Tvorba kvant v 0+1 dimenzi

$$\ddot{\hat{\Phi}} + \omega^2(t)\,\hat{\Phi} = 0 \qquad f_1 \cdot \partial f \cdot f_2 = m(\dot{f}_1 f_2 - f_1 \dot{f}_2)\big|_t$$

↑ z akce vstoupí do $\langle , \rangle$

$$\omega \xrightarrow{t\to-\infty} \omega_i \qquad \omega \xrightarrow{t\to+\infty} \omega_f$$

přirozená volba vakua v i a f oblasti
– nejnižší stav Hamilt.

fukv. rozštěpení v i a f oblasti:

$$u_i^+ \xrightarrow{t\to-\infty} \frac{1}{\sqrt{2m\omega_i}} \exp(-i\omega_i t)$$

$$u_f^+ \xrightarrow{t\to+\infty} \frac{1}{\sqrt{2m\omega_f}} \exp(-i\omega_f t)$$

u_i^+, u_i^- tvoří C-bázi v $\mathcal{S}^{C}$

$u_i = u_i^+ + u_i^-$ tvoří R-bázi v $\mathcal{S}_{J_i}$

obdobně u_f^+, u_f^-, u

máme tedy Bogoljubovovu transf.

$$u_f^+ = \alpha\, u_i^+ + \beta\, u_i^-$$

$$u_f^- = \beta^* u_i^+ + \alpha^* u_i^-$$

normování

$$1 = \langle u_f, u_f\rangle_f = |\alpha|^2 - |\beta|^2 \qquad 1 = \langle u_i, u_i\rangle_i$$

kreační a anih. oper

$$\hat{a} = \alpha^* \hat{a}_i - \beta^* \hat{a}_i^+$$

$$\hat{a}_f^+ = \alpha\, \hat{a}_i^+ - \beta\, \hat{a}_i$$

vakua (obdob. odvoz. vzorce se zám. f ↔ i)

$$|i\,\mathrm{vac}\rangle = \frac{1}{\sqrt{\alpha}} \exp\Big(-\frac{1}{2}\frac{\beta^*}{\alpha}\, \underbrace{\hat{a}_f^+\, \hat{a}_f^+}_{\frac{1}{2}\sum_{kl} V_{kl}\, \hat{a}_{fk}^+ \hat{a}_{fl}}\Big) |f\,\mathrm{vac}\rangle \quad \Leftarrow V_{kl} = -\sum_m \alpha^{-1}_{km} \beta^*_{ml}$$

počet kreov. část.

$$\langle i\,\mathrm{vac}|\hat{N}_f|i\,\mathrm{vac}\rangle = |\beta|^2$$

příklad

$$\omega(t) = \omega_i \qquad t<0$$
$$= \omega_f \qquad t>0$$

$$u_f^+ = \frac{1}{\sqrt{2m\omega_f}} \exp(-i\omega_f t) \qquad t>0$$

$$u_f^+ = \frac{\alpha}{\sqrt{2m\omega_i}} \exp(-i\omega_i t) + \frac{\beta}{\sqrt{2m\omega_i}} \exp(i\omega_i t) \qquad t<0$$

spojitost pro $t=0$

$$\frac{1}{\sqrt{\omega_f}} = \frac{\alpha}{\sqrt{\omega_i}} + \frac{\beta}{\sqrt{\omega_i}}$$

spojitost derivace

$$-i\sqrt{\omega_f} = -i\alpha\sqrt{\omega_i} + i\beta\sqrt{\omega_i}$$

$$\Downarrow \quad \sqrt{\frac{\omega_i}{\omega_f}} = \alpha+\beta \qquad \sqrt{\frac{\omega_f}{\omega_i}} = \alpha-\beta$$

$$\Downarrow \quad \alpha = \frac{1}{2}\left(\sqrt{\frac{\omega_i}{\omega_f}} + \sqrt{\frac{\omega_f}{\omega_i}}\right) \qquad \beta = \frac{1}{2}\left(\sqrt{\frac{\omega_i}{\omega_f}} - \sqrt{\frac{\omega_f}{\omega_i}}\right)$$

$$\frac{\beta^*}{\alpha} = \frac{\omega_i-\omega_f}{\omega_i+\omega_f} \qquad |\beta|^2 = \frac{(\omega_i-\omega_f)^2}{4\omega_i\omega_f}$$

Pohybující zrcadlo v $d=2$, $m=0$

$d=2$ Mink. prostor

$u = t - z$

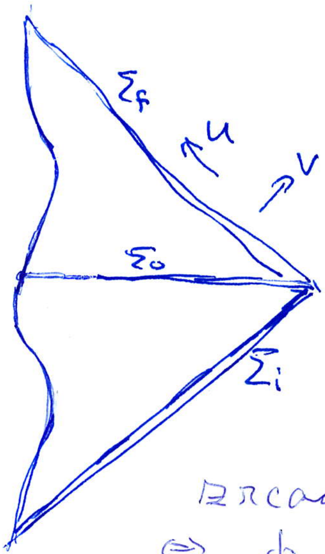
$v = t + z$

$g = -dt^2 + dz^2 = -\frac{1}{2} du \vee dv \qquad g^{\frac{1}{2}} = dt\,dz = \frac{1}{2} du\,dv$

