Rindlerovy souřadnice

inerc. souř. t, z

nulové souř. u, v

$$u = t - z \qquad t = \frac{u+v}{2}$$
$$v = t + z \qquad z = \frac{v-u}{2}$$

nulové Rindlerovy souř. U, V

$$u = s_u \exp(-U) \qquad U = -\log|u| \qquad s_u = \operatorname{sign} u$$
$$v = s_v \exp(V) \qquad V = \log|v| \qquad s_v = \operatorname{sign} v$$

Rindlerovy souř. τ, b

$$U = \tau - \ln b \qquad b > 0$$
$$V = \tau + \ln b \qquad \tau \in \mathbb{R}$$

$$t^2 - z^2 = uv = s_u s_v \exp(V-U) = s_u s_v b^2 = \overset{F,P}{\underset{L,R}{\pm}} b^2$$

R, L oblasti

$$t = \frac{v+u}{2} = \overset{R}{\underset{L}{\pm}} \frac{1}{2}(e^V - e^{-U}) = \pm b \,\mathrm{sh}\, \tau \qquad \mathrm{th}\,\tau = \frac{t}{z}$$

$$z = \frac{v-u}{2} = \pm \frac{1}{2}(e^V + e^{-U}) = \pm b \,\mathrm{ch}\, \tau \qquad b^2 = z^2 - t^2$$

F, P oblasti

$$t = \frac{v+u}{2} = \overset{F}{\underset{P}{\pm}} (e^V + e^{-U}) = \pm b \,\mathrm{ch}\, \tau \qquad \mathrm{th}\,\tau = \frac{z}{t}$$

$$z = \frac{v-u}{2} = \pm (e^V - e^{-U}) = \pm b \,\mathrm{sh}\, \tau \qquad b^2 = t^2 - z^2$$

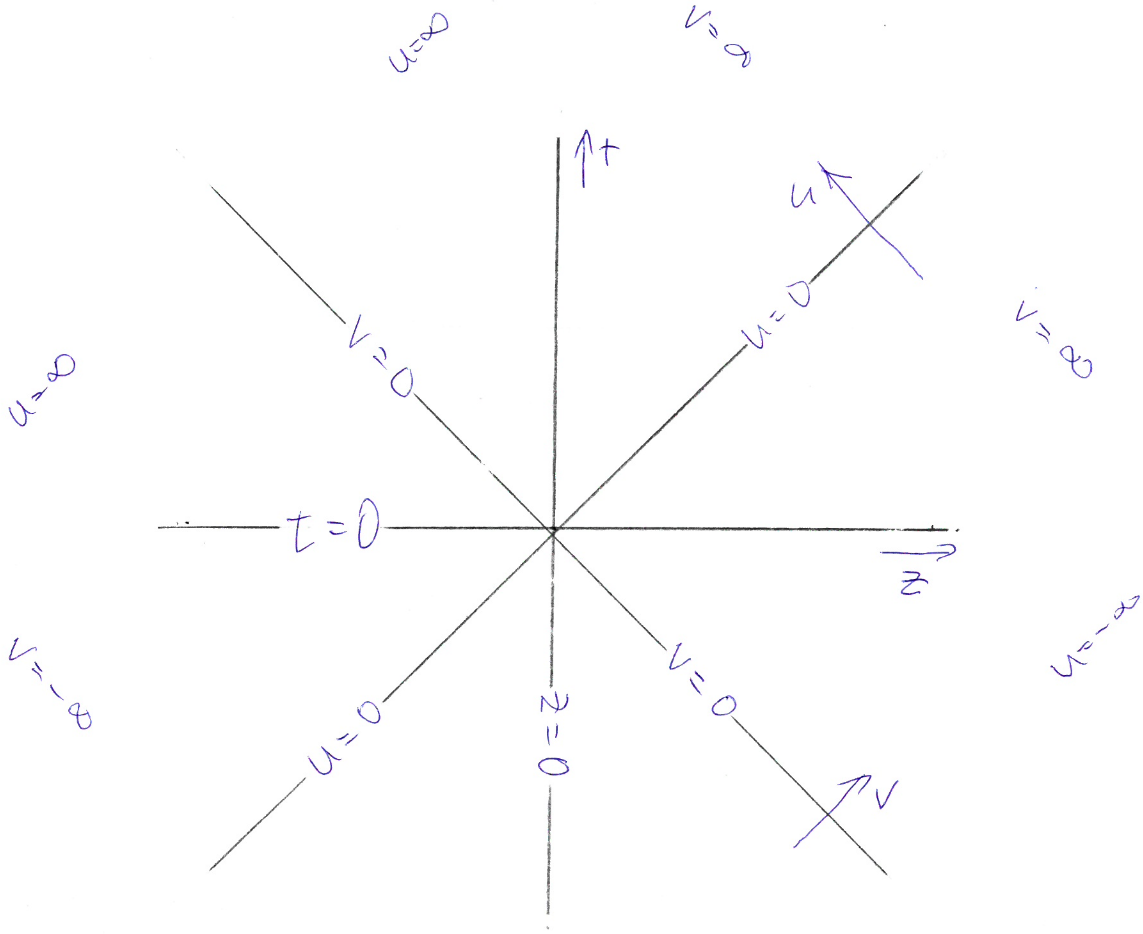
Killingův vekt. boostu

$$\xi = \frac{\partial}{\partial \tau} = z\frac{\partial}{\partial t} + t\frac{\partial}{\partial z} = v\frac{\partial}{\partial v} - u\frac{\partial}{\partial u} = \frac{\partial}{\partial V} + \frac{\partial}{\partial U}$$

