Koherentní stavy

mějme zvolenou částicovou repr.

koher. stav

$$|coh:\phi\rangle = \exp\left(\tfrac{1}{2}\langle\phi,\phi\rangle\right)\hat{W}[\phi]\,|vac\rangle$$

shiftoperátor

$$\hat{W}[\phi] = \exp(i\,\phi\cdot\partial f\cdot\hat{\Phi}) = \exp(i\hat{L}_\phi)$$

$$\hat{W}[\phi]^\dagger\,\hat{\Phi}\,W[\phi] = \hat{\Phi} + \phi\,\hat{\mathbb{1}}$$

důkaz:

$$[\hat{\Phi}, i\phi\cdot\partial f\cdot\hat{\Phi}] = -i[\hat{\Phi},\hat{\Phi}]\cdot\partial f\cdot\phi = \phi \quad\Rightarrow\quad [\hat{\Phi}, F(i\phi\cdot\partial f\cdot\hat{\Phi})] = \phi\,F'(i\phi\cdot\partial f\cdot\hat{\Phi})$$

$$\hat{W}[\phi]^\dagger\hat{\Phi}\hat{W}[\phi] = \hat{\Phi} + \hat{W}[\phi]^\dagger[\hat{\Phi}, \exp(i\phi\cdot\partial f\cdot\hat{\Phi})] = \hat{\Phi} + \phi\,\hat{W}[\phi]^\dagger\hat{W}[\phi]$$

$$= \hat{\Phi} + \phi\,\hat{\mathbb{1}}$$

koherentní stav je tak "posunuté" vakuum

ale! neortonormální, nenormalizované!

pomocné lema

vynechat

$$\hat{W}[\phi_1]\,\hat{W}[\phi_2] = \exp(i\,\phi_1\cdot\partial f\cdot\phi_2)\,\hat{W}[\phi_2]\,\hat{W}[\phi_1]$$

$$\exp(\hat{a}[\phi_1])\exp(\hat{a}[\phi_2]^\dagger) = \exp(\langle\phi_1,\phi_2\rangle)\exp(\hat{a}[\phi_2]^\dagger)\exp(\hat{a}[\phi_1])$$

$$= \exp\left(\tfrac{1}{2}\langle\phi_1,\phi_2\rangle\right)\exp\left(\hat{a}[\phi_1] + \hat{a}[\phi_2]^\dagger\right)$$

důkaz:

důsledek $\exp(\hat{A}+\hat{B}) = \exp\left(-\tfrac{1}{2}[\hat{A},\hat{B}]\right)\exp\hat{A}\,\exp\hat{B}$

pro $[\hat{A},[\hat{A},\hat{B}]] = [\hat{B},[\hat{A},\hat{B}]] = 0$

normální uspořádání shift. oper.

$$\hat{W}[\phi] = \exp(\hat{a}[\phi]^\dagger - \hat{a}[\phi]) =$$

$$= \exp\left(-\tfrac{1}{2}\langle\phi,\phi\rangle\right)\exp(\hat{a}[\phi]^\dagger)\exp(-\hat{a}[\phi])$$

vztah koherentních a částicových stavů

$$|coh\,\phi\rangle = \exp(\hat{a}[\phi]^\dagger)\,|vac\rangle$$
$$= \sum_n \frac{1}{\sqrt{n!}}\,|\phi : n\rangle$$

- superpozice částic v módu ϕ
- všechny počty částic (včetně vakua)

neortonormalita a nenormovanost

$$\langle coh\,\phi_1 | coh\,\phi_2\rangle = \exp(\langle\phi_1|\phi_2\rangle)$$

$$\Uparrow\quad \exp(\hat{a}[\phi_1])\exp(\hat{a}[\phi_2]^\dagger) = \exp(\langle\phi_1,\phi_2\rangle)\exp(\hat{a}[\phi_2]^\dagger)\exp(\hat{a}[\phi_1])$$

