Normální uspořádání

Jak přiřadit pozorov. na $\mathcal{S}$ operátor na $\mathcal{H}$

„stejné funkcionální závislosti na $\hat{\Phi}$"

$$\hat{B} \overset{?}{=} B(\hat{\Phi})$$

– problém s uspoř. součinů

$$[\hat{\Phi}^x, \hat{\Phi}^y] \neq 0$$

normální uspořádání

rozštěpení $\hat{\Phi}$ na neg a pos. fr. část

$$\hat{\Phi} = \hat{\Phi}^- + \hat{\Phi}^+ \qquad [\hat{\Phi}^-, \hat{\Phi}^-] = 0 \qquad [\hat{\Phi}^+, \hat{\Phi}^+] = 0$$

uspořádat $\hat{\Phi}^-$ vždy nalevo od $\hat{\Phi}^+$

příklad

$$K(\phi) = \frac{1}{2}\phi \cdot \overset{\text{symetr.}}{k} \cdot \phi = \frac{1}{2}(\phi^+ + \phi^-) \cdot k \cdot (\phi^+ + \phi^-)$$

$$= \frac{1}{2}\phi^+ \cdot k \cdot \phi^+ + \frac{1}{2}(\phi^- \cdot k \cdot \phi^-) + \phi^- \cdot k \cdot \phi^+$$

$$\hat{K} = :K(\hat{\Phi}): = \frac{1}{2}\hat{\Phi}^+ \cdot k \cdot \hat{\Phi}^+ + \frac{1}{2}\hat{\Phi}^- \cdot k \cdot \hat{\Phi}^- + \hat{\Phi}^- \cdot k \cdot \hat{\Phi}^+$$

!! viz extra stránky s poznámkami o realitě, analit., holomorf., …

obecně

$B : \mathcal{S} \to \mathbb{C}$ analytická

$B(\phi)$ holomorf. rozš. na $\mathcal{S}^{\mathbb{C}}$

$$B(\phi) = B(\phi^-, \phi^+) \qquad \text{kde } \phi = \phi^- + \phi^+$$

$$B(\phi_1, \phi_2) \overset{*}{=} B(\phi_1^-, \phi_2^+) = B(\phi_1^- + \phi_2^+)$$

* automatická projekce na ϕ^- a ϕ^+ v 1. a 2. arg díky ∃ antihol/hol funkce B

definuje přiřazení $B(\phi) \leftrightarrow B(\phi_1, \phi_2)$

$B(\phi_1, \phi_2)$ J-antihol./J-hol fce na $\mathcal{S} \times \mathcal{S}$

holomorfní v ϕ_1, ϕ_2 na $\mathcal{S}^{\mathbb{C}}$

normální uspoř.

$$\hat{B} = :B(\hat{\Phi}): = :B(\hat{\Phi}, \hat{\Phi}):$$

$= B(\hat{\Phi}^-, \hat{\Phi}^+)$ kde $\hat{\Phi}^-$ se místí nalevo od $\hat{\Phi}^+$

analytické pozorov. na $\mathcal{S}$

$B : \mathcal{S} \to \mathbb{C}$ analyt.

$B(\phi)$ $\mathcal{S}^{\mathbb{C}} \to \mathbb{C}$ holom. rozšíř. na $\mathcal{S}^{\mathbb{C}}$

$B(\phi^*)$ $\mathcal{S}^{\mathbb{C}} \to \mathbb{C}$ a-hol. rozš. na $\mathcal{S}^{\mathbb{C}}$

pos/neg fr. rozštěpení dané s-ple. str J

$\mathcal{S}^{\mathbb{C}} = \mathcal{S}^{+} + \mathcal{S}^{-}$

$\phi = \phi^{+} + \phi^{-}$ $\qquad \phi^{\pm} = P^{\pm} \cdot \phi$ $\qquad \phi^{\pm *} = \phi^{* \mp}$

pro $\phi \in \mathcal{S}$ $\qquad \phi^{\pm *} = \phi^{\mp}$

J-ahol/hol fce na $\mathcal{S} \times \mathcal{S}$

$B(\phi_1, \phi_2) : \mathcal{S} \times \mathcal{S} \to \mathbb{C}$

J-ahol v ϕ_1 = ex. hol. rozš. v ϕ_1 závislé jen na ϕ_1^-

J-hol v ϕ_2 = ex. hol. rozš. v ϕ_2 závislé je na ϕ_2^+

$B(\phi_1, \phi_2)$ hol. rozš. $\mathcal{S}^{\mathbb{C}} \times \mathcal{S}^{\mathbb{C}} \to \mathbb{C}$