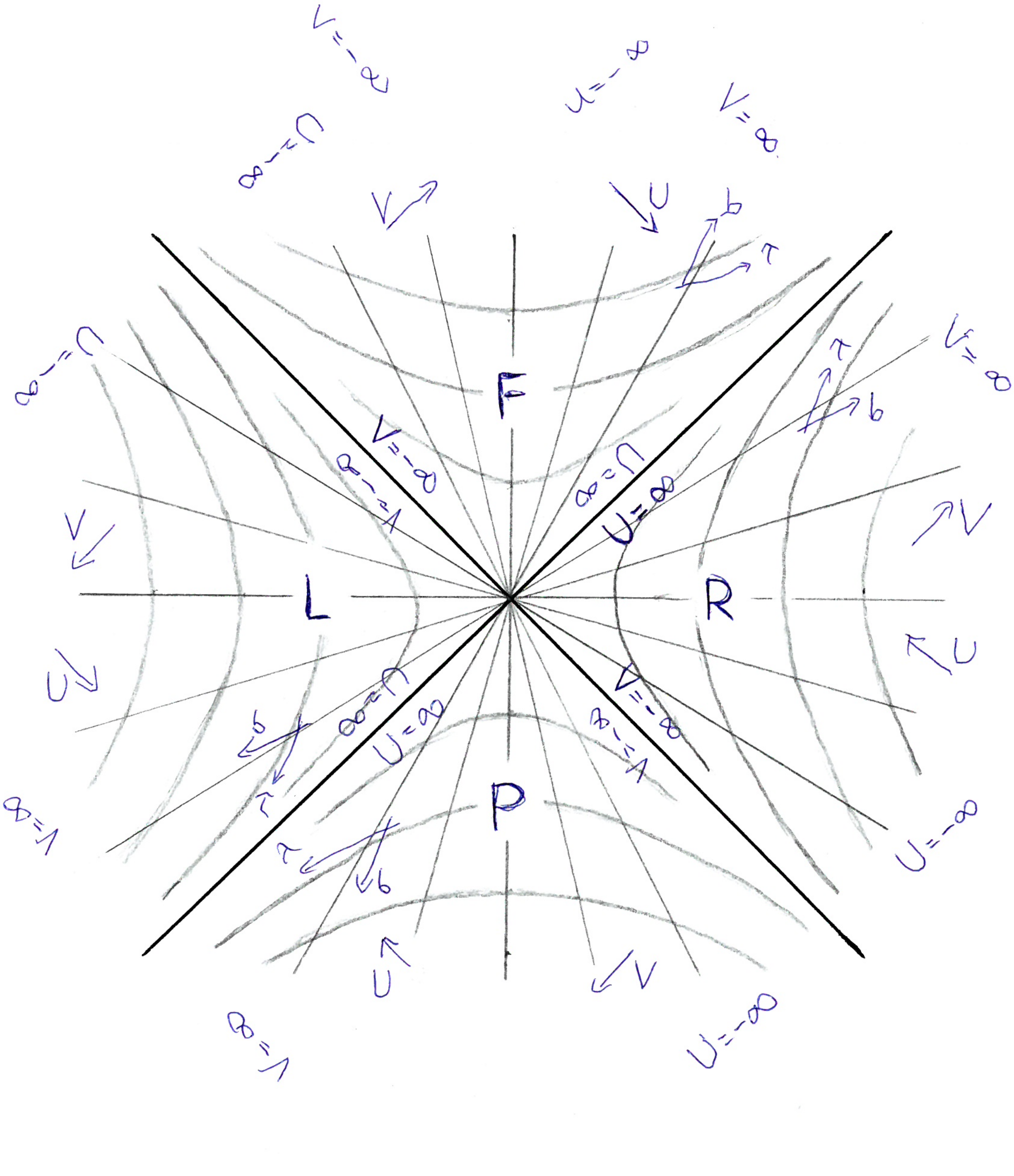
metrika

$$g = -dt^2 + dz^2 + dx^2 + dy^2 = \overset{R,L/F,P}{\mp} b^2 d\tau^2 \pm db^2 + dx^2 + dy^2$$
$$= -\frac{1}{2} du \vee dv + dx^2 + dy^2 = \frac{1}{2} s_u s_v b^2 dU \vee dV + dx^2 + dy^2$$

$$\Box\phi = \left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]\phi = \left[-4\frac{\partial}{\partial u}\frac{\partial}{\partial v} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]\phi \qquad \hat{F} = [-\Box + m^2]$$



u=∞
V=∞
t
u
V=0
u=0
V=∞
u=∞
t=0
z
u=-∞
V=-∞
V=0
u=0
z=0
V



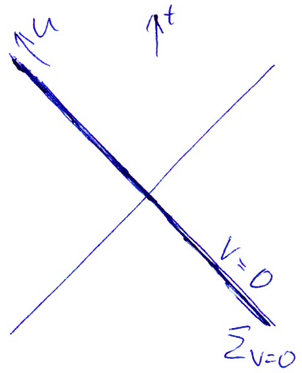
V=-∞
u=-∞
V=∞
U=-∞
V
U
b
r
F
V=∞
V=-∞
U=∞
L
R
P
U=-∞
V=∞