- nejsou kolmé i pro kolmé módy $\langle\phi_1,\phi_2\rangle = 0$
  (obsahuje vakuum)
- nejsou normované i pro norm. módy
  $\langle coh\,\phi | coh\,\phi\rangle = \exp(\langle\phi|\phi\rangle)$

relace úplnosti

$$\hat{\mathbb{1}} = \int_{\phi\in\varphi} dh(\phi)\,|coh\,\phi\rangle\langle coh\,\phi|$$

$$dh(\phi) = \exp(-\langle\phi,\phi\rangle)\,d\Gamma$$

$$d\Gamma = \left(\mathrm{Det}\,\frac{\partial f}{2\pi}\right)^{\frac{1}{2}} \underset{\uparrow\ \det J = 1}{=} \left(\mathrm{Det}\,\frac{\bar{\partial} f}{2\pi}\right)^{\frac{1}{2}} = \prod_x \frac{d\varphi^x\,d\pi_x}{2\pi}$$

viz dále

Koherentní stav v částicové bázi

$\mathbf{u} = \{u_k\}$ norm. báze v $\mathcal{G}_J$

$|\mathbf{u}:\mathbf{m}\rangle$ částicová báze

$$|\mathrm{coh}\,\phi\rangle = \sum_{\mathbf{m}} \frac{1}{\sqrt{\mathbf{m}}} \underbrace{\langle \mathbf{u},\phi\rangle^{\mathbf{m}}}_{\prod_k \langle u_k,\phi\rangle^{m_k}} |\mathbf{u}:\mathbf{m}\rangle$$

důkaz:

$$|\mathrm{coh}\,\phi\rangle = \exp(\underbrace{\langle\hat{\Phi},\phi\rangle}_{\hat{a}[\phi]^\dagger})|\mathrm{vac}\rangle =$$

$$= \exp\Big(\sum_k \langle\hat{\Phi},u_k\rangle\langle u_k,\phi\rangle\Big)|\mathrm{vac}\rangle$$

$$= \prod_k \exp(\langle u_k,\phi\rangle\,\hat{a}_k^\dagger)|\mathrm{vac}\rangle$$

$$= \prod_k \sum_{m_k} \frac{1}{m_k!} \langle u_k,\phi\rangle^{m_k}\,\hat{a}_k^{\dagger m_k}|\mathrm{vac}\rangle$$

$$= \sum_{\mathbf{m}} \underbrace{\frac{1}{\sqrt{\mathbf{m}!}}\langle\mathbf{u},\phi\rangle^{\mathbf{m}}}_{\prod_k \frac{1}{\sqrt{m_k!}}\langle u_k,\phi\rangle^{m_k}}\ \underbrace{|\mathbf{u}:\mathbf{m}\rangle}_{\frac{1}{\sqrt{\mathbf{m}!}}\hat{a}[\mathbf{u}]^{\dagger\mathbf{m}}|\mathrm{vac}\rangle = \prod_k \frac{1}{\sqrt{m_k!}}\hat{a}_k^{\dagger m_k}|\mathrm{vac}\rangle}$$

vlastní vektory anihil. operátorů

$$\hat{a}[\phi_0]\,|coh:\phi\rangle = \langle\phi_0,\phi\rangle\,|coh\,\phi\rangle$$

důkaz

$$\langle\phi_0,\hat{\Phi}\rangle \exp(\tfrac{1}{2}\langle\phi,\phi\rangle)\,\hat{W}[\phi]\,|vac\rangle =$$

$$= \exp(\tfrac{1}{2}\langle\phi,\phi\rangle)\,\hat{W}[\phi]\,\langle\phi_0,\hat{\Phi}+\phi\hat{\mathbb{1}}\rangle|vac\rangle$$

$$= \langle\phi_0,\phi\rangle \exp(\tfrac{1}{2}\langle\phi,\phi\rangle)\,\hat{W}[\phi]|vac\rangle$$

$$= \langle\phi_0,\phi\rangle\,|coh\,\phi\rangle$$

neboli $|coh\,\phi\rangle$ je vl. vekt. neg. fr. části $\hat{\Phi}^-$

$$\hat{\Phi}^-|coh\,\phi\rangle = \phi^-\,|coh\,\phi\rangle$$

akce kreačních oper.