$g^{-1} = -\frac{\partial}{\partial t}^2 + \frac{\partial}{\partial z}^2 = -2 \frac{\partial}{\partial u} \vee \frac{\partial}{\partial v}$

$\vec{F} = 4 \partial_u \partial_v = \partial_t^2 - \partial_z^2$

$\Rightarrow$ řešení $\phi_<(v) + \phi_>(u)$



zrcadlo $\Rightarrow$ $\phi = 0$

zrcadlo pohybující se podle

$t = T(\tau) \qquad$ tj $\qquad u = U(\tau)$

$z = Z(\tau) \qquad\qquad v = V(\tau)$

$$\left.\begin{aligned} \downarrow\; u &= \mathcal{U}(v) \\ v &= \mathcal{V}(u) \end{aligned}\right\} \text{ popis zrcadla}$$

Klein Gord. pole

$$\phi_1 \cdot \partial F[\Sigma_0] \cdot \phi_2 = \int_0^\infty dz \left(\partial_t \phi_1\, \phi_2 - \phi_1 \partial_t \phi_2\right)\Big|_{t=0}$$

$$\phi_1 \cdot \partial F[\Sigma_i] \cdot \phi_2 = \int_{-\infty}^{\infty} dv \left(\partial_v \phi_1\, \phi_2 - \phi_1 \partial_v \phi_2\right)\Big|_{u=-\infty}$$

* viz samostatné stránka

in módy

lze ignorovat na $u=-\infty$

$$u^+_{in\,\omega} = \frac{1}{\sqrt{4\pi\omega}} \left(\exp(-i\omega v) - \exp(-i\omega \mathcal{V}(u))\right)$$

$$u^+_{out\,\omega} = \frac{1}{\sqrt{4\pi\omega}} \left(\exp(-i\omega u) - \exp(-i\omega \mathcal{U}(v))\right)$$

lze ignorovat na $v=\infty$

správná normalizace

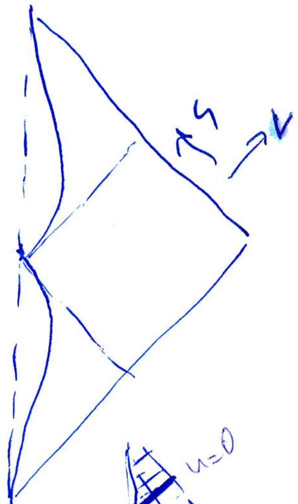
$$\langle u_{ik}, u_{il} \rangle_i = \frac{-i}{4\pi\sqrt{kl}} \int_{-\infty}^{\infty} dv \left(ik \exp(i(k-l)v) + il \exp(i(k-l)v)\right)$$

$$= \frac{1}{2} \frac{1}{\sqrt{kl}} (k+l)\, \delta(k-l) = \delta(k-l)$$

Příklad

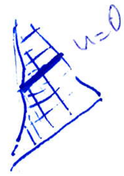
$$U(\tau) = \exp\left(\pm\frac{\beta}{2}\right)\tau \qquad \pm = \operatorname{sign}\tau$$

$$V(\sigma) = \exp\left(\mp\frac{\beta}{2}\right)\tau$$



tj. $\mathcal{U}(v) = \exp(s_v\beta)\, v \qquad s_v = \operatorname{sign} v$

$\mathcal{V}(u) = \exp(-s_u\beta)\, u \qquad s_u = \operatorname{sign} u$



$$u^+_{ik} = \frac{1}{\sqrt{4\pi k}}\left(\exp(-ikv) - \exp(-ik e^{-s_u\beta} u)\right)$$

$$u^+_{fl} = \frac{1}{\sqrt{4\pi l}}\left(\exp(-ilu) - \exp(-il e^{s_v\beta} v)\right)$$

chceme spočíst

$$\beta_{kl} = i\, u^+_{ik} \cdot \partial F[\xi] \cdot u^+_{fl}$$

nezávisí na Σ $\to$ zvolme $U = -\infty$

nedělat - viz alternativní výpočet na Σ_0

$$u^+_{ik} \overset{u=-\infty}{\approx} \frac{1}{\sqrt{4\pi k}} \exp(-ikv)$$

$$\partial_v u^+_{ik} \overset{u=-\infty}{\approx} -\frac{i}{\sqrt{4\pi}}\sqrt{k}\exp(-ikv)$$