Minkowského částicová interpretace

positivně frekv. módy

$$\frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{\omega}}\exp(-i\omega t + i\vec{k}\cdot\vec{r})$$

$$\omega^2 - \vec{k}^2 = m^2$$



positivně frekv. vůči t na $\Sigma_{v=0}$

$\Updownarrow$

positivně frekv. vůči u na $\Sigma_{v=0}$

(na $v=0$ platí $u = 2t$)

$\Updownarrow$

řešení holomorfní v polorovině $\mathrm{Im}\, u < 0$

($\exp(-i\omega u) \to 0$ pro $\omega > 0$ $\mathrm{Im}\, u < 0$ $|u| \to \infty$)

Paley-Wienerova věta

$$\psi(\tau) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \tilde{\psi}(\omega)\, e^{-i\omega t}\, d\omega$$

$\psi(\tau)$ holomorfní v dolní polorovině komplexního τ, tj. $\mathrm{Im}\,\tau < 0$

kvadr. integrabilní podél $\tau = t - i\beta$ $t \in \mathbb{R}$ $\beta > 0$

$\Updownarrow$

$\tilde{\psi}(\omega)$ kvadraticky integr. podél $\omega \in \mathbb{R}^+$

Rindlerovské částicové interpretace

oblast R

- statická s Kill. vekt ξ
- kauzálně samostatná
- lze zvolit pos. fr. módy vůči ξ defin. jen v R

$$w^+_{Rk} = \exp(-i\Omega_k \tau)\, \vartheta_{Rk}(b, x, y) \qquad \Omega_k > 0$$

derivace podél ξ

$$\mathcal{L}_\xi w^+_{Rk} = -i\Omega_k w^+_{Rk} \qquad \Omega_k > 0$$

na $\Sigma_{v=0}$ je $\xi = -u\frac{\partial}{\partial u} = \frac{\partial}{\partial U}$

⇒ Rindl. pos. fr. módy v R se na $\Sigma_{v=0}$ chovají

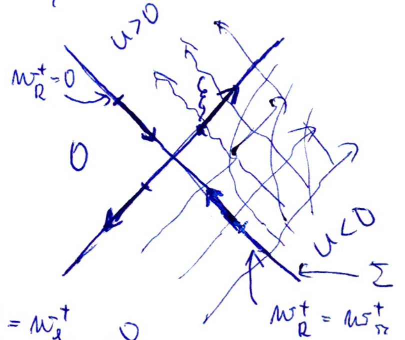
$$w^+_{Rk} \underset{\Sigma_{v=0}}{\approx} \exp(-i\Omega_k U)\, f_k(x,y)$$

lze rozšířit do celého prostoročasu

oblasti F, P - nenulové pomocí poh. rov.

oblast L - nulové

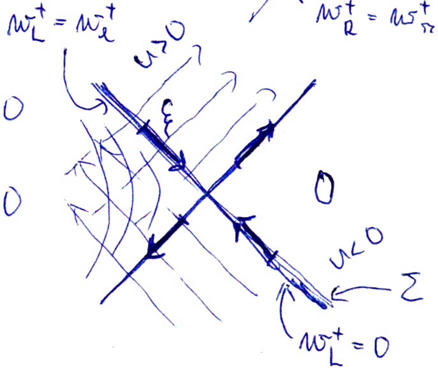
$$w^+_{Rk} \underset{\Sigma_{v=0}}{\approx} \begin{cases} \exp(-i\Omega_k U)\, f_k(x,y) & u<0 \\ 0 & u>0 \end{cases}$$



obdobné módy v L

$$w^+_{Lk} \underset{\Sigma_{v=0}}{\approx} \begin{cases} 0 & u<0 \\ \exp(i\Omega_k U)\, f^*_k(x,y) & u>0 \end{cases}$$

opačné znaménko kvůli opačnému směru ξ v oblasti L



báze pos. fr. módů v celém prostoročasu

w^+_{Lk} $\quad w^+_{Rk}$ $\qquad$ index $[\Omega k]$ (+ x, y směry)

jiné rozštěpení než standardní Minkowského

$$\varphi^+_{RL} \neq \varphi^+_M$$

Vezmeme do $\mathcal{G}^+_{0r}$ patří kombinace

$$m^+_{Rk} = \frac{1}{\sqrt{1-e^{-2\pi\Omega_k}}}\left(w^+_{Rk} + e^{-\pi\Omega_k} w^-_{Lk}\right)$$

$$= \frac{1}{\sqrt{2\,\mathrm{sh}\,\pi\Omega_k}}\left(e^{\frac{\pi\Omega_k}{2}} w^+_{Rk} + e^{-\frac{\pi\Omega_k}{2}} w^-_{Lk}\right)$$

$$\overset{\Sigma}{=} \frac{1}{\sqrt{e^{2\pi\Omega_k}-1}} \exp(-i\Omega_k U)\left(e^{\pi\Omega_k}\Theta(-u) + \Theta(u)\right) f_k$$

$$= \frac{1}{\sqrt{e^{2\pi\Omega_k}-1}} \exp\left[i\Omega_k(\ln|u| - i\pi\,\Theta(-u))\right] f_k$$

$$= \frac{1}{\sqrt{e^{2\pi\Omega_k}-1}} \exp\left[i\Omega_k \ln(u-i0)\right] f_k$$