$$\hat{a}[\phi_0]^\dagger\,|coh\,\phi\rangle = \phi_0\cdot\frac{\delta}{\delta\phi}\,|coh\,\phi\rangle$$

důkaz:

$$\hat{a}[\phi_0]^\dagger|coh\,\phi\rangle = \hat{a}[\phi_0]^\dagger \exp(\hat{a}[\phi]^\dagger)\,|vac\rangle =$$

$$= -i\,\hat{\Phi}^-\cdot\partial\mathcal{F}\cdot\phi_0\,\exp(-i\,\hat{\Phi}^-\cdot\partial\mathcal{F}\cdot\phi)\,|vac\rangle =$$

$$= \phi_0\cdot\frac{\delta}{\delta\phi}\exp(-i\,\hat{\Phi}^-\cdot\partial\mathcal{F}\cdot\phi)\,|vac\rangle = \phi_0\cdot\frac{\delta}{\delta\phi}|coh:\phi\rangle$$

střední počet částic

$$\frac{\langle coh\,\phi|\,\hat{n}[\phi_0]\,|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \frac{\langle\phi,\phi_0\rangle\langle\phi_0,\phi\rangle}{\langle\phi_0,\phi_0\rangle}$$ – ortogonální módy nepřispívají!

tj.

$$\frac{\langle coh\,\phi|\,\hat{n}[\phi]\,|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \langle\phi,\phi\rangle$$

důkaz

$$\hat{n}[\phi_0] = \frac{\hat{a}[\phi_0]^\dagger\,\hat{a}[\phi_0]}{\langle\phi_0,\phi_0\rangle} \qquad \hat{a}[\phi_0]|coh\,\phi\rangle = \langle\phi_0,\phi\rangle|coh\,\phi\rangle$$

celkový počet částic

$$\frac{\langle coh\,\phi|\,\hat{N}\,|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \langle\phi,\phi\rangle$$ – všechny částice v módu ϕ

střední hodnota pole

$$\frac{\langle coh\,\phi|\hat{\Phi}|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \phi$$

důkaz:

$$\frac{\langle coh\,\phi|\hat{\Phi}|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \frac{\langle coh|\hat{\Phi}^- + \hat{\Phi}^+|coh\,\phi\rangle}{\langle coh\,\phi|coh\,\phi\rangle} = \phi^- + \phi^+ = \phi$$

díky $\hat{\Phi}^+|coh\,\phi\rangle = \phi|coh\,\phi\rangle \qquad \langle coh\,\phi|\hat{\Phi}^+ = \phi^+\langle coh\,\phi|$

komplementarita s část. popisem

$$\frac{\langle\phi:n|\hat{\Phi}|\phi:n\rangle}{\langle\phi:n|\phi:n\rangle} = 0$$

či

$$\langle u:m|\hat{\Phi}|u:m\rangle = 0$$

asi vynechat

jeden mód ϕ_0 $\qquad \langle \phi_0, \phi_0 \rangle = 1$

$$\hat{a} = \hat{a}[\phi_0] \qquad [\hat{a}, \hat{a}^+] = \hat{\mathbb{1}} \qquad \hat{n} = \hat{a}^+ \hat{a}$$

