In-out formalismus

experiment mezi dvěmi oblastmi s dobře definovaným pojmem částice

např. asymptoticky statické oblasti

in-oblast $\quad J_i \quad P_i^\pm \quad \phi_i^\pm \quad u_k \quad u_k^\pm \quad G_i^\pm \quad G_i^H$

$\hat{a}_i[\phi] \quad \hat{a}_{ik} = \hat{a}_i[u_k] \quad |i\,\text{vac}\rangle$

out-oblast $\quad J_f \quad P_f^\pm \quad \phi_f^\pm \quad v_k \quad v_k^\pm \quad G_f^\pm \quad G_f^H$

$\hat{a}_f[\phi] \quad \hat{a}_{fk} = \hat{a}_f[v_k] \quad |f\,\text{vac}\rangle$

in-out Green funkce

$$G^+ = \frac{\langle f\,\text{vac}|\hat{\Phi}\hat{\Phi}|i\,\text{vac}\rangle}{\langle f\,\text{vac}|i\,\text{vac}\rangle} \qquad G^- = G^{+T} \qquad G^H = G^+ + G^-$$

$$G^F = \frac{\langle f\,\text{vac}|T\hat{\Phi}\hat{\Phi}|i\,\text{vac}\rangle}{\langle f\,\text{vac}|i\,\text{vac}\rangle} = \frac{1}{2}G^H - i\,G_{sym} = G^\pm - i\,G_{adv/ret}$$

pozor: $\quad G^{+*} \neq G^- \quad G^{H*} \neq G^H \quad G^{F*} \neq G^F$

$$G^+ = P_f^+ \cdot G^+ = G^+ \cdot P_i^- \qquad G^+ = \frac{\langle f\,\text{vac}|\hat{\Phi}_f^+\hat{\Phi}_i^-|i\,\text{vac}\rangle}{\langle f\,\text{vac}|i\,\text{vac}\rangle}$$

$$G^- = P_i^- \cdot G^- = G^- \cdot P_f^+$$

obecně

$$|f\,\text{vac}\rangle \neq |i\,\text{vac}\rangle$$

$$\langle f\,\text{vac}|i\,\text{vac}\rangle \neq 1$$

uvidíme, že 1-částicové ampl. zmizí

$$\langle f\,\text{vac}|\hat{a}_i^\dagger[\phi]|i\,\text{vac}\rangle = 0$$

$$\langle f\,\text{vac}|\hat{a}_f[\phi]|i\,\text{vac}\rangle = 0$$

2-částicové amplitudy

$$\langle f\,vac | \hat{a}^{+}_{in}[\phi_1]\, \hat{a}^{+}_{in}[\phi_2] | in\,vac \rangle$$

$$= \langle f\,vac | \; i\, \phi^{+}_{1\,in} \cdot \partial f \cdot \hat{\Phi} (-i)\, \hat{\Phi} \cdot \partial f \cdot \phi^{+}_{2\,in} \; | in\,vac \rangle$$

$$= \phi^{+}_{1\,in} \cdot \partial f \cdot \langle f\,vac | \hat{\Phi}\, \hat{\Phi} | in\,vac \rangle \cdot \partial f \cdot \phi^{+}_{2\,in}$$

$$= \phi_1 \cdot \Lambda \cdot \phi_2 \quad \langle f\,vac | in\,vac \rangle$$

$$\Lambda = P^{+}_{in} \cdot \partial f \cdot G^{+} \cdot \partial f \cdot P^{+}_{in}$$

$$= \partial f \cdot P^{-}_{in} \cdot G^{+} \cdot P^{-}_{in} \cdot \partial f$$

$$\langle f\,vac | \hat{a}_{f}[\phi_1]\, \hat{a}_{f}[\phi_2] | in\,vac \rangle$$

$$= \phi_1 \cdot V \cdot \phi_2 \quad \langle f\,vac | in\,vac \rangle$$

$$V = P^{-}_{f} \cdot \partial f \cdot G^{+} \cdot \partial f \cdot P^{-}_{f} =$$

$$= \partial f \cdot P^{+}_{f} \cdot G^{+} \cdot P^{+}_{f} \cdot \partial f$$

$$\langle f\,vac | \hat{a}_{f}[\phi_1]\, \hat{a}^{+}_{in}[\phi_2] | in\,vac \rangle$$

$$= \phi_1 \cdot I \cdot \phi_2 \quad \langle f\,vac | in\,vac \rangle$$

$$I = - P^{-}_{f} \cdot \partial f \cdot G^{+} \cdot \partial f \cdot P^{+}_{in} =$$

$$= - \partial f \cdot P^{+}_{f} \cdot G^{+} \cdot P^{-}_{in} \cdot \partial f =$$

$$= - \partial f \cdot G^{+} \cdot \partial f$$

Bogoljubovova transformace

in-módy $\quad u_k \quad u_k^+ \equiv u_{ki}^+ \quad u_k^- = u_{ki}^-$

$|i\,vac\rangle \quad \hat{a}_{ik} \equiv \hat{a}_i[u_k] \quad \hat{a}^+_{ik} = \hat{a}^+_i[u_k]$

out-módy $\quad V_k \quad V_k^+ \equiv V_{kf}^+ \quad V_k^- = V_{kf}^-$

$|f\,vac\rangle \quad \hat{a}_{fk} \equiv \hat{a}_f[V_k] \quad \hat{a}^+_{fk} = \hat{a}^+_f[V_k]$

$$V_k^+ = \sum_\ell \left(U_\ell^+ \alpha_{\ell k} + U_\ell^- \beta_{\ell k} \right)$$

$$V_k^- = \sum_\ell \left(U_\ell^- \alpha^*_{\ell k} + U_\ell^+ \beta^*_{\ell k} \right)$$

$$\delta_{k\ell} = -i\, V_k^- \cdot \partial f \cdot V_\ell^+ = \sum_m \left(\alpha^*_{mk} \alpha_{m\ell} - \beta^*_{mk} \beta_{m\ell} \right)$$

$$0 = -i\, V_k^+ \cdot \partial f \cdot V_\ell^+ = \sum_m \left(\beta_{mk} \alpha_{m\ell} - \alpha_{mk} \beta_{m\ell} \right)$$

$$U_k^+ = \sum_\ell \left(V_\ell^+ \alpha^*_{k\ell} - V_\ell^- \beta_{k\ell} \right)$$

$$U_k^- = \sum_\ell \left(V_\ell^- \alpha_{k\ell} - V_\ell^+ \beta^*_{k\ell} \right)$$

$$\alpha_{k\ell} = -i\, U_k^- \cdot \partial f \cdot V_\ell^+ = (-i\, V_\ell^- \cdot \partial f \cdot U_k^+)^* \overset{inv}{=} \alpha^*_{\ell k}$$

$$\beta_{k\ell} = i\, U_k^+ \cdot \partial f \cdot V_\ell^+ = -i\, V_\ell^+ \cdot \partial f \cdot U_k^+ \overset{inv}{=} -\beta_{\ell k}$$

$$\hat{\Phi} = \sum_k \left(U_k^- \hat{a}^+_{ik} + U_k^+ \hat{a}_{ik} \right) = \sum_\ell \left(V_\ell^- \hat{a}^+_{f\ell} + V_\ell^+ \hat{a}_{f\ell} \right)$$

$$= \sum_{k,\ell} \left(U_k^- \left(\alpha^*_{k\ell} \hat{a}^+_{f\ell} + \beta_{k\ell} \hat{a}_{f\ell} \right) + U_k^+ \left(\alpha_{k\ell} \hat{a}_{f\ell} + \beta^*_{k\ell} \hat{a}^+_{f\ell} \right) \right)$$