$B(\phi_1, \phi_2) = B(\phi_1^-, \phi_2) = B(\phi_1, \phi_2^+) = B(\phi_1^-, \phi_2^+)$

korespondence

B anal. na $\mathcal{S}$ je v jednoznačné koresp. s

B J-ahol/J-hol na $\mathcal{S} \times \mathcal{S}$

$B(\phi) \leftrightarrow B(\phi_1, \phi_2)$

$B(\phi) = B(\phi, \phi)$ $\qquad = B(\phi^-, \phi^+) = B(\phi^- + \phi^+)$

holom. rozš. B

$B(\phi_1, \phi_2) = B(\phi_1^- + \phi_2^+)$

holomorf rozš. B

konzistence

$B \to B(\phi_1, \phi_2) = B(\phi_1^- + \phi_2^+) \to \tilde{B}(\phi) = B(\phi, \phi) = B(\phi^- + \phi^+) = B(\phi)$ ✓

$B \to B(\phi) = B(\phi, \phi) \to \tilde{B}(\phi_1, \phi_2) = B(\phi_1^- + \phi_2^+) = B(\phi_1^- + \phi_2^+, \phi_1^- + \phi_2^+) =$

$= B(\phi_1^-, \phi_2^+) = B(\phi_1, \phi_2)$ ✓

ČJnT-12b extra

komplexní sdružení a realita

B analytické na $\mathcal{S}$

= polynom nekonečného stupně

B^* analytické na $\mathcal{S}$

= polynom nek. stup. s komplexně sdr. koef

$$B^*(\phi) = B(\phi)^* \qquad \phi \in \mathcal{S} \qquad (\mathrm{I})$$

holomorfní rozšíření na $\mathcal{S}^c$

$B(\phi)$ hol. rozš. B na $\mathcal{S}^c$

$B(\phi^*)$ ahol. rozš. B na $\mathcal{S}^c$

$B^*(\phi)$ hol. rozš. B^* na $\mathcal{S}^c$

$B^*(\phi^*)$ ahol. rozš. B^* na $\mathcal{S}^c$

$$B^*(\phi^*) = B(\phi)^* \qquad \phi \in \mathcal{S}^c \qquad (\mathrm{II})$$

J-ahol/hol tvar

$$B \rightarrow B(\phi_1, \phi_2) = \underbrace{B(\phi_1^- + \phi_2^+)}_{\text{hol. rozš. } B}$$

$$B^* \rightarrow B^*(\phi_1, \phi_2) = \underbrace{B^*(\phi_1^- + \phi_2^+)}_{\text{hol. rozš. } B^*}$$

na $\mathcal{S}^c$: $B^*(\phi_1, \phi_2) = B^*(\phi_1^- + \phi_2^+) \overset{(\mathrm{II})}{=} B(\phi_1^{-*} + \phi_2^{+*})^* = B(\phi_2^{*-} + \phi_1^{*+})^*$

$$B^*(\phi_1, \phi_2) = B(\phi_2^*, \phi_1^*)^* \qquad (\mathrm{III})$$

$$\text{na } \mathcal{S} : \quad B^*(\phi_1, \phi_2) = B(\phi_2, \phi_1)^* \qquad (\mathrm{IV})$$

reálné pozorovatelné

$B = B^*$ tj $B(\phi) = B(\phi)^*$ na $\mathcal{S}$ tj $B(\phi^*) = B(\phi)^*$ na $\mathcal{S}^c$

$$B(\phi) = B(\phi)^* \quad \text{na } \mathcal{S} \qquad \Leftarrow (\mathrm{I})$$