částic. stavy

$$|n\rangle = |\phi_0 : n\rangle = \frac{1}{\sqrt{n!}} \hat{a}^{+n} |vac\rangle \qquad \langle n | n \rangle = 1$$

koh. stav $\qquad \phi = \kappa \phi_0 \qquad \langle \phi, \phi \rangle = |\kappa|^2$

$$|koh_{\kappa\phi_0}\, \phi\rangle = \exp\left(\tfrac{1}{2}|\kappa|^2\right) \hat{W}[\phi] |vac\rangle$$

$$= \exp\left(\hat{a}[\phi]^+\right) |vac\rangle =$$

$$= \exp\left(\kappa\, \hat{a}^+\right) |vac\rangle$$

$$= \sum_n \frac{1}{\sqrt{n!}} \kappa^n |n\rangle$$

$$\langle koh\, \phi | koh\, \phi \rangle = \exp\left(|\kappa|^2\right)$$

$$|\widetilde{koh}\, \phi\rangle = \hat{W}[\phi] |vac\rangle =$$

$$= \exp\left(-\tfrac{1}{2}|\kappa^2|\right) \sum_n \frac{1}{\sqrt{n!}} \kappa^n |n\rangle$$

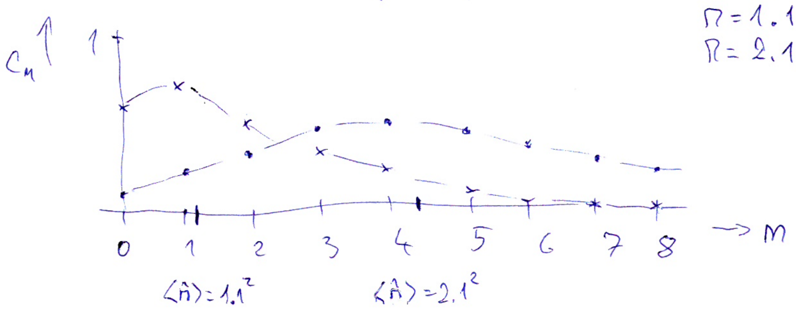
$$\langle \widetilde{koh}\, \phi | \widetilde{koh}\, \phi \rangle = 1$$

$$\langle \widetilde{koh}\, \phi | \hat{n} | \widetilde{koh}\, \phi \rangle = \frac{\langle koh\, \phi | \hat{n} | koh\, \phi \rangle}{\langle koh\, \phi | koh\, \phi \rangle} = \langle \phi, \phi \rangle = |\kappa|^2$$

obsazovací koefic pro $|\widetilde{koh}\, \kappa\phi_0\rangle = \sum c_n |n\rangle$

$\kappa = 1.1$ $\quad \times \times \times$

$\kappa = 2.1$ $\quad \bullet \bullet \bullet$



Holomorfní reprezentace

přehled pojmů

$f: \mathbb{C} \to \mathbb{C}$ holomorfní $\qquad f(z)$

- existuje komplexní derivace podle z
- f nezávisí na z^*
- f lze napsat jako (nekon.) řadu v z

$F: V \to \mathbb{C}$ holomorfní na complx. vekt. pr. V

$\equiv F(\phi + z\psi)$ holomorfní v z $\quad \forall \phi, \psi$

pokud $V = V_R^{\mathbb{C}} = V_R \oplus i V_R$, pak F holomorf. znamená i že F závisí na ϕ a ne na ϕ^*

$F: V_R \to \mathbb{C}$ analytická na reál. vekt. pr. V_R

$\equiv$ existuje holomorf. rozšíření na $V = V_R^{\mathbb{C}}$
(ne nutně globálně - póly atd.)

$F: \mathcal{S} \to \mathbb{C}$ J-holomorfní

$\equiv F(\phi + z * \psi)$ holomorfní v z $\quad \forall \phi, \psi$

Platí:

F analytická na $\mathcal{S}$

F je J-holomorf. pokud $F(\phi) = F(\phi^+)$

F je J-antiholomorf. pokud $F(\phi) = F(\phi^-)$

kde $F(\phi)$ je holom. rozš. F na $\mathcal{S}^{\mathbb{C}}$

důkaz

$(z * \phi)^+ = z\phi^+ \qquad (z * \phi)^- = z^* \phi^- \qquad \Rightarrow \quad z * \phi = z\phi^+ + z^*\phi^-$

$F(\phi) = F(z\phi^+ + z^*\phi^-) \quad \Rightarrow$ závislost na z / z^* = záv. na ϕ^+ / ϕ^-

holomorfnost koh. st.