$\Downarrow$

$$\hat{a}^+_{ik} = \sum_\ell \left(\alpha^*_{k\ell} \hat{a}^+_{f\ell} + \beta_{k\ell} \hat{a}_{f\ell} \right) \qquad / \alpha_{km} \; +$$

$$\hat{a}_{ik} = \sum_\ell \left(\alpha_{k\ell} \hat{a}_{f\ell} + \beta^*_{k\ell} \hat{a}^+_{f\ell} \right) \qquad / -\beta_{km}$$

$\Downarrow$

$$\hat{a}^+_{f\ell} = \sum_m \left(\hat{a}^+_{im} \alpha_{m\ell} - \hat{a}_{im} \beta_{m\ell} \right)$$

$$\hat{a}_{f\ell} = \sum_m \left(\hat{a}_{im} \alpha^*_{m\ell} - \hat{a}^+_{im} \beta^*_{m\ell} \right)$$

$\alpha_{k\ell}, \beta_{k\ell}$ Bogoljubovovy koeficienty

$|f\,vac\rangle \neq |i\,vac\rangle \iff \beta_{k\ell}$ nemizí

$$\hat{a}_{fk} |i\,vac\rangle = \sum_\ell \left(\hat{a}_{i\ell} \alpha^*_{\ell k} - \hat{a}^+_{i\ell} \beta^*_{\ell k} \right) |i\,vac\rangle = -\sum_\ell \hat{a}^+_{i\ell} \beta^*_{\ell k} |i\,vac\rangle$$

$= 0$ pouze pro $\beta_{k\ell} = 0$

$$\hat{a}_{ik} = \sum_f \left(\alpha_{kf}\, \hat{a}_{ff} + \beta^*_{kf}\, \hat{a}^+_{ff}\right)$$

⇓

$$\hat{a}_{fk} = \sum_f \left(\alpha^{-1}_{kf}\, \hat{a}_{if} - \sum_m \alpha^{-1}_{km}\, \beta^*_{mf}\, \hat{a}^+_{ff}\right)$$

$$= \sum_\ell \left(\alpha^{-1}_{k\ell}\, \hat{a}_{i\ell} + V_{k\ell}\, \hat{a}^+_{f\ell}\right)$$

$$\sum_m \alpha^{-1}_{km}\alpha_{m\ell} = \delta_{k\ell} \qquad V_{k\ell} = -\sum_m \alpha^{-1}_{km}\beta^*_{m\ell} \qquad V_{k\ell} = V_{\ell k}$$

$\Uparrow \alpha\cdot\beta = \beta\cdot\alpha$

1-částicové ampl. (kreace 1 částice)

$$\langle f\ \mathrm{vac}|\, \hat{a}_{fk}\, |i\ \mathrm{vac}\rangle = \sum_\ell \langle f\ \mathrm{vac}|\, \alpha^{-1}_{k\ell}\hat{a}_{i\ell} + V_{k\ell}\hat{a}^+_{f\ell}\, |i\ \mathrm{vac}\rangle = 0$$

obdobně anihilace 1. částice

$$\langle f\ \mathrm{vac}|\, \hat{a}^+_{ik}\, |i\ \mathrm{vac}\rangle = 0$$

2-částicové ampl.

kreace 2 částic — symetrické v k,l

$$\langle f\ \mathrm{vac}|\, \hat{a}_{fk}\hat{a}_{f\ell}\, |i\ \mathrm{vac}\rangle = \sum_m \langle f\ \mathrm{vac}|\, \hat{a}_{fk}\left(\alpha^{-1}_{\ell m}\hat{a}_{im} + V_{\ell m}\hat{a}^+_{fm}\right)|i\ \mathrm{vac}\rangle$$

← plyne symetrie $V_{k\ell}$

$$= \sum_m V_{\ell m}\langle f\ \mathrm{vac}|\, \hat{a}_{fk}\hat{a}^+_{fm}\, |i\ \mathrm{vac}\rangle = V_{k\ell}\langle f\ \mathrm{vac}|i\ \mathrm{vac}\rangle$$

$$V_{\ell k} = V_{k\ell} = -\sum_m \alpha^{-1}_{km}\beta^*_{m\ell} \qquad V_{k\ell} = v_k\cdot V\cdot v_\ell$$

anihilace 2 částic

$$\langle f\ \mathrm{vac}|\, \hat{a}^+_{ik}\hat{a}^+_{i\ell}\, |i\ \mathrm{vac}\rangle = \Lambda_{k\ell}\langle f\ \mathrm{vac}|i\ \mathrm{vac}\rangle$$

$$\Lambda_{k\ell} = \Lambda_{\ell k} = \sum_m \beta_{km}\alpha^{-1}_{m\ell} \qquad \Lambda_{k\ell} = u_k\cdot\Lambda\cdot u_\ell$$

1-1 částicová propagace

$$\langle f\ \mathrm{vac}|\, \hat{a}_{fk}\hat{a}^+_{i\ell}\, |i\ \mathrm{vac}\rangle = \sum_m \langle f\ \mathrm{vac}|\left(\alpha^{-1}_{km}\hat{a}_{im} + V_{km}\hat{a}^+_{fm}\right)\hat{a}^+_{i\ell}\, |i\ \mathrm{vac}\rangle$$

$$= \sum_m \alpha^{-1}_{km}\langle f\ \mathrm{vac}|\, \hat{a}_{im}\hat{a}^+_{i\ell}\, |i\ \mathrm{vac}\rangle = \alpha^{-1}_{k\ell}\langle f\ \mathrm{vac}|i\ \mathrm{vac}\rangle$$

$$= I_{k\ell}\langle f\ \mathrm{vac}|i\ \mathrm{vac}\rangle \qquad I_{k\ell} = \alpha^{-1}_{k\ell} \qquad I_{k\ell} = v_k\cdot I\cdot u_\ell$$