$$B(\phi) = B(\phi^*)^* \quad \text{na } \mathcal{S}^c \qquad \Leftarrow (\mathrm{II})$$

$$B(\phi_1, \phi_2) = B(\phi_2, \phi_1)^* \quad \text{na } \mathcal{S} \qquad \Leftarrow (\mathrm{IV})$$

$$B(\phi_1, \phi_2) = B(\phi_2^*, \phi_1^*)^* \quad \text{na } \mathcal{S}^c \qquad \Leftarrow (\mathrm{III})$$

komplexní sdružení

$$(B(\phi))^* = B^*(\phi^*) \quad \text{na } \mathcal{S}^{\mathbb{C}}$$

$$\Downarrow$$

$$(B(\phi_1, \phi_2))^* = B^*(\phi_2, \phi_1) \quad \text{na } \mathcal{S}$$

$$(B(\phi_1, \phi_2))^* = B^*(\phi_2^*, \phi_1^*) \quad \text{na } \mathcal{S}^{\mathbb{C}} \quad \Leftarrow$$

$\phi_1, \phi_2 \in \mathcal{S}^{\mathbb{C}}$ $\quad \phi_i^{\pm *} = \phi_i^{*\mp}$

$$(B(\phi_1, \phi_2))^* = (B(\phi_1^- + \phi_2^+))^* = B^*(\phi_2^{*-} + \phi_1^{*+}) = B^*(\phi_2^*, \phi_1^*)$$

B reálné na $\mathcal{S}$ $\quad \Leftrightarrow \quad B = B^*$

$$\Rightarrow B(\phi_1, \phi_2)^* = B(\phi_2, \phi_1) \quad \text{na } \mathcal{S}$$

$$\Rightarrow B(\phi_1, \phi_2)^* = B(\phi_2^*, \phi_1^*) \quad \text{na } \mathcal{S}^{\mathbb{C}}$$

hermitovskost

B reálné

$$\Downarrow$$

$\hat{B} = :B(\hat{\Phi}):$ hermitovský

$$:B(\hat{\Phi}):^\dagger = :B(\hat{\Phi}^-, \hat{\Phi}^+):^\dagger = B^*(\hat{\Phi}^{+\dagger}, \hat{\Phi}^{-\dagger})$$ nejprve. s $\hat{\Phi}^{-\dagger}$ napravo od $\hat{\Phi}^{+\dagger}$

obecné

$$= :B^*(\hat{\Phi}^-, \hat{\Phi}^+): = :B^*(\hat{\Phi}):$$

$\hat{\Phi}^{-\dagger} = \hat{\Phi}^+$

B reálné $\quad B = B^* \quad \Rightarrow \quad \hat{B} = \hat{B}^\dagger$

Lze formulovat opačně?
Jak přiřadit operátoru $\hat{B}$ pozorovatelnou B?

normální tvar

$$\hat{B} = B_{op}(\hat{\Phi}) = B_{op}(\hat{\Phi}^- + \hat{\Phi}^+)$$

└ nějaký konkrétní předpis na operátorech

lze vždy prokomutovat $\hat{\Phi}^-$ nalevo od $\hat{\Phi}^+$
⇒ lze zapsat v tvaru

$$\hat{B} = :B(\hat{\Phi}^-, \hat{\Phi}^+): \quad \text{pro nějakou } B(\phi_1, \phi_2)$$

tj. máme přiřazení

$$\hat{B} \to B(\phi_1, \phi_2) \to B(\phi) = B(\phi^-, \phi^+)$$

explicitně pomocí anih. a kr. operátorů

$$\hat{\Phi} = \underbrace{\sum_k \hat{a}_k^+ u^-}_{\hat{\Phi}^-} + \underbrace{\sum_k \hat{a}_k u^+}_{\hat{\Phi}^+}$$

$$\hat{B}_{op} = B_{op}\left(\sum_k \hat{a}_k^+ u^- + \sum_k \hat{a} u^+\right)$$