$|\mathrm{coh}\,\phi\rangle$ je J-holomorfní

$\langle \mathrm{coh}\,\phi|$ je J-antiholomorfní

$|\mathrm{coh}\,\phi\rangle = \exp(\hat{a}[\phi]^\dagger)|\mathrm{vac}\rangle = \exp(-i\hat{\Phi}\cdot\omega\cdot\phi^+)|\mathrm{vac}\rangle$

holomorfní repr.

$F[|\text{stav}\rangle](\phi) = \langle \text{coh}\,\phi|\text{stav}\rangle$ J-antihol.

$F[\langle\text{stav}|](\phi) = \langle\text{stav}|\text{coh}\,\phi\rangle$ J-hol.

$F[\hat{A}](\phi_1,\phi_2) = \langle\text{coh}\,\phi_1|\hat{A}|\text{coh}\,\phi_2\rangle$ J-antihol/J-hol.

věrná repr. - neztrácí informaci
(viz normální uspoř. dále)

vlastnosti:

$$F[\hat{a}[\phi]^\dagger|\text{stav}\rangle](\phi_1) = \langle\phi_1,\phi\rangle\, F[|\text{stav}\rangle](\phi_1)$$

$$F[\hat{a}[\phi]|\text{stav}\rangle](\phi_1) = \phi\cdot\frac{\delta}{\delta\phi} F[|\text{stav}\rangle](\phi_1)$$

příklady

$$F[|\text{vac}\rangle](\phi_1) = 1$$

$$F[\hat{a}[\phi]^{\dagger n}|\text{vac}\rangle](\phi_1) = \langle\phi_1,\phi\rangle^n$$

$$F[|\text{coh}\,\phi\rangle](\phi_1) = \exp(\langle\phi_1,\phi\rangle)$$

$$F[|\text{vac}\rangle\langle\text{vac}|](\phi_1,\phi_2) = 1$$

$$F[\hat{1}](\phi_1,\phi_2) = \exp(\langle\phi_1,\phi_2\rangle)$$

$$F[\hat{n}[\phi]](\phi_1,\phi_2) = \frac{\langle\phi_1,\phi\rangle\langle\phi,\phi_2\rangle}{\langle\phi,\phi\rangle}\exp(\langle\phi_1,\phi_2\rangle)$$

$$F[\hat{N}](\phi_1,\phi_2) = \langle\phi_1,\phi_2\rangle\exp(\langle\phi_1,\phi_2\rangle)$$

$$F[|\text{coh}\,\phi\rangle\langle\text{coh}\,\phi|](\phi_1,\phi_2) = \exp(\langle\phi_1,\phi\rangle + \langle\phi,\phi_2\rangle)$$

$$F[\hat{W}[\phi]](\phi_1,\phi_2) = \exp\left(-\tfrac{1}{2}\langle\phi,\phi\rangle + \langle\phi_1,\phi\rangle - \langle\phi,\phi_2\rangle + \langle\phi_1,\phi_2\rangle\right)$$

částicové amplitudy v holomorfní repr.

$$\langle u : m \mid \text{stav} \rangle = \frac{1}{\sqrt{m!}} (u \cdot \delta)^m \, \mathcal{F}[\,|\text{stav}\rangle\,](0)$$

$$\uparrow \prod_k (u_k \cdot \delta)^{m_k}$$

důkaz:

$$\langle \text{coh}\, \phi \,|\, \hat{a}[\phi_0] \,|\, \text{stav} \rangle = \phi_0 \cdot \frac{\delta}{\delta\phi} \langle \text{coh} : \phi \mid \text{stav} \rangle$$

$$\langle \text{coh}\, \phi \,|\, \hat{a}[u]^m \,|\, \text{stav} \rangle = (u \cdot \delta)^m \langle \text{coh} : \phi \mid \text{stav} \rangle$$

$$\langle u : m \mid \text{stav} \rangle = \frac{1}{\sqrt{m!}} \langle \text{vac} \,|\, \hat{a}[u]^m \,|\, \text{stav} \rangle = \frac{1}{\sqrt{m!}} (u \cdot \delta)^m \langle \text{coh} : \phi \mid \text{stav} \rangle \Big|_{\phi = 0}$$

$$= \frac{1}{\sqrt{m!}} (u \cdot \delta)^m \, \mathcal{F}[\,|\text{stav}\rangle\,](0)$$

Rekonstrukce stavů a operátorů z holom. repr.