Vyjádření Λ, V, I pomocí $\Lambda_{kl}, V_{kl}, I_{kl}$

níže:

$$P_i^{\pm} = \mp i \sum_k u_k^{\pm} u_k^{\mp} \cdot \partial F$$

$$P_f^{\pm} = \mp i \sum_k v_k^{\pm} v_k^{\mp} \cdot \partial F$$

$$\Lambda = P_i^{+} \cdot \Lambda \cdot P_i^{+} = \sum_{k,l} \partial F \cdot u_k^{-} u_k^{+} \cdot \Lambda \cdot u_l^{+} u_l^{-} \cdot \partial F =$$

$$= \partial F \cdot \left(\sum_{kl} u_k^{-} \Lambda_{kl} u_l^{-} \right) \cdot \partial F$$

$$V = P_F^{-} \cdot V \cdot P_F^{-} = \sum_{kl} \partial F \, v_k^{+} v_k^{-} \cdot V \cdot v_l^{-} v_l^{+} \cdot \partial F =$$

$$= \partial F \cdot \left(\sum_{kl} v_k^{+} V_{kl} v_l^{+} \right) \cdot \partial F$$

$$I = P_f^{-} \cdot I \cdot P_i^{+} = -\sum_{kl} \partial F \, v_k^{+} v_k^{-} \cdot I \cdot u_l^{+} u_l^{-} \cdot \partial F$$

$$= -\partial F \cdot \left(\sum_{kl} v_k^{+} I_{kl} u_l^{-} \right) \cdot \partial F$$

Střední počty částic

$$\langle in\,vac|\,\hat{n}_f[v_k]\,|in\,vac\rangle =$$

$$= \langle in\,vac|\,\hat{a}^{+}_{fk}\,\hat{a}_{fk}\,|in\,vac\rangle$$

$$= \sum_{m,n} \langle in\,vac|(-\beta_{mk})\,\hat{a}_{in}\,\hat{a}^{+}_{in}\,(-\beta^{*}_{nk})\,|in\,vac\rangle$$

$$= \sum_{n} |\beta_{nk}|^2$$

$$\langle in\,vac|\,\hat{N}_f\,|in\,vac\rangle$$

$$= \sum_{m} \langle in\,vac|\,\hat{n}_{fm}[\phi]\,|in\,vac\rangle$$

$$= \sum_{m,n} |\beta_{m,n}|^2$$

J-linearita

$$L\cdot J = J\cdot L$$

$$P^{\pm}\cdot L = L\cdot P^{\pm}$$

hermit. sdruž.

$$\langle L^{\langle +\rangle}\cdot\phi_1,\phi_2\rangle = \langle\phi_1, L\cdot\phi_2\rangle$$

$$L^{\langle +\rangle} = -G_c\cdot L\cdot\partial\hat{f}$$

J-antilineární

$$A\cdot J = -J\cdot A$$

$$P^{\pm}\cdot A = A\cdot P^{\mp}$$

transpozice

$$\langle A^{\langle T\rangle}\cdot\phi_1,\phi_2\rangle^{*} = \langle\phi_1, A\cdot\phi_2\rangle$$

$$A^{\langle T\rangle} = G_c\cdot A\cdot\partial\hat{f}$$

$$a(\phi_1,\phi_2) = \phi_1\cdot a\cdot\phi_2 = \langle A\cdot\phi_1,\phi_2\rangle$$

$$a^{T}(\phi_1\phi_2) = \phi_1\cdot a^{T}\cdot\phi_2 = \langle A^{\langle T\rangle}\cdot\phi_1,\phi_2\rangle$$

$$\phi_1\cdot a\cdot\phi_2 = \phi_2\cdot a^{T}\cdot\phi_1$$

Operátor přechodu

zadání i a f módů definuje identifikaci
J_i a J_f struktury

operátor přechodu (transition operator)

$$V_k = S \cdot u_k$$

převádí číslování módů v i-experimentu na analogické v f-experimentu

isomorfismus z J_i na J_f strukturu

$$\langle S\cdot\phi_1 , S\cdot\phi_2\rangle_f = \langle\phi_1 , \phi_2\rangle_i$$

$$J_f \cdot S = S \cdot J_i$$

$$S \cdot \partial F \cdot S = \partial F$$

plyne z ortonorm. bazi u_k a v_k

obecně nelze volit $S = \delta$ protože módy definující J_f a J_i musejí být odlišné

Bogoljubovovy operátory

$$s = \alpha + \beta \qquad s = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$$

$\alpha \cdot J_i = J_i \cdot \alpha$ (i-lineární) $\qquad \beta \cdot J_i = - J_i \cdot \beta$ (i-antilineární)

$$\alpha = \frac{1}{2}(s - J_i \cdot s \cdot J_i) = P_i^+ \cdot s \cdot P_i^+ + P_i^- \cdot s \cdot P_i^-$$

$$\beta = \frac{1}{2}(s + J_i \cdot s \cdot J_i) = P_i^+ \cdot s \cdot P_i^- + P_i^- \cdot s \cdot P_i^+$$

platí

$$P_f^+ \cdot s = s \cdot P_i^+ = (\alpha + \beta) \cdot P_i^+ =$$

$$= P_i^+ \cdot \alpha + P_i^- \cdot \beta$$

$$P_f^- \cdot s = P_i^- \cdot \alpha + P_i^+ \cdot \beta$$

$\Downarrow$

$$\hat{a}_f[s \cdot \phi] = \hat{a}_i[\alpha \cdot \phi] - \hat{a}_i^+[\beta \cdot \phi]$$

$$-i(s\cdot\phi)_f^+ \cdot \partial f \cdot \hat{\Phi} = -i(\alpha\cdot\phi)_i^- \cdot \partial f \cdot \hat{\Phi} - i(\beta\cdot\phi)_i^+ \cdot \partial f \cdot \hat{\Phi}$$

$$\hat{a}_f^+[s \cdot \phi] = \hat{a}_i^+[\alpha \cdot \phi] - \hat{a}_i[\beta \cdot \phi]$$

α, β srovnávají in a out módy

$$V_k^+ = s \cdot u_k^+ \qquad \Leftarrow P_f^+ \cdot V_k = P_f^+ \cdot s \cdot u_k = s \cdot P_i^+ \cdot u_k$$

$$V_k^+ = (\alpha \cdot u_k)_i^+ + (\beta \cdot u_k)_i^-$$

$$= \sum_l \left(u_l^+ \alpha_{lk} + u_l^- \beta_{lk} \right)$$

kde $(\alpha \cdot u_k)_i^+ = \sum_l u_l^+ \alpha_{lk}$

$$\alpha_{lk} = \langle u_l, \alpha \cdot u_k \rangle_i = -i\, u_l^- \cdot \partial f \cdot V_k^+$$

$$\beta_{lk} = \langle \beta \cdot u_k, u_l \rangle_i = i\, u_l^+ \cdot \partial f \cdot V_k^+$$

$$\beta_{lk}^* = \langle u_l, \beta \cdot u_k \rangle_i = -i\, u_l^- \cdot \partial f \cdot V_k^-$$