holomorfní v polorovině $\mathrm{Im}\,u<0$

$\Rightarrow$ pozitivně frekv. vůči u

$$m^+_{Lk} = \frac{1}{\sqrt{1-e^{-2\pi\Omega_k}}}\left(w^+_{Lk} + e^{-\pi\Omega_k} w^-_{Rk}\right)$$

$$= \frac{1}{\sqrt{2\,\mathrm{sh}\,\pi\Omega_k}}\left(e^{\frac{\pi\Omega_k}{2}} w^+_{Lk} + e^{-\frac{\pi\Omega_k}{2}} w^-_{Rk}\right)$$

$$= \frac{1}{\sqrt{1-e^{-2\pi\Omega_k}}} \exp(i\Omega_k U)\left(\Theta(u) + e^{-\pi\Omega_k}\Theta(-u)\right) f^*_k$$

$$= \frac{1}{\sqrt{1-e^{-2\pi\Omega_k}}} \exp\left[-i\Omega_k(\ln|u| - i\pi\,\Theta(u))\right] f^*_k$$

$$= \frac{1}{\sqrt{1-e^{-2\pi\Omega_k}}} \exp\left[-i\Omega_k \ln(u-i0)\right] f^*_k$$

též holomorfní v polorov. $\mathrm{Im}\,u<0$

lze přepsat

$$m^+_{Rk} = \mathrm{ch}\,\chi_k\, w^+_{Rk} + \mathrm{sh}\,\chi_k\, w^-_{Lk} \qquad \mathrm{ch}\,\chi_k = \frac{1}{\sqrt{1-e^{-2\pi\Omega_k}}}$$

$$m^+_{Lk} = \mathrm{ch}\,\chi_k\, w^+_{Lk} + \mathrm{sh}\,\chi_k\, w^-_{Rk} \qquad \exp\pi\Omega_k = \coth\chi_k$$

Bogoljubovove transf.

$$m^+_{Rk} = \mathrm{ch}\,\chi_k\, w^+_{Rk} + \mathrm{sh}\,\chi_k\, w^-_{Lk}$$

$$m^+_{Lk} = \mathrm{ch}\,\chi_k\, w^+_{Lk} + \mathrm{sh}\,\chi_k\, w^-_{Rk}$$

$$\mathrm{ch}\,\chi_k = \frac{1}{\sqrt{1-e^{-\beta\Omega_k}}} \qquad \exp\left(\frac{\beta\Omega_k}{2}\right) = \coth\chi_k \qquad \beta = 2\pi$$

$$\alpha_{RkRl} = \mathrm{ch}\,\chi_k\,\delta_{kl} \qquad \alpha_{LkLl} = \mathrm{ch}\,\chi_k\,\delta_{kl} \qquad \alpha_{RkLl} = \alpha_{LkRl} = 0$$

$$\beta_{RkLl} = \mathrm{sh}\,\chi_k\,\delta_{kl} \qquad \beta_{LkRl} = \mathrm{sh}\,\chi_k\,\delta_{kl} \qquad \beta_{RkRl} = \beta_{LkLl} = 0$$

M a R, L Fock báze

$$\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_L$$

$|LR\,vac\rangle = |L\,vac\rangle \otimes |R\,vac\rangle$ vakuum odpovídající módům w_{Ak}

$\hat{a}_{Lk} = \hat{a}_{Lk} \otimes \mathbb{1}_R \qquad \hat{a}_{Rk} = \mathbb{1}_L \otimes \hat{a}_{Rk}$ anih. & kr. op. na $\mathcal{H}_L$ a $\mathcal{H}_R$

$|M\,vac\rangle$ vakuum odpovídající módům m_{Ak}

$$|M\,vac\rangle = \mathcal{O}^* \exp\left(\frac{1}{2}\hat{\Phi}\cdot\Lambda^*\cdot\hat{\Phi}\right)|LR\,vac\rangle$$

$$\frac{1}{2}\hat{\Phi}\cdot\Lambda^*\cdot\hat{\Phi} = \frac{1}{2}\sum_{k,l}\left(\Lambda^*_{RkRl}\,\hat{a}^+_{Rk}\hat{a}^+_{Rl} + \Lambda^*_{LkLl}\,\hat{a}^+_{Lk}\hat{a}^+_{Ll}\right) + \sum_{k,l}\Lambda^*_{LkRl}\,\hat{a}^+_{Lk}\hat{a}^+_{Rl}$$

$$\hat{\Phi} = \sum_k\left(\hat{a}^+_{Lk}\,w^-_{Lk} + \hat{a}^+_{Rk}\,w^-_{Rk} + \hat{a}_{Lk}\,w^+_{Lk} + \hat{a}_{Rk}\,w^+_{Rk}\right)$$

$$\Lambda^*_{RkLl} = \mathrm{th}\,\chi_k\,\delta_{kl} \qquad \Lambda^*_{RkRl} = \Lambda^*_{LkLl} = 0 \quad \Leftarrow \Lambda_{AkBl} = \sum_{M,m}\beta_{AkMm}\,\alpha^{-1}_{MmBl}$$

$$\Downarrow$$

$$\frac{1}{2}\hat{\Phi}\cdot\Lambda^*\cdot\hat{\Phi} = \sum_k \mathrm{th}\,\chi_k\,\hat{a}^+_{Lk}\hat{a}^+_{Rk} = \sum_k \exp\left(-\frac{\beta\Omega_k}{2}\right)\hat{a}^+_{Lk}\hat{a}^+_{Rk}$$