$$u^+_{fl} \overset{u=-\infty}{\approx} -\frac{1}{\sqrt{4\pi l}} \exp(-il e^{s_v\beta} v)$$

$$\partial_v u^+_{fl} \overset{u=-\infty}{\approx} \frac{i}{\sqrt{4\pi}}\sqrt{l}\, e^{s_v\beta}\exp(-il e^{s_v\beta} v)$$

$$\beta_{kl} = -\frac{1}{4\pi}\int_{-\infty}^{\infty} \exp(-i(k+l e^{s_v\beta})v)\left(\sqrt{\frac{k}{l}} - \sqrt{\frac{l}{k}}\, e^{s_v\beta}\right) dv$$

$$= -\frac{1}{4\pi}\int_0^{\infty} \exp(i(k+l e^{-\beta})v)\left(\sqrt{\frac{k}{l}} - \sqrt{\frac{l}{k}}\, e^{-\beta}\right) dv$$

$$-\frac{1}{4\pi}\int_0^{\infty} \exp(-i(k+l e^{\beta})v)\left(\sqrt{\frac{k}{l}} - \sqrt{\frac{l}{k}}\, e^{\beta}\right) dv = *$$

Platí

$$\int_0^\infty \exp(\pm i\xi z)\,dz = \frac{\pm i}{\xi \pm i0} = \frac{1}{\mp i\xi + 0}$$

$$= \pm i\left(P\frac{1}{\xi} \mp i\pi\delta(\xi)\right) = \pi\delta(\xi) \pm i P\frac{1}{\xi}$$

tj.

$$* \quad \beta_{kl} = -\frac{1}{4\pi}\left[\left(\pi\,\cancelto{0}{\delta(k+le^{-\beta})} + iP\frac{1}{k+le^{-\beta}}\right)\frac{k-le^{-\beta}}{\sqrt{kl}}\right.$$

$$\left. + \left(\pi\,\cancelto{0}{\delta(k+le^{\beta})}_{k,l>0} - iP\frac{1}{k+le^{\beta}}\right)\frac{k-le^{\beta}}{\sqrt{kl}}\right]$$

$$= \frac{i}{4\pi}\frac{1}{\sqrt{kl}}\left[\frac{k-le^{\beta}}{k+le^{\beta}} + \frac{l-ke^{\beta}}{l+ke^{\beta}}\right]$$

$$= -\frac{i}{2\pi}\frac{1}{\sqrt{kl}}\frac{e^{\beta}-e^{-\beta}}{e^{\beta}+e^{-\beta}+\frac{k^2+l^2}{kl}} = -\frac{i}{2\pi}\frac{1}{\sqrt{kl}}\frac{\mathrm{sh}\,\beta}{\mathrm{ch}\,\beta + \frac{k^2+l^2}{2kl}}$$

$$n_k = \int_0^\infty |\beta_{k\ell}|^2 d\ell = \frac{1}{4\pi^2}\int_0^\infty \frac{1}{k\ell}\frac{\mathrm{sh}^2\beta}{(\mathrm{ch}\beta + \frac{k^2+\ell^2}{2k\ell})^2} d\ell = *$$

$$\ell = k\exp\alpha \qquad d\ell = k\exp\alpha\, d\alpha = \ell\, d\alpha \qquad \frac{k^2+\ell^2}{2k\ell} = \frac{1+\exp 2\alpha}{2\exp\alpha} = \mathrm{ch}\,\alpha$$

$$* = \frac{1}{4\pi^2}\frac{\mathrm{sh}^2\beta}{k}\int_{-\infty}^{\infty}\frac{1}{(\mathrm{ch}\beta + \mathrm{ch}\alpha)^2}d\alpha =$$

$$= \frac{1}{2\pi^2}\frac{1}{k}\left(\frac{\beta}{\mathrm{th}\beta} - 1\right) \approx \frac{1}{2\pi^2}\frac{1}{k}\left(\overset{\mathrm{ch}\beta}{(1+\tfrac{1}{2}\beta^2)}\;\overset{\frac{\beta}{\mathrm{sh}\beta}}{(1-\tfrac{1}{6}\beta^2)} - 1\right)$$

$$= \frac{1}{2\pi^2}\frac{1}{k}\frac{1}{3}\beta^2 \qquad \beta \ll 1$$

Obdobně

$$\alpha_{k\ell} = -i\, u_{ik}\cdot\partial f\cdot u^{+}_{f\ell} = \frac{i}{4\pi k\ell}\left(\frac{k+\ell e^\beta}{k-\ell e^\beta + i0} + \frac{\ell + k e^\beta}{\ell - k e^\beta + i0}\right)$$

$$= \frac{1}{2}\left(\delta(\ell e^{\frac{\beta}{2}} - k e^{-\frac{\beta}{2}}) + \delta(k e^{\frac{\beta}{2}} - \ell e^{-\frac{\beta}{2}})\right)$$