$$= \langle u_k, \beta^{(T)} \cdot u_l \rangle_i$$

Bogoljubovovy koeficienty

Vlastnosti Bogoljub. operátorů

inverzní operátor přechodu

$$S^{-1} = \alpha^{\langle+\rangle} - \beta^{\langle T\rangle}$$

Důk: $\partial\mathcal{F} = S\cdot\partial\mathcal{F}\cdot S \quad\Rightarrow\quad S^{-1} = -G_c\cdot S\cdot\partial\mathcal{F}$

$$S^{-1} = -G_c\cdot S\cdot\partial\mathcal{F} = -G_c\cdot\alpha\cdot\partial\mathcal{F} - G_c\cdot\beta\cdot\partial\mathcal{F} = \alpha^{\langle+\rangle} - \beta^{\langle T\rangle}$$

inverzní Bogoljub. oper

$\alpha^{\langle+\rangle}$ a $-\beta^{\langle T\rangle}$ hrají roli Bogoljub. oper. pro přechod $\mathcal{I}_f\to\mathcal{I}_i$

netriviální, že jsou $\mathcal{I}_f$-lin a $\mathcal{I}_f$-antilin.

podmínky na Bogoljub. oper.

$S\cdot S^{-1} = \delta \Downarrow$ $\qquad\qquad$ $S^{-1}\cdot S = \delta \Downarrow$

$\alpha\cdot\alpha^{\langle+\rangle} - \beta\cdot\beta^{\langle T\rangle} = \delta$ $\qquad\qquad$ $\alpha^{\langle+\rangle}\cdot\alpha - \beta^{\langle T\rangle}\cdot\beta = \delta$ ← lin část

$\beta\cdot\alpha^{\langle+\rangle} = \alpha\cdot\beta^{\langle T\rangle}$ $\qquad\qquad$ $\alpha^{\langle+\rangle}\cdot\beta = \beta^{\langle T\rangle}\cdot\alpha$ ← alin část

díky $\alpha\alpha^{\langle+\rangle} = \delta + \beta\beta^{\langle T\rangle}$ má vždy α inverzi $\Rightarrow$

$$\alpha^{-1\langle+\rangle} = \alpha - \beta\cdot\alpha^{-1}\cdot\beta$$

$\alpha^{-1}\cdot\beta = \beta^{\langle T\rangle}\cdot\alpha^{-1\langle+\rangle}$ $\qquad\qquad$ $\beta\cdot\alpha^{-1} = \alpha^{-1\langle+\rangle}\cdot\beta^{\langle T\rangle}$

$(\alpha^{-1}\cdot\beta)^{\langle T\rangle} = \alpha^{-1}\cdot\beta$ $\qquad\qquad$ $(\beta\cdot\alpha^{-1})^{\langle T\rangle} = \beta\cdot\alpha^{-1}$

vztahy mezi α, β, S

$$S\cdot\alpha^{-1} = \delta + \beta\cdot\alpha^{-1}$$

$$\alpha^{-1\langle+\rangle}S^{-1} = \delta - \beta\cdot\alpha^{-1}$$

$$S\cdot\alpha^{-1}\cdot\beta\cdot S^{-1} = \beta\cdot\alpha^{-1}$$

Důk:

$$S\cdot\alpha^{-1} = (\alpha+\beta)\cdot\alpha^{-1} = \delta + \beta\cdot\alpha^{-1}$$

$$\alpha^{-1\langle+\rangle}\cdot S^{-1} = \alpha^{\langle+\rangle-1}(\alpha^{\langle+\rangle} - \beta^{\langle T\rangle}) = \delta - \alpha^{-1\langle+\rangle}\beta^{\langle T\rangle} = \delta - (\beta\cdot\alpha^{-1})^{\langle T\rangle} = \delta - \beta\cdot\alpha^{-1}$$

$$S\cdot\alpha^{-1}\cdot\beta\cdot S^{-1} = (\delta + \beta\cdot\alpha^{-1})\cdot\beta\cdot(\alpha^{\langle+\rangle} - \beta^{\langle T\rangle}) =$$

$$= \beta\alpha^{\langle+\rangle} - \beta\beta^{\langle T\rangle} + (\alpha - \alpha^{-1\langle+\rangle})(\alpha^{\langle+\rangle} - \beta^{\langle T\rangle}) =$$

$$= (\alpha\alpha^{\langle+\rangle} - \beta\beta^{\langle T\rangle} - \delta) + (\beta\alpha^{\langle+\rangle} - \alpha\beta^{\langle T\rangle}) + \alpha^{-1\langle+\rangle}\beta^{\langle T\rangle}$$

$$= (\beta\alpha^{-1})^{\langle T\rangle} = \beta\alpha^{-1}$$

Různé operátory přechodu

S je isomorf. z J_i struktury do J_f struktury

není jednoznačné
  může se lišit isomorf. J_i struktury,
  případně isomorf. J_f struktury

$$S = S_0 \cdot U_i = U_f \cdot S_0$$

S, S_0 — operátory přechodu
U_i — J_i-unitární oper. J_i-lin.
U_f — J_f-unitární oper. J_f-lin.

U_i, U_f "pouze" mění bázi beze změny J_i a J_f

závislost Bogoljub. oper.
  na volbě U_i resp. U_f

$$S = \alpha + \beta \qquad S_0 = \alpha_0 + \beta_0$$

$$\alpha = \alpha_0 \cdot U_i \qquad \Leftarrow \quad \tfrac{1}{2}(S - J_i \cdot S \cdot J_i) = \tfrac{1}{2}(S_0 - J_i \cdot S_0 \cdot J_i) \cdot U_i$$

$$\beta = \beta_0 \cdot U_i \qquad \Leftarrow \quad \text{podobně}$$

některé kombinace S, α, β nezávisí na U_i, U_f

$$\beta \cdot \alpha^{-1} = \beta_0 \cdot \alpha_0^{-1} \qquad \beta_0 \cdot U_i \cdot U_i^{-1} \cdot \alpha_0^{-1}$$

$$S \cdot \alpha^{-1} = S_0 \cdot \alpha_0^{-1} \qquad S_0 \cdot U_i \cdot U_i^{-1} \cdot \alpha_0^{-1}$$

$$\beta \cdot S^{-1} = \beta_0 \cdot S^{-1} \qquad \beta_0 \cdot U_i\, U_i^{-1} \cdot S_0^{-1}$$

$$\alpha \cdot \alpha^{\langle + \rangle} = \alpha_0 \cdot \alpha_0^{\langle + \rangle} \qquad \alpha_0 \cdot U_i \cdot U_i^{\langle + \rangle} \cdot \alpha_0^{\langle + \rangle}$$

$$\beta \cdot \beta^{\langle T \rangle} = \beta_0 \cdot \beta_0^{\langle T \rangle} \qquad \beta_0 \cdot U_i \cdot U_i^{\langle + \rangle} \cdot \beta_0^{\langle T \rangle}$$