Minkowského vakuum v Rindlerovské interpretaci

$$|M\,vac\rangle = O^* \exp\left(\tfrac{1}{2}\hat{\Phi}\cdot\Lambda^*\cdot\hat{\Phi}\right)|LR\,vac\rangle$$

$$= O^* \exp\Big[\sum_k \exp\left(-\tfrac{\beta\Omega_k}{2}\right)\hat{a}^+_{Lk}\hat{a}^+_{Rk}\Big]|LR\,vac\rangle$$

$$= O^* \prod_k \exp\Big[e^{-\frac{\beta\Omega_k}{2}}\hat{a}^+_{Lk}\hat{a}^+_{Rk}\Big]|LR\,vac\rangle$$

$$= O^* \prod_k \sum_{m_k} \frac{1}{m_k!} e^{-\frac{\beta m_k\Omega_k}{2}}\hat{a}^{+m_k}_{Lk}\hat{a}^{+m_k}_{Rk}|LR\,vac\rangle$$

$$= O^* \sum_{\underbrace{m}_{\{m_k\}}} \frac{1}{m!}\exp\Big(-\tfrac{\beta}{2}\underbrace{E_m}_{\sum_k m_k\Omega_k}\Big)a_L^{+m}a_R^{+m}|LR\,vac\rangle$$

$$= O^* \sum_m \exp\left(-\tfrac{\beta}{2}E_m\right)|L\,w_k : m\rangle|R\,w_k : m\rangle$$

entanglovaný stav provazující stavy v $\mathcal{H}_L$ a $\mathcal{H}_R$

zúžení na oblast R

$$\hat{D}_R = \mathrm{Tr}_{\mathcal{H}_L}|M\,vac\rangle\langle M\,vac|$$

$$= |O|^2\sum_m\langle L\,w_k : m|\Big[\sum_m e^{-\frac{\beta E_m}{2}}|L\,w_k : m\rangle|R\,w_k : m\rangle\otimes$$

$$\otimes\sum_{m'} e^{-\frac{\beta E_{m'}}{2}}\langle L\,w_k : m'|\langle R\,w_k : m'|\Big]|L\,w_k : m\rangle$$

$$= |O|^2\sum_m e^{-\beta E_m}|R\,w_k : m\rangle\langle R\,w_k : m|$$

$$= \frac{1}{Z}\exp\left(-\beta\hat{H}_R\right)$$

kde $\hat{H}_R = \sum_m E_m |R\,w_k : m\rangle\langle R\,w_k : m|$

$$Z = \mathrm{Tr}\,\exp\left(-\beta\hat{H}_R\right) = \Big[\prod_k\left(1-e^{-\beta\Omega_k}\right)\Big]^{-1}$$

$$T = \frac{1}{\beta} = \frac{1}{2\pi} \quad \text{teplota}$$

dostáváme gaussovský stav o teplotě T v oblasti R

Normovací faktor

$$O = \left(\det_{\varphi_i} \alpha_{AkBl}\right)^{-\frac{1}{2}} = \left(\prod_k \operatorname{ch}\chi_k \operatorname{ch}\chi_k\right)^{-\frac{1}{2}}$$

$$= \left(\prod_k \left(1 - e^{-\beta\Omega_k}\right)\right)^{\frac{1}{2}}$$

$$|O|^2 = \prod_k \left(1 - e^{-\beta\Omega_k}\right)$$

$$Z = \operatorname{Tr} \exp(-\beta\hat{H}) = \sum_m \exp(-\beta E_m)$$

$$= \prod_k \sum_{m_k} \left(\exp(-\beta\Omega_k)\right)^{m_k} = \prod_k \frac{1}{1 - e^{-\beta\Omega_k}} = \frac{1}{|O|^2}$$

energie (hamiltonián) vůči času t

$\to$ vlastní čas pro pozorovatele

ve vzdál. $b = 1$

přeškálování na obecnou vzdálenost

$$b = \frac{1}{\alpha_{\kappa \text{ zrychlení}}} \qquad T = \frac{\alpha}{2\pi} \left(= \frac{\hbar\alpha}{2\pi c k_B}\right)$$

$$\frac{T}{1\,°K} \approx \frac{\alpha}{2{,}6 \cdot 10^{19}\, m\, s^{-2}}$$

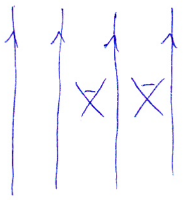
neměřitelné pro rozumná zrychlení

Analytická struktura Gr. fce

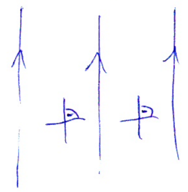
Wickova rotace v různých časech
– prodlužování v prostoru komplexních metrik(?)

Minkowského Gr. fce

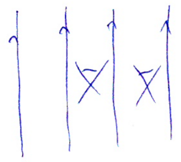
vakuová G_M | euklid. Gr. fce na rovině



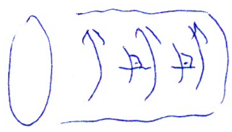
$\rightsquigarrow$



termální $G_{M\beta}$ | euklidovské Gr. fc. na válci



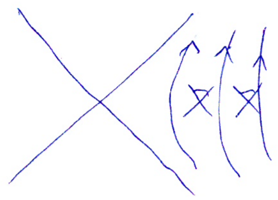
$\rightsquigarrow$



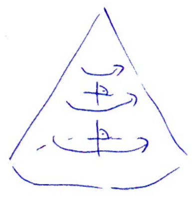
perioda β
(obvod cyl.)