$$+ \frac{1}{4\pi\sqrt{k\ell}}\, \mathcal{P}\left(\frac{\ell + k e^\beta}{\ell - k e^\beta} + \frac{k + \ell e^\beta}{k - \ell e^\beta}\right)$$

$$\alpha_{kl} = -i\, u^-_{ik} \cdot \partial f \cdot u^+_{fl}$$

$$= -\frac{i}{4\pi\sqrt{kl}} \int_{-\infty}^{\infty} dv \Big[ik \exp(ikv) \exp(-il\, e^{s_v\beta} v) + il\, e^{s_v\beta} \exp(ikv) \exp(-il\, e^{s_v\beta} v) + il\, e^{s_v\beta}\, \underbrace{v\, \cancel{2\delta(v)}}_{\text{z derivace } s_v} \exp(-il\, e^{s_v\beta} v) \Big]$$

$$= \frac{1}{4\pi\sqrt{kl}} \int_0^\infty (k + l e^{-\beta}) \exp(-i(k - l e^{-\beta})v)\, dv$$

$$+ \frac{1}{4\pi\sqrt{kl}} \int_0^\infty (k + l e^{\beta}) \exp(i(k - l e^{\beta})v)\, dv$$

$$= \frac{1}{4\pi\sqrt{kl}} (k + l e^{-\beta}) \left(\pi\, \delta(k - l e^{-\beta}) - i P \frac{1}{k - l e^{-\beta}} \right)$$

$$+ \frac{1}{4\pi\sqrt{kl}} (k + l e^{\beta}) \left(\pi\, \delta(k - l e^{\beta}) + i P \frac{1}{k - l e^{\beta}} \right)$$

$$e^\beta \delta(l - k e^\beta)$$

$$= \frac{1}{2}\sqrt{\frac{k}{l}} \left(\delta(k - l e^{-\beta}) + (k - l e^{\beta}) \right)$$

$$+ \frac{1}{4\pi\sqrt{kl}}\, P \Big(\underbrace{\frac{k + l e^{\beta}}{k - l e^{\beta}} - \frac{k + l e^{-\beta}}{k - l e^{-\beta}}}_{\frac{k + l e^{\beta}}{k - l e^{\beta}} + \frac{l + k e^{\beta}}{l - k e^{\beta}}} \Big)$$

$$= \frac{1}{2} \left(\delta(l e^{\frac{\beta}{2}} - k e^{-\frac{\beta}{2}}) + \delta(k e^{\frac{\beta}{2}} - l e^{-\frac{\beta}{2}}) \right)$$

$$+ \frac{1}{4\pi\sqrt{kl}}\, P \left(\frac{l + k e^{\beta}}{l - k e^{\beta}} + \frac{k + l e^{\beta}}{k - l e^{\beta}} \right)$$

$$= \frac{i}{4\pi\sqrt{kl}} \left(\frac{k + l e^{\beta}}{k - l e^{\beta} + i0} + \frac{l + k e^{\beta}}{l - k e^{\beta} + i0} \right)$$

Alternativní výpočet na Σ_0

$$u^+_{ik} = \frac{1}{\sqrt{4\pi k}}\left(\exp(-ikv) - \exp(-ike^{-s_u\beta}u)\right)$$

$$u^+_{f\ell} = \frac{1}{\sqrt{4\pi \ell}}\left(\exp(-i\ell u) - \exp(-i\ell e^{s_v\beta}v)\right)$$

$t=0 \qquad u=-z \qquad v=z \qquad z\in\mathbb{R}^+$

$\qquad s_u=-1 \qquad s_v=+1$

$$u^+_{ik}\big|_\Sigma = \frac{1}{\sqrt{4\pi k}}\left(\exp(-ikz) - \exp(ike^{\beta}z)\right)$$

$$\partial_t u^+_{ik}\big|_\Sigma = \frac{-i\sqrt{k}}{\sqrt{4\pi}}\left(\exp(-ikz) - e^{\beta}\exp(ike^{\beta}z)\right)$$

$$u^+_{f\ell}\big|_\Sigma = \frac{1}{\sqrt{4\pi \ell}}\left(\exp(i\ell z) - \exp(-i\ell e^{\beta}z)\right)$$

$$\partial_t u^+_{f\ell}\big|_\Sigma = \frac{-i\sqrt{\ell}}{\sqrt{4\pi}}\left(\exp(i\ell z) - e^{\beta}\exp(-i\ell e^{\beta}z)\right)$$

$$\beta_{k\ell} = i\, u^+_{ik} \cdot \partial f[\Sigma_0] \cdot u^+_{f\ell} =$$

$$= \frac{1}{4\pi}\int_0^\infty dz \Big[\sqrt{\frac{k}{\ell}} \left(\exp(-ikz) - e^{\beta}\exp(ike^{\beta}z)\right)\left(\exp(i\ell z) - \exp(-i\ell e^{\beta} z)\right)$$

