponenciálně s $\omega_k \to \infty$

tj. pro

$$-\beta < \operatorname{Im} z < 0$$

G^{-}_{β} holomorfní pro

$$0 < \operatorname{Im} z < \beta$$



platí

$$G^{+}(\Delta t) = G^{-}(\Delta t) \quad \text{pro} \quad \Delta t \in \mathbb{R} \quad |\Delta t| < s$$

$\Rightarrow$ $G^{\pm}$ analyt. prodl. jedné fce

$$G^{\pm}(\Delta t) = G(\Delta t \mp i0)$$

platí

$$q^{+}_{k}(z - i\beta) = q^{-}_{k}(z)$$

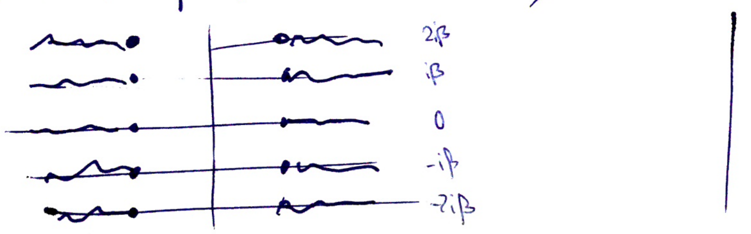
$q^{+}_{k}(z) = \exp(-i\omega_k z) + \exp(i\omega_k(z+i\beta))$

$q^{-}_{k}(z) = \exp(-i\omega_k(z-i\beta)) + \exp(i\omega_k z)$

$\Rightarrow$ periodicita

Termální analyt. prodl. Gr. fce je periodická

$$G_{\beta}(z + iN\beta) = G(z) \qquad N \in \mathbb{Z}$$



Wickova rotace

$$G^S_\beta(\tau) = G_\beta(-i\tau)$$

řeší eliptickou úlohu na riemannovské varietě s periodou β v euklidovském čase

Singulární struktura Gr. fcí

Hadamardův parametrix:

$$G(x|x') = \frac{u}{\sigma} + v \ln \sigma + w$$

$u(x|x')$ $v(x|x')$ $w(x,x')$ hladké pro blízké x, x'

$\sigma(x|x')$ kvadrát geodetické vzdálenosti

Ultrastatický prostoročas $N=1$

$\sigma(x|x') = -\tau^2 + s^2$ "Pythagorova věta"

$\tau = t - t'$ s prostorová geod. vzd.

u, v, w závisí pouze na τ, s

$$G(\tau) = \frac{u}{-\tau^2+s^2} + v \ln(-\tau^2+s^2) + w$$

↑ póly v $\tau = \pm s$ ↑ řezy pro $|\tau| > s$

Wightmant fce $\tau \equiv \Delta t$

$\frac{1}{x \pm i0} = P\frac{1}{x} \mp i\pi\delta(x)$

$\ln(x \pm i0) = \ln|x| \pm i\pi\Theta(-x)$

$$G^{\pm}(\Delta t) = G(\Delta t \mp i0)$$

$$= \frac{u}{2s}\left(\frac{+1}{\Delta t + s \mp i0} + \frac{-1}{\Delta t - s \mp i0}\right)$$

$$+ v\left(\ln(s + \Delta t \mp i0) + \ln(s - \Delta t \pm i0)\right)$$

$$+ w$$

$$= \frac{u}{2s}\left(P\frac{1}{\Delta t + s} \pm i\pi\delta(\Delta t + s) - P\frac{1}{\Delta t - s} \mp i\pi\delta(\Delta t - s)\right)$$

$$+ v\left(\ln|s + \Delta t| \mp i\pi\Theta(-\Delta t - s) + \ln|s - \Delta t| \pm i\pi\Theta(\Delta t - s)\right)$$

$$+ w$$

$$= \frac{u}{2s} P\frac{1}{-\Delta t^2 + s^2} \pm i\pi\frac{u}{2s}\left(\delta(\Delta t + s) - \delta(\Delta t - s)\right)$$

$$+ v\ln|-\Delta t^2 + s^2| \mp i\pi v\left(\Theta(-\Delta t - s) - \Theta(\Delta t - s)\right)$$

$$+ w$$

Hadamard

$$G^{H}(\Delta t) = \frac{u}{s} P\frac{1}{-\Delta t^2 + s^2} + 2v\ln|-\Delta t^2 + s^2| + 2w$$

Klasické gr. fc.

$$G_c = i G^+ - i G^- =$$

$$= 2\pi \frac{u}{2s} \left(\delta(\Delta t - s) - \delta(\Delta t + s) \right) - 2\pi v \left(\Theta(\Delta t - s) - \Theta(-\Delta t - s) \right)$$

$$G_{ret} = G_c \Theta(\Delta t)$$

$$= \frac{2\pi u}{2s} \delta(\Delta t - s) - 2\pi v \Theta(\Delta t - s)$$

$$G_{adv} = - G_c \Theta(-\Delta t) = G_{ret}(-\Delta t)$$

$$= \frac{2\pi u}{2s} \delta(-\Delta t - s) - 2\pi v \Theta(-\Delta t - s)$$

$$G_{avg} = 2\pi u \, \delta(-\Delta t^2 + s^2) - 2\pi v \, \Theta(\Delta t^2 - s^2)$$

$$= 2\pi u \, \delta(\sigma) - 2\pi v \, \Theta(-\sigma)$$

Př: plochý prostor

$$\tilde{F} = g^{\frac{1}{2}}\left[-\Box + m^2\right]$$

$$G = \frac{i m^2}{8\pi} H_1^{(2)}(z) \qquad z^2 = m^2(+\Delta t^2 - s^2)$$

$$= \frac{m^2}{4\pi^2}\left(-\frac{1}{z^2} J_0(z) + \ln z \,\frac{J_1(z)}{z} + \mathrm{anal}(z^2)\right)$$

kde

$$J_0(z) = \mathrm{anal}(z^2)$$

$$\frac{J_1(z)}{z} = \mathrm{anal}(z^2)$$

Besselovy fce

$$H_1^2$$

Hankelova fce

$$G = \frac{1}{4\pi^2}\left(\frac{1}{-\Delta t^2 + s^2} J_0(z) + \ln(-\Delta t^2 + s^2)\, m^2 \frac{J_1(z)}{2z} + \mathrm{anal}(z^2)\right)$$

$$u = \frac{1}{4\pi^2} J_0(z)$$

$$v = \frac{m^2}{4\pi^2}\frac{J_1(z)}{z}$$

$\Rightarrow$ gr. fc. G_c, G_{ret}, G_{adv}

$m = 0$ degenerovaný případ
formálně fungují

$$G = \frac{1}{4\pi^2}\frac{1}{-\Delta t^2 + s^2}$$

$$u = \frac{1}{4\pi^2}$$

$$v = 0$$

$$\Rightarrow G_{ret} = \frac{1}{4\pi}\frac{1}{s}\delta(\Delta t - s) \qquad G_{adv} = \frac{1}{4\pi}\frac{1}{s}\delta(-\Delta t - s)$$