$$S \cdot \alpha^{-1} \cdot \beta \cdot S^{-1} = \beta \cdot \alpha^{-1} \qquad \Leftarrow \quad S \cdot \alpha^{-1} \cdot \beta \cdot S^{-1} = (\delta + \beta\alpha^{-1})\beta(\alpha^{\langle + \rangle} - \beta^{\langle T \rangle}) =$$

$$= \beta\alpha^{\langle + \rangle} - \beta\beta^{\langle T \rangle} + (\alpha - \alpha^{-1\langle + \rangle})(\alpha^{\langle + \rangle} - \beta^{\langle T \rangle})$$

$$= \alpha\alpha^{\langle + \rangle} - \beta\beta^{\langle T \rangle} - \delta + \beta\alpha^{\langle + \rangle} - \alpha\beta^{\langle T \rangle} + \alpha^{-1\langle + \rangle}\beta^{\langle T \rangle}$$

$$= (\beta\alpha^{-1})^{\langle T \rangle} = \beta\alpha^{-1}$$

$$\alpha^{-1\langle + \rangle} S^{-1} = \delta - \beta \cdot \alpha^{-1} \qquad \alpha^{-1\langle + \rangle} S^{-1} = \delta - \alpha^{-1\langle + \rangle}\beta^{\langle T \rangle} = \delta - (\beta\alpha^{-1})^{\langle T \rangle}$$

$$= \delta - \beta\alpha^{-1}$$

viz výše

Kanonický Bogoljubův operátor

z vlastností $\alpha, \beta, \alpha^{\langle + \rangle}, \beta^{\langle T \rangle}$ dostaneme

$$[\alpha \cdot \alpha^{\langle + \rangle}, \beta \beta^{\langle T \rangle}] = 0 \qquad \Leftarrow \alpha\alpha^{\langle + \rangle} = \delta + \beta\beta^{\langle T \rangle}$$

$$[\alpha \cdot \alpha^{\langle + \rangle}, \beta \alpha^{-1}] = 0 \qquad \Leftarrow \alpha\alpha^{\langle + \rangle}\beta\alpha^{-1} - \beta\alpha^{-1}\alpha\alpha^{\langle + \rangle} = \alpha\beta^{\langle T \rangle}\alpha\alpha^{-1} - \beta\alpha^{\langle + \rangle} = 0$$

$$[\beta \cdot \beta^{\langle T \rangle}, \beta\alpha^{-1}] = 0 \qquad \Leftarrow \beta\beta^{\langle T \rangle}\beta\alpha^{-1} - \beta\alpha^{-1}\beta\beta^{\langle T \rangle} = \beta\beta^{\langle T \rangle}\alpha^{\langle + \rangle -1}\beta^{\langle T \rangle} - \beta\beta^{\langle T \rangle}\alpha^{\langle + \rangle -1}\beta^{\langle T \rangle} = 0$$

a vazby

$$\alpha \cdot \alpha^{\langle + \rangle} - \beta \cdot \beta^{\langle T \rangle} = \delta$$

$$\beta \cdot \beta^{\langle T \rangle} \cdot (\alpha \cdot \alpha^{\langle + \rangle})^{-1} = (\beta \cdot \alpha^{-1})^2 \qquad \Leftarrow \beta\beta^{\langle T \rangle}\alpha^{\langle + \rangle -1}\alpha^{-1} = \beta\alpha^{-1}\beta\alpha^{-1}$$

lze psát $\alpha\alpha^{\langle + \rangle}, \beta\beta^{\langle T \rangle}, \beta\alpha^{-1}$ jako fce jednoho oper.
vazby lze vyřešit volbou

$$\alpha \cdot \alpha^{\langle + \rangle} = ch^2 \mathcal{X} \qquad s \cdot \alpha^{-1} = \delta + th\, \mathcal{X}$$

$$\beta \cdot \beta^{\langle T \rangle} = sh^2 \mathcal{X} \qquad s \cdot \alpha^{-1} \cdot \beta \cdot s^{-1} = th\, \mathcal{X}$$

$$\beta \cdot \alpha^{-1} = th\, \mathcal{X} \qquad \alpha^{-1\langle + \rangle} s^{-1} = \delta - th\, \mathcal{X}$$

kde operátor Bogoljubův operátor $\mathcal{X}$ splňuje

$$\mathcal{X} \cdot J_i = - J_i \cdot \mathcal{X} \qquad J_i \text{ - antilin} \qquad \Leftarrow \beta \cdot \alpha^{-1} = th\, \mathcal{X}$$

$$\mathcal{X}^{\langle T \rangle} = \mathcal{X} \qquad J_i \text{ symetrie}$$

lze též ukázat J_f-antilin a symetrii
netriviální $\Leftarrow$ z obecné konstrukce $\mathcal{X}$

operátor přechodu s_0 lze zvolit tak, aby

$$\alpha_0^{\langle + \rangle} = \alpha_0 \qquad \beta_0^{\langle T \rangle} = \beta_0$$

tzv. kanonická volba

$$\Downarrow \quad \alpha_0^2 = ch^2 \mathcal{X} \qquad \beta_0^2 = sh^2 \mathcal{X}$$

$$\Downarrow \quad \alpha_0 = ch\, \mathcal{X} \qquad \beta_0 = sh\, \mathcal{X} \qquad \beta \cdot \alpha^{-1} = th\, \mathcal{X}$$

$$s_0 = \alpha_0 + \beta_0 = \exp \mathcal{X} \qquad s_0^{-1} = \alpha_0 - \beta_0 = \exp(-\mathcal{X})$$

obecný operátor přechodu s

$$s = \exp \mathcal{X}\; U_i \qquad \alpha = ch\, \mathcal{X}\; U_i$$

$$\beta = sh\, \mathcal{X}\; U_i$$

Kanonická báze

$$X \circ u_k = x_k u_k$$

$$\alpha_{kl} = \langle u_k, \mathrm{ch}\, X \circ u_l \rangle_i = \mathrm{ch}\, x_1 \delta_{kl}$$

$$\beta_{kl} = \mathrm{sh}\, x_k \delta_{kl}$$

$$V_k = (\exp X_k) u_k$$

$$V_{kf}^{+} = u_{ki}^{+} \mathrm{ch}\, x_k + u_{ki}^{-} \mathrm{sh}\, x_k$$

$$\hat{a}_{fk}^{+} = \hat{a}_{ik}^{+} \mathrm{ch}\, x_k - \hat{a}_{ik} \mathrm{sh}\, x_k$$

$$\alpha_{kl}^{-1} = \frac{1}{\mathrm{ch}\, x_k} \delta_{kl}$$

$$V_{kl} = -\sum_m \alpha_{km}^{-1} \beta_{ml}^{*} = -\frac{\mathrm{sh}\, x_k}{\mathrm{ch}\, x_k} \delta_{kl} = -\mathrm{th}\, x_k \delta_{kl}$$