$$- \sqrt{\frac{\ell}{k}} \left(\exp(-ikz) - \exp(ike^{\beta}z)\right)\left(\exp(i\ell z) - e^{\beta}\exp(-i\ell e^{\beta} z)\right)\Big]$$

$$= \frac{1}{4\pi}\int_0^\infty dz \Big[\exp(-i(k-\ell)z)\left(\sqrt{\frac{k}{\ell}} - \sqrt{\frac{\ell}{k}}\right)$$

$$+ \exp(ie^{\beta}(k-\ell)z)\left(\sqrt{\frac{k}{\ell}} - \sqrt{\frac{\ell}{k}}\right)e^{\beta} \qquad z \to -e^{\beta} z$$

$$- \exp(-i(k+e^{\beta}\ell)z)\left(\sqrt{\frac{k}{\ell}} - e^{\beta}\sqrt{\frac{\ell}{k}}\right)$$

$$- \exp(i(ke^{\beta}+\ell)z)\left(e^{\beta}\sqrt{\frac{k}{\ell}} - \sqrt{\frac{\ell}{k}}\right)\Big]$$

$$= \frac{1}{4\pi}\int_{-\infty}^{\infty} dz\, \exp(-i(k-\ell)z)\, \frac{k-\ell}{\sqrt{k\ell}}$$

$$- \frac{1}{4\pi\sqrt{k\ell}} \int_0^\infty dz \Big[\exp(-i(k+e^{\beta}\ell)z)\,(k-e^{\beta}\ell)$$

$$+ \exp(i(e^{\beta}k+\ell)z)\,(e^{\beta}k-\ell)\Big]$$

$$= \cancel{\frac{1}{2}\delta(k-\ell)\frac{k-\ell}{\sqrt{k\ell}}}^{\,\to 0} - \frac{1}{4\pi\sqrt{k\ell}}\Big[$$

$$\left(\pi\, \cancel{\delta(k+e^{\beta}\ell)}^{\,0} - iP\frac{1}{k+e^{\beta}\ell}\right)(k-e^{\beta}\ell)$$

$$\left(\pi\, \cancel{\delta(e^{\beta}k+\ell)}^{\,0} + iP\frac{1}{e^{\beta}k+\ell}\right)(e^{\beta}k-\ell)\Big]$$

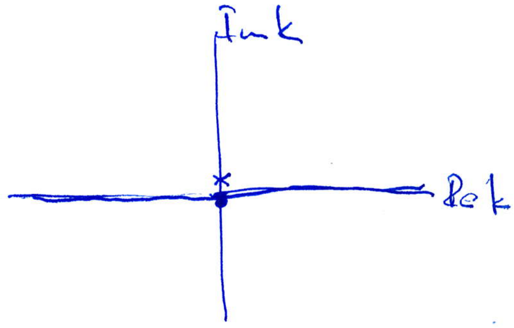
$$= \frac{i}{4\pi\sqrt{k\ell}}\left[\frac{k-e^{\beta}\ell}{k+e^{\beta}\ell} - \frac{e^{\beta}k-\ell}{e^{\beta}k+\ell}\right]$$

$$= -\frac{i}{2\pi}\frac{1}{\sqrt{k\ell}}\, \frac{\sinh\beta}{\cosh\beta + \frac{k^2+\ell^2}{2k\ell}}$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \Theta(z)\exp(-ikz)\,dz =$$

$$= \frac{1}{\sqrt{2\pi}}\,\frac{i}{k}\Big[\exp(-ikz)\Big]_0^{\infty} =$$

$$= \frac{1}{\sqrt{2\pi}}\,\frac{-i}{k-i0}$$



$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\,\frac{-i}{k-i0}\exp(ikz)\,dz =$$

$$= \frac{-i}{2\pi}\int_{-\infty}^{\infty}\frac{1}{k-i0}\exp(ikz)\,dz$$

$$= \left\{\begin{array}{ll} z<0 \Rightarrow \mathrm{Im}\,k<0 & = 0 \\ z>0 \Rightarrow \mathrm{Im}\,k>0 & = 2\pi i\,\frac{-i}{2\pi} \end{array}\right\} = \Theta(z)$$

$$\int_0^{\infty}\exp(\pm ikz)\,dz = \frac{\pm i}{k\pm i0} = \frac{-1}{\pm ik-0} = \frac{1}{\mp ik+0}$$