Rindlerovské Gr. fce v oblasti R

termální $G_{R\beta}$ | euklidovské Gr. fc. na kuželu



$\rightsquigarrow$



perioda β
(vrchol. úhel)

pro $T=0$ $\beta=\infty$
nekonečný kužel

pro $\beta=2\pi$
vrchol kužele hladký
→ euklidovská rovina
→ shodné s Mink. vakuem

Módy v 1+1 dimenzi

$$[-\Box + m^2]\phi = 0 \qquad \text{tj.} \quad [\partial_t^2 - \partial_z^2 + m^2]\phi = 0$$

R-oblast

$$g = -b^2 d\tau^2 + db^2 \qquad g^{\frac{1}{2}} = b\, d\tau\, db$$

$$\Box\phi = g^{-\frac{1}{2}}\partial\cdot(g^{\frac{1}{2}}\, g^{-1}\cdot\partial\phi) = -\frac{1}{b}\left(b\frac{1}{b^2}\phi_{,\tau}\right)_{,\tau} + \frac{1}{b}(b\,\phi_{,b})_{,b}$$

$$= \left[\partial_b^2 + \frac{1}{b}\partial_b - \frac{1}{b^2}\partial_\tau^2\right]\phi$$

separace $\quad \phi = F(b)\exp(-i\Omega\tau)$

$$\Box\phi = \left(F_{,bb} + \frac{1}{b}F_{,b} + \frac{\Omega^2}{b^2}F\right)e^{-i\Omega\tau}$$

$\Downarrow$

$$F_{,bb} + \frac{1}{b}F_{,b} + \left(\frac{\Omega^2}{b^2} - m^2\right)F = 0 \qquad \text{Besselova rovnice}$$

řešení - kombinace Besselových fcí

$$K_{i\Omega}(mb) \quad \text{a} \quad I_{i\Omega}(mb)$$

potřeba provést analýzu pro $b\to 0$

$$K_{i\Omega},\ I_{i\Omega} \sim b^{\pm i\Omega}$$

→ chování na plochách $u=0$ a $v=0$

→ identifikace s předchozí diskuzí

Případ $m = 0$

pro $m^2 = 0$ volné šíření v 1+1 di–

$$\phi = \phi_>(u) + \phi_<(v) \qquad \phi_>, \phi_< \text{ libovolné}$$

řešení v τ, b souřadnicích

$$\phi = F(b)\exp(-i\Omega\tau)$$

$$F_{,bb} + \frac{1}{b}F_{,b} + \frac{\Omega^2}{b^2}F = 0 \iff b(bF_{,b})_{,b} + \Omega^2 F = 0$$

$$\Downarrow \quad F = N b^{\pm i\Omega}$$

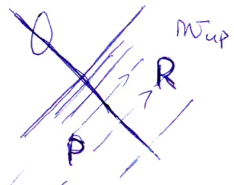
$$\phi = N\exp(-i\Omega(\tau \mp \ln b)) \qquad N = \frac{1}{\sqrt{4\pi\Omega}}$$

↑ souřadnice U nebo V

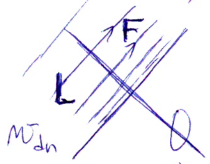
$\Rightarrow$ řešení $\phi_>$ nebo $\phi_<$

definujeme módy

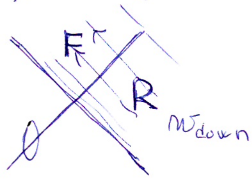
$$w^+_{up\Omega} = N_\Omega \exp(-i\Omega U)\,\Theta(-u) \qquad \neq 0 \text{ v } RP$$



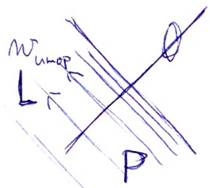
$$w^+_{dn\Omega} = N_\Omega \exp(i\Omega U)\,\Theta(u) \qquad \neq 0 \text{ v } LF$$



$$w^+_{down\Omega} = N_\Omega \exp(-i\Omega V)\,\Theta(v) \qquad \neq 0 \text{ v } RF$$



$$w^+_{umop\Omega} = N_\Omega \exp(i\Omega V)\,\Theta(-v) \qquad \neq 0 \text{ v } LP$$



definují Rindlerovskou část. interpretaci

w^+_{up}, w^+_{down} poz. fr. módy v oblasti R

$|R\, vac\rangle \quad \hat{a}_{up\Omega} \quad \hat{a}_{down\Omega} \quad \dots$

w^+_{dn}, w^+_{umop} poz. fr. módy v oblasti L

$|L\, vac\rangle \quad \hat{a}_{dn\Omega} \quad \hat{a}_{umop\Omega} \quad \dots$

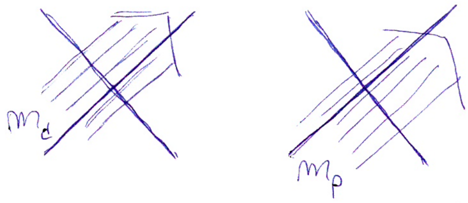
celý p.č.