$$\Lambda_{kl} = \sum_n \beta_{kn} \alpha_{nl}^{-1} = \mathrm{th}\, x_k \delta_{kl}$$

$$I_{kl} = \alpha_{kl}^{-1} = \frac{1}{\mathrm{ch}\, x_k} \delta_{kl}$$

$$\frac{\langle f\, vac | \hat{a}_{fk} \hat{a}_{fl} | i\, vac \rangle}{\langle f\, vac | i\, vac \rangle} = V_{kl} = -\mathrm{th}\, x_k \delta_{kl}$$

$$\langle i\, vac | \hat{n}_f [V_k] | i\, vac \rangle = \sum_m \mathrm{sh}^2 x_k \delta_{mk} \delta_{mk} = \mathrm{sh}^2 x_k$$

$$\langle i\, vac | \hat{N}_f | i\, vac \rangle = \sum_k \mathrm{sh}^2 x_k$$

2-částicové amplitudy obecných módů pomocí α, β, X

vztah kr/anih. operátorů

$$\hat a_f[s\cdot\phi] = \hat a_i[\alpha\cdot\phi] - \hat a_i[\beta\cdot\phi]^\dagger \qquad \Downarrow\ \phi \to \alpha^{-1}\cdot\phi \qquad \dagger$$

$$\hat a_i[\phi]^\dagger = \hat a_f[s\cdot\alpha^{-1}\cdot\phi]^\dagger + \hat a_i[\beta\cdot\alpha^{-1}\cdot\phi]$$

vztah Bogoljub. oper. a koef.

$$\alpha_{kl} = \langle u_k, \alpha\cdot u_l\rangle_i \qquad \alpha^*_{kl} = \langle u_l, \alpha^{(+)}\cdot u_k\rangle_i$$

$$\beta_{kl} = \langle \beta\cdot u_l ; u_k\rangle_i \qquad \beta^*_{kl} = \langle u_k, \beta\cdot u_l\rangle_i$$

2 částicové amplitudy

Λ: $$\phi_1\cdot\Lambda\cdot\phi_2 = \frac{\langle f\,vac|\hat a_i[\phi_1]^\dagger \hat a_i[\phi_2]^\dagger|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle} = \frac{\langle f\,vac|\hat a_i[\beta\cdot\alpha^{-1}\cdot\phi_1]\,\hat a_i[\phi_2]^\dagger|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle}$$

$$= \langle \beta\cdot\alpha^{-1}\cdot\phi_1, \phi_2\rangle_i = \langle \operatorname{th} X\cdot\phi_1, \phi_2\rangle_i$$

$$u_k\cdot\Lambda\cdot u_l = \Lambda_{kl} = \sum_m \beta_{km}\,\alpha^{-1}_{ml} \qquad \Lambda = \partial F\cdot\Big(\sum_{kl} u^-_k\,\Lambda_{kl}\,u^-_l\Big)\cdot\partial F$$

$\phi_1\cdot\Lambda\cdot\phi_2 = \phi_2\cdot\Lambda\cdot\phi_1$ bi-J_i-lineární sym. forma citlivé na $\phi^+_{1,2\,i}$ $\Lambda_{kl} = \Lambda_{lk}$

V: $$\phi_1\cdot V\cdot\phi_2 = \frac{\langle f\,vac|\hat a_f[\phi_1]\,\hat a_f[\phi_2]|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle} = \frac{\langle f\,vac|(\hat a_i[\alpha\cdot s^{-1}\cdot\phi_1] - \hat a_i[\beta\cdot s^{-1}\cdot\phi_1]^\dagger)\,\hat a_i[\beta\cdot s^{-1}\cdot\phi_2]^\dagger|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle} \ \text{nie dôkaz}$$

$$= -\langle s^{-1}\cdot\phi_1, \alpha^{-1}\cdot\beta\cdot s^{-1}\cdot\phi_2\rangle_i = -\langle \phi_1, \beta\cdot\alpha^{-1}\cdot\phi_2\rangle_f = -\langle\phi_1, \operatorname{th} X\cdot\phi_2\rangle_f$$

$$v_k\cdot V\cdot v_l = V_{kl} = -\sum_m \alpha^{-1}_{km}\,\beta^*_{ml} \qquad V = \partial F\cdot\Big(\sum_{kl} v^+_k\,V_{kl}\,v^+_l\Big)\cdot\partial F$$

$\phi_1\cdot V\cdot\phi_2 = \phi_2\cdot V\cdot\phi_1$ bi-J_f-antilin. forma citlivé na $\phi^-_{1,2\,f}$ $V_{kl} = V_{lk}$

Důk: $$\phi_1\cdot s\cdot V\cdot s\cdot\phi_2 = \frac{\langle f\,vac|\hat a_f[s\cdot\phi_1]\,\hat a_f[s\cdot\phi_2]|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle} = -\frac{\langle f\,vac|(\hat a_i[\alpha\cdot\phi_1] - \hat a_i[\beta\cdot\phi_1]^\dagger)\,\hat a_i[\beta\cdot\phi_2]^\dagger|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle}$$

$$= \phi_1\cdot\beta\cdot\Lambda\cdot\beta\cdot\phi_2 - \langle\alpha\cdot\phi_1, \beta\cdot\phi_2\rangle_i = \langle(\beta\cdot\alpha^{-1}\cdot\beta - \alpha)\cdot\phi_1, \beta\cdot\phi_2\rangle_i$$

$$= -\langle\alpha^{-1(+)}\cdot\phi_1, \beta\cdot\phi_2\rangle_i = -\langle\phi_1, \alpha^{-1}\cdot\beta\cdot\phi_2\rangle_i =$$

$$= -\langle s\cdot\phi_1, s\cdot\alpha^{-1}\cdot\beta\cdot s^{-1}\cdot s\cdot\phi_2\rangle_f = -\langle s\cdot\phi_1, \beta\cdot\alpha^{-1}\cdot s\cdot\phi_2\rangle_f = -\langle s\cdot\phi_1, \operatorname{th} X\cdot s\cdot\phi_2\rangle_f$$

I: $$\phi_1\cdot I\cdot\phi_2 = \frac{\langle f\,vac|\hat a_f[\phi_1]\,\hat a_i[\phi_2]^\dagger|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle}$$

$$= \langle\phi_1, s\cdot\alpha^{-1}\cdot\phi_2\rangle_f = \langle\phi_1, (\delta + \operatorname{th} X)\,\phi_2\rangle_f$$

$$= \langle s^{-1}\cdot\phi_1, \alpha^{-1}\cdot\phi_2\rangle_i = \langle(\delta - \operatorname{th} X)\cdot\phi_1, \phi_2\rangle_i$$

$$v_k\cdot I\cdot u_l = I_{kl} = \alpha^{-1}_{kl} \qquad I = -\partial F\cdot\Big(\sum_{kl} v^+_k\,I_{kl}\,u^-_l\Big)\cdot\partial F$$