$$= \pm i\left(P\frac{1}{k} \mp i\pi\,\delta(k)\right) =$$

$$= \pi\,\delta(k) \pm i\,P\frac{1}{k}$$

$$\frac{1}{x\pm i0} = P\frac{1}{x} \mp i\pi\,\delta(x)$$

*

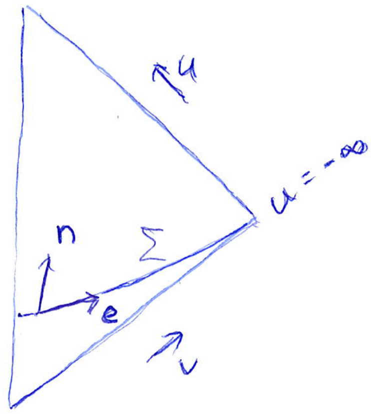
K-G forma na ploše $u=-\infty$

Σ_i jako limita plochy Σ

$u = u(v)$ $\quad$ u klesající

$v = v$ $\quad$ tj $u'<0$

budeme limitit $u \to -\infty$



tečný vektor k Σ

$$\tilde{e} = \left(\frac{\partial}{\partial v} + u'\frac{\partial}{\partial u}\right)$$

$$\tilde{e}^2 = -u' > 0 \qquad\qquad g = -\tfrac{1}{2}(du\,dv + dv\,du)$$

normované diáda

$$e = \left(\frac{1}{\sqrt{-u'}}\frac{\partial}{\partial v} - \sqrt{-u'}\frac{\partial}{\partial u}\right)$$

$$n = \left(\frac{1}{\sqrt{-u'}}\frac{\partial}{\partial v} + \sqrt{-u'}\frac{\partial}{\partial u}\right)$$

duální kovektorová diáda

$$\epsilon = \frac{1}{2}\left(\sqrt{-u'}\,dv - \frac{1}{\sqrt{-u'}}\,du\right)$$

$$\nu = \frac{1}{2}\left(\sqrt{-u'}\,dv + \frac{1}{\sqrt{-u'}}\,du\right)$$

restrikce ϵ na Σ

$$\epsilon|_\Sigma = \frac{1}{2}\left(\sqrt{-u'}\,dv - \frac{1}{\sqrt{-u'}}\,u'dv\right) = \sqrt{-u'}\,dv$$

K-G. forma na Σ

$$\phi_1 \cdot \partial f[\Sigma] \cdot \phi_2 = \int_\Sigma (n\cdot d\phi_1\,\phi_2 - \phi_1\,n\cdot d\phi_2)\,\epsilon|_\Sigma$$

$$= \int_\Sigma \left[\left(\frac{1}{\sqrt{-u'}}\frac{\partial\phi_1}{\partial v} + \sqrt{-u'}\frac{\partial\phi_1}{\partial u}\right)\phi_2 - \phi_1\left(\frac{1}{\sqrt{-u'}}\frac{\partial\phi_2}{\partial v} + \sqrt{-u'}\frac{\partial\phi_2}{\partial u}\right)\right]\sqrt{-u'}\,dv$$

$$= \int_\Sigma \left[\left(\frac{\partial\phi_1}{\partial v} - u'\frac{\partial\phi_1}{\partial u}\right)\phi_2 - \phi_1\left(\frac{\partial\phi_2}{\partial v} - u'\frac{\partial\phi_2}{\partial u}\right)\right]dv$$

limita $u \to -\infty = \text{konst}$ $\quad$ $u' \to 0$

$$= \int_\Sigma \left[\frac{\partial\phi_1}{\partial v}\,\phi_2 - \phi_1\frac{\partial\phi_2}{\partial v}\right]dv$$

Kosmologická kreace částic

FRW prostorově plochý model

$$g = -dt^2 + a^2(t)\,q = a^2(\eta)\left(-d\eta^2 + q\right)$$

η konformní čas $\qquad dt = a(\eta)\,d\eta$

q 3D euklidovské metrika

$\dot{} \equiv \frac{\partial}{\partial t} \qquad \mathring{} \equiv \frac{\partial}{\partial \eta} \qquad \vec{x}$ bod v 3D

konformní interakce se křivostí + hmotnost

$$\Box + m^2 + \xi R$$

kde $\Box = \partial_t^2 + \frac{3\dot{a}}{a}\partial_t - \frac{1}{a^2}\Delta \qquad \Delta$ Laplace vůči q

módy

vlastní funkce Δ

$$\exp(i\vec{k}\cdot\vec{x}) \qquad \vec{k} \text{ vlnový vektor}$$

módy

$$u^{+}_{\vec{k}}(t,\vec{x}) = \frac{1}{a}\,\chi_k \exp(i\vec{k}\cdot\vec{x})$$

pohybová rovnice pro χ_k

$$\mathring{\mathring{\chi}}_k + \omega_k^2(\eta)\,\chi_k = 0$$

kde $\omega_k^2(\eta) = \vec{k}^2 + m^2 a^2(\eta) - (1-6\xi)\frac{\mathring{\mathring{a}}}{a}(\eta)$