Relace úplnosti

$$\hat{\mathbb{1}} = \int_{\phi} dh \, |coh\, \phi\rangle\langle coh\, \phi|$$

$$dh = \exp(-\langle\phi,\phi\rangle)\, d\Gamma(\phi) = \exp(-\tfrac{1}{2}\phi\cdot\partial\mathcal{F}\cdot\phi)\, d\Gamma(\phi)$$

$$d\Gamma = \left(\mathrm{Det}\,\frac{\partial\mathcal{F}}{2\pi}\right)^{\frac{1}{2}} = \left(\mathrm{Det}\,\frac{\partial\mathcal{F}}{2\pi}\right)^{\frac{1}{2}}$$

$\Downarrow$

$$|stav\rangle = \int dh(\phi)|coh\,\phi\rangle\langle coh\,\phi|stav\rangle = \int dh(\phi)|coh\,\phi\rangle\, F[|stav\rangle](\phi)$$

$$\hat{A} = \int dh(\phi_1)\,dh(\phi_2)\,|coh\,\phi_1\rangle\langle coh\,\phi_1|\hat{A}|coh\,\phi_2\rangle\langle coh\,\phi_2| =$$

$$= \int dh(\phi_1)\,dh(\phi_2)\,|coh\,\phi_1\rangle\, F[\hat{A}](\phi_1,\phi_2)\,\langle coh\,\phi_2|$$

[Inr-11b

relace úplnosti

$$F\left[\int_\phi dh\,|\mathrm{coh}\,\phi\rangle\langle\mathrm{coh}\,\phi|\right](\phi_1,\phi_2) =$$

$$= \int_\phi d\Gamma\,\exp\left(-\langle\phi,\phi\rangle + \langle\phi_1,\phi\rangle + \langle\phi,\phi_2\rangle\right)$$

$$= \int_\phi \left(\mathrm{Det}\,\frac{\bar\partial F}{2\pi}\right)^{\frac12} \exp\left(-\frac12\,\phi\cdot\bar\partial F\cdot\phi + \phi\cdot\bar\partial F\cdot(\phi_1^- + \phi_2^+)\right) =$$

$$= \exp\left(\frac12\,(\phi_1^- + \phi_2^+)\cdot\bar\partial F\cdot(\phi_1^- + \phi_2^+)\right)$$

$$= \exp\left(\langle\phi_1,\phi_2\rangle\right) =$$

$$= F[\hat{1}](\phi_1,\phi_2)$$

$$\langle\phi,\psi\rangle = \phi\cdot\frac12(\bar\partial F - i\partial F)\cdot\psi = \phi^-\cdot\bar\partial F\cdot\psi^+$$

$$dh(\phi) = \exp(-\langle\phi,\phi\rangle)\,d\Gamma(\phi)$$

$$d\Gamma(\phi) = \left(\mathrm{Det}\,\frac{\partial F}{2\pi}\right)^{\frac12} = \left(\mathrm{Det}\,\frac{\bar\partial F}{2\pi}\right)^{\frac12} = \prod_x \frac{d\varphi^x\,d\pi_x}{2\pi}$$

[det] = 1

$$dh(\phi) = \exp\left(-\frac12\,\phi\cdot\bar\partial F\cdot\phi\right)\,\mathrm{Det}^{\frac12}\,\frac{\bar\partial F}{2\pi}$$

$\bar\partial F$ pos. def. kvadr. forma