$\phi_1\cdot I\cdot\phi_2$ J_f-antilin. v ϕ_1, J_i-lineární v ϕ_2 citlivé na ϕ^-_{1f}, ϕ^+_{2i}

Důk: $$\phi_1\cdot s\cdot I\cdot\phi_2 = \frac{\langle f\,vac|\hat a_f[s\cdot\phi_1]\,\hat a_i[\phi_2]^\dagger|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle} = \frac{\langle f\,vac|\hat a_f[s\cdot\phi_1]\,\hat a_f[s\cdot\alpha^{-1}\cdot\phi_2]^\dagger|i\,vac\rangle}{\langle f\,vac|i\,vac\rangle}$$

$$= \langle s\cdot\phi_1, s\cdot\alpha^{-1}\cdot\phi_2\rangle_f = \langle\phi_1, \alpha^{-1}\cdot\phi_2\rangle_i$$

S-matice

překlad in-módů na out-módy

$$V_k = S \cdot U_k$$

S transition opr. (oper. přechodu)

převádí "číslování" módů v in-exper. na analogické v out-exper.

tento přechod funguje i na kv. úrovni

→ unitární transf na $\mathcal{H}$

$\hat{S}^{-1} = \hat{S}^{\dagger}$

$$|f\,vac\rangle = \hat{S}^{\dagger}\,|i\,vac\rangle$$

$$\hat{a}_f^{\dagger}[S\cdot\phi]\,|f\,vac\rangle = \hat{S}^{\dagger}\,\hat{a}_i^{\dagger}[\phi]\,|i\,vac\rangle$$

} ⇒ obdobně pro vícečásticové stavy

$$|f\,coh\,S\cdot\phi\rangle = \hat{S}^{\dagger}\,|i\,coh\,\phi\rangle$$

tj.

$$\hat{a}_f[S\cdot\phi] = \hat{S}^{\dagger}\,\hat{a}_i[\phi]\,\hat{S}$$

$$\hat{a}_f^{\dagger}[S\cdot\phi] = \hat{S}^{\dagger}\,\hat{a}_i^{\dagger}[\phi]\,\hat{S}$$

⇕

$$S\cdot\hat{\Phi} = \hat{S}\cdot\hat{\Phi}\cdot\hat{S}^{\dagger}$$

důkaz: $\langle S\cdot\phi, \hat{\Phi}\rangle_f = \hat{S}^{\dagger}\langle\phi,\hat{\Phi}\rangle_i\hat{S}$

⇓

$\langle S\cdot\phi, \hat{S}\hat{\Phi}\hat{S}^{\dagger}\rangle_f = \langle\phi,\hat{\Phi}\rangle_i = \langle S\cdot\phi, S\cdot\hat{\Phi}\rangle_f$

$S\cdot\phi \to \phi$

⇓

$\Leftrightarrow \langle\phi, \hat{S}\hat{\Phi}\hat{S}^{\dagger}\rangle_f = \langle\phi, S\cdot\hat{\Phi}\rangle_f$

$\Leftrightarrow \langle\hat{S}\hat{\Phi}\hat{S}^{\dagger}, \phi\rangle_f = \langle S\cdot\hat{\Phi}, \phi\rangle_f$

$\Leftarrow$ (pozor na pořadí $\hat{S}$, $\hat{S}^{\dagger}$)

připomenutí:

z ortonorm. u_k a v_k dostáváme

$$\langle S\cdot\phi_1, S\cdot\phi_2\rangle_f = \langle\phi_1,\phi_2\rangle_i$$

⇓

$$S\cdot\partial f\cdot S = \partial f \qquad J_f\cdot S = S\cdot J_i$$

S je isomorf J_i a J_f struktury

pozor: obecně nelze zvolit $S = \delta$

tj. módy musejí být odlišné

S-matice

$$\hat{a}_f[S\cdot\phi] = \hat{S}^\dagger \hat{a}_i[\phi]\hat{S}$$

$\Downarrow$

$$\hat{S}\left(\hat{a}_i[\alpha\cdot\phi] - \hat{a}_i[\beta\cdot\phi]^\dagger\right) = \hat{a}_i[\phi]\hat{S}$$

$$\hat{S}\left(\hat{a}_i[\alpha\cdot\phi]^\dagger - \hat{a}_i[\beta\cdot\phi]\right) = \hat{a}_i[\phi]^\dagger\hat{S}$$

i-holomorfní repr.

$$\left(\left[\langle\alpha\cdot\phi,\phi_2\rangle_i - \phi\cdot\beta\cdot\delta_2\right]f_i[\hat{S}]\right)(\phi_1,\phi_2) = \phi\cdot(\delta_1 f_i[\hat{S}])(\phi_1,\phi_2)$$

$$\left(\left[\phi\cdot\alpha\cdot\delta_2 - \langle\beta\cdot\phi_1,\phi_2\rangle_i\right]f_i[\hat{S}]\right)(\phi_1,\phi_2) = \langle\phi_1,\phi\rangle_i f_i[\hat{S}](\phi_1,\phi_2)$$

$\Downarrow$

$$\phi\cdot\delta_1 f_i[\hat{S}] = \left(\langle\phi_1,\alpha^{-1}\cdot\phi_2\rangle_i - \langle\phi_1,\alpha^{-1}\cdot\beta\cdot\phi\rangle_i\right)f_i[\hat{S}]$$

$$\phi\cdot\delta_2 f_i[\hat{S}] = \left(\langle\phi_1,\alpha^{-1}\cdot\phi\rangle_i + \langle\beta\cdot\alpha^{-1}\cdot\phi,\phi_2\rangle_i\right)f_i[\hat{S}]$$

$\Downarrow$

$$f_i[\hat{S}](\phi_1,\phi_2) = o\,\exp\Big[-\tfrac{1}{2}\langle\phi_1,\alpha^{-1}\cdot\beta\cdot\phi_1\rangle_i + \tfrac{1}{2}\langle\beta\cdot\alpha^{-1}\cdot\phi_2,\phi_2\rangle_i + \langle\phi_1,\alpha^{-1}\cdot\phi_2\rangle_i\Big]$$

normalizace o z podmínky

$$\hat{S}\hat{S}^\dagger = \hat{1}$$

$$\Downarrow\ o = (\det_{\mathcal{S}_i}\mathrm{ch}\,\mathfrak{X})^{-\frac{1}{2}} = \Big(\prod_{\varkappa\in\mathfrak{I}}\mathrm{ch}\,X_\varkappa\Big)^{-\frac{1}{2}}$$

můžeme napsat $\hat{S}$ v normálním uspoř.