normalizace

$$\langle u_{\vec{k}}, u_{\vec{l}}\rangle = -i\int \frac{1}{a}\left(\mathring{u}^{-}_{\vec{k}}\,u^{+}_{\vec{l}} - u^{-}_{\vec{k}}\,\mathring{u}^{+}_{\vec{l}}\right) a^3\, d^3\vec{x} =$$

$$= -i\left(\mathring{\chi}^*_k \chi_l - \chi^*_k \mathring{\chi}_l\right)\int \exp(-i(\vec{k}-\vec{l})\cdot\vec{x})\, d^3\vec{x}$$

$$= -i\left(\mathring{\chi}^*_k \chi_k - \chi^*_k \mathring{\chi}_k\right)(2\pi)^3\,\delta(\vec{k}|\vec{l})$$

$$\langle u_{\vec{k}}, u_{\vec{l}}\rangle = \delta_{\vec{k},\vec{l}} \iff -iV\left(\mathring{\chi}^*_k \chi_k - \chi^*_k \mathring{\chi}_k\right) = 1$$

$\Rightarrow$ normalizace $\chi_k \qquad V = (2\pi)^3\,\delta(\vec{k}|\vec{k}) = \int d^3\vec{x}$

in-out úloha

$$\omega(\eta) \xrightarrow{\eta \to \pm\infty} \omega_{f/i} = \text{konst} \qquad \text{(předpoklad)}$$

$$\chi_{f/i\,k} \to \frac{1}{\sqrt{2V\omega_{f/i}}} \exp(-i\omega_{f/i}\,\eta)$$

viz normaliz.   pos.-frekv. závislost

Bogoljubovova transformace

$$u^+_{f\vec{k}} = \sum_{\vec{l}} \left(\alpha_{\vec{k}\vec{l}}\, u^+_{i\vec{l}} + \beta_{\vec{k}\vec{l}}\, u^-_{i\vec{l}} \right)$$

$$\frac{1}{a} \chi_{fk} \exp(i\vec{k}\cdot\vec{x}) = \sum_{\vec{l}} \frac{1}{a} \left(\alpha_{\vec{k}\vec{l}}\, \chi_{il} \exp(i\vec{l}\cdot\vec{x}) + \beta_{\vec{k}\vec{l}}\, \chi^*_{il} \exp(-i\vec{l}\cdot\vec{x}) \right)$$

prostorová závislost $\Rightarrow$

$\vec{k} = \vec{l} \quad \rightsquigarrow \quad \alpha$ člen

$\vec{k} = -\vec{l} \quad \rightsquigarrow \quad \beta$ člen

$\Downarrow$

$$\alpha_{\vec{k}\vec{l}} = \alpha_k\, \delta_{\vec{k},\vec{l}} \qquad \beta_{\vec{k}\vec{l}} = \beta_k\, \delta_{\vec{k},-\vec{l}}$$

α_k, β_k závisí jen na $k = |\vec{k}|$ platí $|\alpha_k|^2 - |\beta_k|^2 = 1$

"diagonalizovaná" Bogoljub. transf.

$$u^+_{f\vec{k}} = \alpha_k\, u^+_{i\vec{k}} + \beta_k\, u^-_{i-\vec{k}}$$

$$\hat{a}_{f\vec{k}} = \alpha_k\, \hat{a}_{i\vec{k}} + \beta^*_k\, \hat{a}^+_{i-\vec{k}}$$

α_k, β_k závisí na konkrétním tvaru $a(\eta)$ skrze pohybovou rovnici pro χ_k

2-částicové amplitudy (pouze nenulové)

$$V_{\vec{k},-\vec{k}} = -\frac{\beta^*_k}{\alpha_k} \qquad \text{kreace částic s opačnou hyb.}$$

$$\Lambda_{\vec{k},-\vec{k}} = \frac{\beta_k}{\alpha_k} \qquad \text{anihilace částic s opač. hyb.}$$

$$I_{\vec{k},\vec{k}} = \frac{1}{\alpha_k} \qquad \text{zachování částice s danou hyb.}$$

interpretace $|i\,vac\rangle$ v řeči f-částic

$$|i\,vac\rangle = \langle f\,vac|i\,vac\rangle \exp\left(\frac{1}{2}\sum_{k,l} V_{kl}\, \hat{a}^{+}_{fk}\, \hat{a}^{+}_{fl}\right)|f\,vac\rangle$$

$$= \langle f\,vac|i\,vac\rangle \exp\left(-\frac{1}{2}\sum_{k} \frac{\beta^{*}_{k}}{\alpha_{k}}\, \hat{a}^{+}_{fk}\, \hat{a}^{+}_{f-k}\right)|f\,vac\rangle$$

$$\langle f\,vac|i\,vac\rangle = \left(\prod_{k} \alpha_{k}\right)^{-\frac{1}{2}}$$