↑ sumy součinů $\hat{\Phi}$
= sumy součinů $\hat{a}_k^{+\,m_k}$ a $\hat{a}_l^{\,m_l}$ pro vše k, l, m_k, m_l
→ lze převést do norm. tvaru

$$= \sum_{m,n} b_{m,n} \underbrace{\hat{a}[u]^{+\,m}}_{\langle \hat{\Phi}, u\rangle^m} \underbrace{\hat{a}[u]^n}_{\langle u, \hat{\Phi}\rangle^n}$$

definujeme

$$B(\phi_1, \phi_2) = \sum_{m,n} b_{m,n} \langle \phi_1, u\rangle^m \langle u, \phi_2\rangle^n$$

zřejmě

$$\hat{B} = :B(\hat{\Phi}, \hat{\Phi}):$$

Platí

$B(\phi_1, \phi_2)$ - nezávisí na u_k
- definováno jednoznačně

$$B(\phi_1, \phi_2) = \frac{\langle coh\, \phi_1 | \hat{B} | coh\, \phi_2 \rangle}{\langle coh\, \phi_1 | coh\, \phi_2 \rangle}$$

$$= F[\hat{B}](\phi_1 \phi_2) \exp(-\langle \phi_1, \phi_2 \rangle)$$

důkaz:

$$\langle coh\, \phi_1 | \hat{B} | coh\, \phi_2 \rangle =$$

$$= \sum_{m,n} b_{mn} \langle coh\, \phi_1 | \hat{a}[u]^{+m} \hat{a}[u]^n | coh\, \phi_2 \rangle$$

$$= \sum_{m,n} b_{m,n} \langle \phi_1, u \rangle^m \langle u | \phi_2 \rangle^n \langle coh\, \phi_1 | coh\, \phi_2 \rangle$$

$$= B(\phi_1, \phi_2) \langle coh\, \phi_1 | coh\, \phi_2 \rangle$$

věrnost holomorf. repr. oper.

$$F[\hat{B}](\phi_1; \phi_2) = 0 \implies \hat{B} = :0: = 0$$

Př: $\hat{N}$ v norm. uspoř.

$$\hat{N} = :N(\hat{\Phi}): = :\mathcal{N}(\hat{\Phi}^-, \hat{\Phi}^+):$$

$$\mathcal{N}(\phi_1, \phi_2) = \frac{\langle \mathrm{coh}\,\phi_1 | \hat{N} | \mathrm{coh}\,\phi_2 \rangle}{\langle \mathrm{coh}\,\phi_1 | \mathrm{coh}\,\phi_2 \rangle} \overset{*}{=} \langle \phi_1, \phi_2 \rangle$$

$$\hat{N} = :\langle \hat{\Phi}, \hat{\Phi} \rangle: = -i\hat{\Phi}^- \cdot \partial\mathcal{F} \cdot \hat{\Phi}^+$$

$$= \sum_{k,l} \hat{a}_k^+ \langle \phi_k, \phi_l \rangle \hat{a}_l = \sum_k \hat{a}_k^+ \hat{a}_k$$

$$* = \frac{\sum_k \langle \mathrm{coh}\,\phi_1 | \hat{a}_k^+ \hat{a}_k | \mathrm{coh}\,\phi_2 \rangle}{\langle \mathrm{coh}\,\phi_1 | \mathrm{coh}\,\phi_2 \rangle} = \sum_k \langle \phi_1 | u_k \rangle \langle u_k | \phi_2 \rangle = \langle \phi_1, \phi_2 \rangle$$

Př: $\langle \mathrm{coh}\,\phi_1 | \mathrm{coh}\,\phi_2 \rangle = \exp(\langle \phi_1, \phi_2 \rangle)$

$$= \langle \mathrm{vac} | \exp(\hat{a}[\phi_1]) \exp(\hat{a}^+[\phi_2]) | \mathrm{vac} \rangle$$

$$= \exp(\tfrac{1}{2}\langle \phi_1, \phi_2 \rangle) \langle \mathrm{vac} | \exp(\hat{a}[\phi_1] + \hat{a}[\phi_2]^+) | \mathrm{vac} \rangle$$

$$= \exp(\langle \phi_1, \phi_2 \rangle) \langle \mathrm{vac} | \exp(\hat{a}[\phi_1]^+) \exp(\hat{a}[\phi_2]) | \mathrm{vac} \rangle$$