$$\hat{S} = :f_i[\hat{S}](\hat{\Phi},\hat{\Phi})\ \exp(-\langle\hat{\Phi},\hat{\Phi}\rangle):_i$$

$$(\beta\cdot\phi)\cdot\delta_2 F_i[\hat{S}] = \left(\langle\phi_1, \alpha^{-1}\cdot\beta\cdot\phi\rangle_i + \langle\beta\cdot\alpha^{-1}\cdot\beta\cdot\phi, \phi_2\rangle_i\right) F_i[\hat{S}]$$

$$\Downarrow$$

$$\phi\cdot\delta_1 F_i[\hat{S}] = \left(\langle\alpha\cdot\phi_1, \phi_2\rangle_i - \langle\beta\cdot\alpha^{-1}\cdot\beta\cdot\phi_1, \phi_2\rangle_i - \langle\phi_{11}, \alpha^{-1}\cdot\beta\cdot\phi\rangle_i\right) F_i[\hat{S}]$$

$$\alpha^{-1\langle +\rangle} = \alpha - \beta\cdot\alpha^{-1}\cdot\beta \quad \Leftarrow \quad \begin{aligned} &\alpha\cdot\alpha^{\langle +\rangle} - \beta\cdot\beta^{\langle +\rangle} = \delta \\ &\beta^{\langle +\rangle}\cdot\alpha^{-1\langle +\rangle} = \alpha^{-1}\cdot\beta \end{aligned}$$

$$\Downarrow$$

$$\phi\cdot\delta_1 F[\hat{S}] = \left(\langle\phi_1 \alpha^{-1}\cdot\phi_2\rangle_i - \langle\phi_{11}, \alpha^{-1}\cdot\beta\cdot\phi\rangle_i\right) F_i[\hat{S}]$$

S-matice v i-normálním uspořádání

$$\hat S = :\varphi_i(\hat\Phi,\hat\Phi):_i \qquad \varphi_i(\phi_1,\phi_2) = \frac{\langle icoh\,\phi_1|\hat S|icoh\,\phi_2\rangle}{\langle icoh\,\phi_1|icoh\,\phi_2\rangle} = f_i[\hat S](\phi_1,\phi_2)\exp(-\langle\phi_1,\phi_2\rangle)$$

$$= O :\exp\left(-\frac12\langle\hat\Phi^-_i,\alpha^{-1}\cdot\beta\cdot\hat\Phi^-_i\rangle_i + \frac12\langle\beta\cdot\alpha^{-1}\cdot\hat\Phi^+_i,\hat\Phi^+_i\rangle_i + \langle\hat\Phi^-_i,\alpha^{-1}\cdot\hat\Phi^+_i\rangle_i - \langle\hat\Phi^-_i,\hat\Phi^+_i\rangle_i\right):_i$$

$$= O\exp\left(\frac12\hat\Phi^-_i\cdot s\cdot V\cdot s\cdot\hat\Phi^-_i\right):\exp\left(\hat\Phi^-_i\cdot s\cdot I\cdot\hat\Phi^+_i - \langle\hat\Phi^-_i,\hat\Phi^+_i\rangle_i\right):_i\exp\left(\frac12\hat\Phi^+_i\cdot\Lambda\cdot\hat\Phi^+_i\right)$$

$$\hat\Phi^-_i = \sum_k \hat a^+_{ik}u^-_k \qquad \hat\Phi^+_i = \sum_l \hat a_{il}u^+_l \qquad V^+_k = s\cdot u^+_k$$

$$= O\exp\left(\frac12\sum_{k,l}V_{kl}\hat a^+_{ik}\hat a^+_{il}\right):\exp\left(\sum_{kl}I_{kl}\hat a^+_{ik}\hat a_{il}\right)\underbrace{\exp\left(-\sum_j\hat a^+_{ij}\hat a_{ij}\right):_i}\exp\left(\frac12\sum_{k,l}\Lambda_{kl}\hat a_{ik}\hat a_{il}\right)$$

$$:\exp\left(-\sum_j\hat a^+_{ij}\hat a_{ij}\right):_i = |i\,vac\rangle\langle i\,vac|$$

S-matice v f-normálním uspořádání

$$\hat S^+\hat a_{ik}\hat S = \hat a_{fk} \qquad \hat S^+\hat a^+_{ik}\hat S = \hat a_{fk} \qquad \text{díky } V_k = s\cdot u_k$$

$$\hat S = \hat S^+\hat S\hat S$$

$$:\exp\left(-\sum_j\hat a^+_{fj}\hat a_{fj}\right):_f = |f\,vac\rangle\langle f\,vac|$$

$$= O\exp\left(\frac12\sum_{k,l}V_{kl}\hat a^+_{fk}\hat a^+_{fl}\right):\exp\left(\sum_{kl}I_{kl}\hat a^+_{fk}\hat a_{fl}\right)\overbrace{\exp\left(-\sum_j\hat a^+_{fj}\hat a_{fj}\right):_f}\exp\left(\frac12\sum_{kl}\Lambda_{kl}\hat a_{fk}\hat a_{fl}\right)$$

$$\hat\Phi^-_f = \sum_k\hat a^+_{fk}V^-_k \qquad \hat\Phi^+_f = \sum_l\hat a_{fl}V^+_l \qquad \text{nebo} \qquad \hat S^+\hat\Phi\hat S = s^{-1}\cdot\hat\Phi$$

$$= O\exp\left(\frac12\hat\Phi^-_f\cdot V\cdot\hat\Phi^-_f\right):\exp\left(\hat\Phi^-_f\cdot I\cdot s^{-1}\cdot\hat\Phi^+_f - \langle\hat\Phi^-_f,\hat\Phi^+_f\rangle_f\right):_f\exp\left(\frac12\hat\Phi^+_f\cdot s^{-1}\cdot\Lambda\cdot s^{-1}\cdot\hat\Phi^+_f\right)$$

vakuová amplituda

$$\langle f\,vac|i\,vac\rangle = \langle i\,vac|\hat S|i\,vac\rangle = O = (\det_{\mathcal{H}_1}\operatorname{ch}\mathcal{X})^{-\frac12}$$

vztah i a f vakua

$$|i\,vac\rangle = \hat S|f\,vac\rangle = \exp\left(\frac12\hat\Phi^-\cdot V\cdot\hat\Phi^-\right)|f\,vac\rangle\, O$$

$$= \exp\left(\frac12\sum_{kl}V_{kl}\hat a^+_{fk}\hat a^+_{fl}\right)|f\,vac\rangle\langle f\,vac|i\,vac\rangle$$

$$\langle f\,vac| = \langle i\,vac|\hat S = O\langle i\,vac|\exp\left(\frac12\hat\Phi^+\cdot\Lambda\cdot\hat\Phi^+\right)$$

$$= \langle f\,vac|i\,vac\rangle\langle i\,vac|\exp\left(\frac12\sum_{kl}\Lambda_{kl}\hat a_{ik}\hat a_{il}\right)$$