Kreace částic díky expanzi

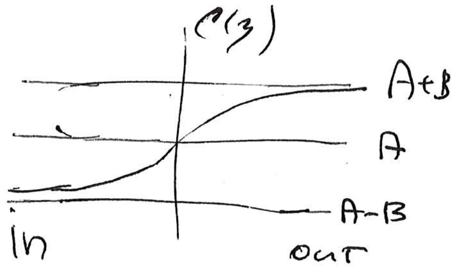
(Birrell-Davies p.61)

$$g = -dt^2 + a(t)\,dx^2$$

$$d\eta = \frac{1}{a}\,dt \qquad \text{1+1 Dimenze}$$

$$g = a^2(\eta)\left(-d\eta^2 + dx^2\right)$$

$$C(\eta) = a^2(\eta)$$



příklad $C(\eta) = A + B\,\mathrm{th}(\rho\eta)$

$$C(\eta) \to A \pm B \qquad \eta \to \pm\infty$$

$$u_k(\eta, x) = \frac{1}{\sqrt{2\pi}}\,\chi_k(\eta)\exp(ikx)$$

$$\frac{d^2}{d\eta^2}\chi_k(\eta) + \left(k^2 + C(\eta)m^2\right)\chi_k(\eta) = 0$$

řešení v řeči hypergeom. fcí

(*)
$$u^{+}_{ik}(\eta, x) = \frac{1}{\sqrt{4\pi\omega_i}}\exp\left[ikx - i\omega_+\eta - i\frac{\omega_-}{\rho}\ln(2\,\mathrm{ch}\,\rho\eta)\right]$$
$$\,_2F_1\left(1 + i\frac{\omega_-}{\rho},\ i\frac{\omega_-}{\rho},\ 1 - i\frac{\omega_i}{\rho};\ \tfrac{1}{2}(1 + \mathrm{th}\,\rho\eta)\right)$$
$$\xrightarrow{\eta \to -\infty} \frac{1}{\sqrt{4\pi\omega_i}}\exp(ikx - i\omega_i\eta)$$

$$u^{+}_{fk}(\eta, x) = \frac{1}{\sqrt{4\pi\omega_f}}\exp\left[ikx - i\omega_+\eta - i\frac{\omega_-}{\rho}\ln(2\,\mathrm{ch}\,\rho\eta)\right]$$
$$\,_2F_1\left(1 + i\frac{\omega_-}{\rho},\ i\frac{\omega_-}{\rho},\ 1 + i\frac{\omega_f}{\rho};\ \tfrac{1}{2}(1 - \mathrm{th}\,\rho\eta)\right)$$
$$\xrightarrow{\eta \to +\infty} \frac{1}{\sqrt{4\pi\omega_f}}\exp(ikx - i\omega_f\eta)$$

porovnáním prostorové závislosti u^+_{ik} u^+_{fk} vede k diagonalizaci Bogoljub. koef. Klein-Gordonova forma dá koef. α_k β_k (s pomocí vzorečků pro hypergeom. fce F_1)

$$u^+_{fk}(\eta, x) = \alpha_k u^+_{ik}(\eta, x) + \beta_k u^-_{i\,-k}(\eta, x)$$

kde

$$\alpha_k = \sqrt{\frac{\omega_f}{\omega_i}} \frac{\Gamma(1 - i\frac{\omega_i}{\varrho})\Gamma(-i\frac{\omega_f}{\varrho})}{\Gamma(-i\frac{\omega_+}{\varrho})\Gamma(1 - i\frac{\omega_+}{\varrho})}$$

$$\beta_k = \sqrt{\frac{\omega_f}{\omega_i}} \frac{\Gamma(1 - i\frac{\omega_i}{\varrho})\Gamma(i\frac{\omega_f}{\varrho})}{\Gamma(i\frac{\omega_-}{\varrho})\Gamma(1 + i\frac{\omega_-}{\varrho})}$$

tj. $\alpha_{kl} = \alpha_k \delta(k-l)$ $\quad \beta_{kl} = \beta_k \delta(k+l)$

$$|\alpha_k|^2 = \frac{\operatorname{sh}^2 \frac{\pi\omega_+}{\varrho}}{\operatorname{sh}\frac{\pi\omega_i}{\varrho}\operatorname{sh}\frac{\pi\omega_f}{\varrho}}$$

$$|\beta_k|^2 = \frac{\operatorname{sh}^2(\frac{\pi\omega_-}{\varrho})}{\operatorname{sh}\frac{\pi\omega_i}{\varrho}\operatorname{sh}\frac{\pi\omega_f}{\varrho}}$$

$*$

$$\omega_i^2 = k^2 + m^2(A-B) \qquad \omega_i = \omega_+ - \omega_-$$

$$\omega_f^2 = k^2 + m^2(A+B) \qquad \omega_f = \omega_+ + \omega_-$$

$$\omega_\pm = \frac{1}{2}(\omega_f \pm \omega_i)$$