$$|LR\, vac\rangle = |L\, vac\rangle \otimes |R\, vac\rangle$$

Minkowského módy

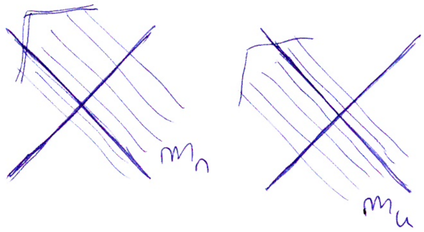
$m^+_{d\Omega} = \operatorname{ch}\chi_\Omega\, w^+_{dn\Omega} + \operatorname{sh}\chi_\Omega\, w^-_{up\Omega}$

$m^+_{p\Omega} = \operatorname{ch}\chi_\Omega\, w^+_{up\Omega} + \operatorname{sh}\chi_\Omega\, w^-_{dn\Omega}$



$m^+_{u\Omega} = \operatorname{ch}\chi_\Omega\, w^+_{unop\Omega} + \operatorname{sh}\chi_\Omega\, w^-_{down\Omega}$

$m^+_{n\Omega} = \operatorname{ch}\chi_\Omega\, w^+_{down\Omega} + \operatorname{sh}\chi_\Omega\, w^-_{unop\Omega}$



$\operatorname{ch}\chi_\Omega = \dfrac{1}{\sqrt{1-e^{-\beta\Omega}}}$ $\quad \operatorname{sh}\chi_\Omega = \dfrac{e^{-\frac{\beta\Omega}{2}}}{\sqrt{1-e^{-\beta\Omega}}}$ $\quad \operatorname{th}\chi_\Omega = e^{-\frac{\beta\Omega}{2}}$ $\quad \beta = 2\pi$

ortonormalita m_d a m_p (resp. m_u a m_n)
 - plyne z transf. vlastností Bogoljub. transf.

ortogonalita módů $m_d\, m_p$ vs. m_u, m_n
 - plyne ze závislosti na u ~~vs.~~ závislost na v

vycházi holomorfní závislost v dolní komplexní polorovině souřadnic u resp. v
$\Downarrow$
Minkowského positivně frekvenční módy

vskutku:

$m^+_{d\Omega} = \dfrac{N_\Omega}{\sqrt{1-e^{-\beta\Omega}}}(u-i0)^{-i\Omega}$ $\qquad m^+_{p\Omega} = \dfrac{N_\Omega}{\sqrt{e^{\beta\Omega}-1}}(u-i0)^{i\Omega}$

$m^+_{u\Omega} = \dfrac{N_\Omega}{\sqrt{e^{\beta\Omega}-1}}(v-i0)^{i\Omega}$ $\qquad m^+_{n\Omega} = \dfrac{N_\Omega}{\sqrt{1-e^{-\beta\Omega}}}(v-i0)^{-i\Omega}$

$N_\Omega = \dfrac{1}{\sqrt{2\pi\Omega}}$

mají:

$$m^+_{d\Omega} = \frac{N_\Omega}{\sqrt{1-e^{-\beta\Omega}}} \underbrace{\exp(i\Omega U)}_{-\log|u|} \underbrace{\left(\theta(u) + e^{-\pi\Omega}\theta(-u)\right)}_{\exp(-\pi\theta(-u))}$$

$$= \frac{N_\Omega}{\sqrt{1-e^{-\beta\Omega}}}\exp\left(-i\Omega\left(\log|u| - i\pi\theta(u)\right)\right) =$$

$$= \frac{N_\Omega}{\sqrt{1-e^{-\beta\Omega}}}\exp\left(-i\Omega\log(u-i0)\right) = \frac{N_\Omega}{\sqrt{1-e^{-\beta\Omega}}}(u-i0)^{-i\Omega}$$

$$m_d^+ = \phi_0 \, \mathrm{ch}\,X \left(\overset{m_{dn}^+}{\exp(+i\Omega U)\,\Theta(u)} + \mathrm{th}\,X \overset{m_{up}^-}{\exp(+i\Omega U)\,\Theta(-u)} \right)$$

$$= \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \exp(i\Omega U) \left(\Theta(u) + e^{-\pi\Omega}\Theta(-u) \right)$$

$$\exp(-\pi\Omega\Theta(-u))$$

$$= \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \exp(i\Omega(U + i\pi\,\Theta(-u))$$

$$= \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \exp(-i\Omega(\ln|u| - i\pi\,\Theta(-u))$$

$$= \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \exp\left[-i\Omega \ln(u - i0)\right] =$$

$$= \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} (u - i0)^{-i\Omega}$$

$$m_p^+ = \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \left(\overset{e^{-\pi\Omega}}{\mathrm{th}\,X} \overset{m_{dn}^-}{\exp(-i\Omega U)\Theta(u)} + \overset{m_{up}^+}{\exp(-i\Omega U)}\,\Theta(-u) \right)$$

$$= \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \exp(-i\Omega U \quad - \pi\Omega\Theta(u))$$

$$\frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \exp(i\Omega(\ln|u| + i\pi\,\Theta(-u) + i\pi))$$