$$= \exp(\langle \phi_1, \phi_2 \rangle)$$

Př: $\hat{\mathbb{1}} = \int d\mu(\phi)\, |\mathrm{coh}\,\phi\rangle\langle \mathrm{coh}\,\phi|$

$$d\mu(\phi) = \exp(-\langle \phi, \phi \rangle)\, d\Gamma \qquad d\Gamma = \left(\mathrm{Det}\frac{\partial\mathcal{F}}{2\pi}\right)^{\frac{1}{2}}$$

$$\langle \mathrm{coh}\,\phi_1 | \mathrm{coh}\,\phi_2 \rangle \overset{?}{=} \int d\mu\, \langle \mathrm{coh}\,\phi_1 | \mathrm{coh}\,\phi \rangle \langle \mathrm{coh}\,\phi | \mathrm{coh}\,\phi_2 \rangle$$

$$= \int d\Gamma \exp(-\langle \phi, \phi \rangle + \langle \phi_1, \phi \rangle + \langle \phi, \phi_2 \rangle)$$

$$= \int \left(\mathrm{Det}\frac{\partial\mathcal{F}}{2\pi}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\phi \cdot \partial\mathcal{F} \cdot \phi + \phi \cdot \partial\mathcal{F} \cdot (\phi_1^- + \phi_2^+)\right)$$

$$= \exp\left(\frac{1}{2}(\phi_1^- + \phi_2^+) \cdot \partial\mathcal{F} \cdot (\phi_1^- + \phi_2^+)\right)$$

$$= \exp(\langle \phi_1, \phi_2 \rangle)$$

Př: Vakuum projektor v norm. uspoř.

$$|vac\rangle\langle vac| = :V[\hat{\Phi}]: = :V(\hat{\Phi}, \hat{\Phi}):$$

$$V(\phi_1, \phi_2) = \frac{\langle coh\,\phi_1|vac\rangle\langle vac|coh\,\phi_2\rangle}{\langle coh\,\phi_1|coh\,\phi_2\rangle} = \exp(-\langle\phi_1,\phi_2\rangle)$$

$$V(\phi) = \exp(-\langle\phi,\phi\rangle)$$

$$|vac\rangle\langle vac| = :\exp(-\langle\hat{\Phi},\hat{\Phi}\rangle):$$

ukážeme

$$\hat{a}_k\,|vac\rangle\langle vac| = 0$$

$$|vac\rangle\langle vac|vac\rangle = |vac\rangle$$

$$|vac\rangle\langle vac| = :\exp(-\langle\hat{\Phi},\hat{\Phi}\rangle): =$$

$$= :\exp(-\sum_k \hat{a}_k^+\hat{a}_k): $$

$$= :\prod_k \exp(-\hat{a}_k^+\hat{a}_k):$$

$$= \prod_k \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\hat{a}_k^{+n}\hat{a}_k^n =$$

$$= \sum_{m}\frac{(-1)^m}{m!}\hat{a}[u]^{+m}\,\hat{a}[u]^m$$

$$\hat{a}_l\;:\exp(-\langle\hat{\Phi},\hat{\Phi}\rangle): = :\Big(\prod_{k\neq l}\exp(-\hat{a}_k^+\hat{a}_k)\Big):\Big(\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\hat{a}_l\hat{a}_l^{+n}\hat{a}_l^n\Big)$$

$$= :\Big(\prod_{k\neq l}\exp(-\hat{a}_k^+\hat{a}_k)\Big):\Big(\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\big(\underbrace{\hat{a}_l^{+n}\hat{a}_l^{n+1}}_{\text{vyruší se}} + n\,\hat{a}_l^{+n-1}\underbrace{\hat{a}_l^n}_{n\to n+1}\big)\Big)$$

$$= 0$$

$$:\exp(-\langle\hat{\Phi},\hat{\Phi}\rangle):|vac\rangle = \sum_{m}\frac{(-1)^m}{m!}\hat{a}[u]^{+m}\hat{a}[u]^m|vac\rangle = |vac\rangle$$

$$m = \{0, \ldots\}$$