$$\exp(i\Omega \ln(u - i0) \quad - \pi\Omega)$$

$$= \frac{\phi_0 e^{-\pi\Omega}}{\sqrt{1-e^{-2\pi\Omega}}} (u - i0)^{i\Omega}$$

$$m_u = \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \left(\overset{m_{up}^+}{\exp(i\Omega V)\,\Theta(-v)} + e^{-\pi\Omega} \overset{m_{dn}^-}{\exp(i\Omega V)}\,\Theta(v) \right)$$

$$= \frac{\phi_0}{\sqrt{1-e^{-2\pi\Omega}}} \exp\left(i\Omega V \frac{-\pi\Omega\,\Theta(v)}{+\pi\Omega\,\Theta(-v) - \pi\Omega}\right)$$

$$= \frac{\phi_0 e^{-\pi\Omega}}{\sqrt{1-e^{-2\pi\Omega}}} \exp(+i\Omega(\ln|v| - i\pi\,\Theta(-v)))$$

$$m_n^+ = \frac{\phi_0}{\sqrt{}} \left(\overset{m_{down}^+}{\exp(-i\Omega V)\,\Theta(v)} + e^{-\pi\Omega} \overset{m_{up}^-}{\exp(-i\Omega V)}\,\Theta(-v) \right)$$

$$= \frac{\phi_0}{\sqrt{}} \exp(-i\Omega V - \pi\Omega\,\Theta(-v))$$

$$= \frac{\phi_0}{\sqrt{}} \exp(-i\Omega(\ln|v| - i\pi\,\Theta(-v)))$$

$$\exp(-i\Omega \ln(v - i0)) \quad = \frac{\phi_0}{\sqrt{}} (v - i0)^{-i\Omega}$$

normalizace Rindlerových módů

např. w_{up}

$$\langle w_{up\Omega}, w_{up\Omega'} \rangle_{Rindl} = -i\, w^-_{up\Omega} \cdot \partial\!\!\!\!\!\;\overset{\leftrightarrow}{} \cdot w^+_{up\Omega'}$$

$\Sigma_{v=0}$

$$= -i\frac{1}{2}\int_{-\infty}^{\infty} du \left(\partial_u w^-_{up\Omega}\, w^+_{up\Omega'} - w^-_{up\Omega}\, \partial_u w^+_{up\Omega'} \right) \Theta(-u)$$

pouze $(-\infty,0) \to U \in \mathbb{R}$

$$= -i\frac{1}{2}\int_{-\infty}^{\infty} dU \left(\partial_U w^-_{up\Omega}\, w^+_{up\Omega'} - w^-_{up\Omega}\, \partial_U w^+_{up\Omega'} \right)$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} dU \left[\Omega N_\Omega N^*_{\Omega'} \exp(i(\Omega-\Omega')U) + \Omega' N_\Omega N^*_{\Omega} \exp(i(\Omega-\Omega')U) \right]$$

$$= \frac{1}{2}\left(2\pi\Omega N_\Omega N^*_{\Omega'}\, \delta(\Omega-\Omega') + 2\pi\Omega' N_\Omega N^*_{\Omega'}\, \delta(\Omega-\Omega') \right) =$$

$$= 2\pi\Omega\, |N_\Omega|^2\, \delta(\Omega-\Omega')$$

$$\Rightarrow \quad N_\Omega = \frac{1}{\sqrt{2\pi\Omega}}$$

rozklad K-G formy pro $m=0$

$$\phi = \phi_>(u) + \phi_<(v)$$

$$\partial_t\phi = \phi'_> + \phi'_< \qquad \partial_z\phi = -\phi'_> + \phi'_< \quad \Rightarrow \quad \begin{aligned} \phi'_> &= \partial_t\phi + \partial_z\phi \to \phi_> \\ \phi'_< &= \partial_t\phi - \partial_z\phi \to \phi_< \end{aligned}$$

$$\phi\cdot\partial\!\!\!\!\!\;\overset{\leftrightarrow}{}\cdot\psi = \int(\partial_t\phi\,\psi - \phi\,\partial_t\psi)\,dz = \int\left[(\phi'_> + \phi'_<)(\psi_> + \psi_<) - (\phi_> + \phi_<)(\psi'_> + \psi'_<)\right]dz$$

$$= \int[\phi'_>\psi_> - \phi_>\psi'_>]\,dz + \int[\phi'_<\psi_< - \phi_<\psi'_<]\,dz$$

$$+ \int[\phi'_>\psi_< - \phi_>\psi'_<]\,dz + \int[\phi'_<\psi_> - \phi_<\psi'_>]\,dz$$

$$= \phi_>\cdot\partial\!\!\!\!\!\;\overset{\leftrightarrow}{}_>\cdot\psi_> + \phi_<\cdot\partial\!\!\!\!\!\;\overset{\leftrightarrow}{}_<\cdot\psi_<$$

$$+ \int\underbrace{[-\partial_z\phi_>\,\psi_< - \phi_>\,\partial_z\psi_<]}_{-\partial_z(\phi_>\psi_<)}dz + \int\underbrace{[\partial_z\phi_<\,\psi_> + \phi_<\,\partial_z\psi_<]}_{\partial_z(\phi_<\psi_>)}dz$$

$$= \phi_>\cdot\partial\!\!\!\!\!\;\overset{\leftrightarrow}{}_>\cdot\psi_> + \phi_<\cdot\partial\!\!\!\!\!\;\overset{\leftrightarrow}{}_<\cdot\psi_<$$

$$\phi_>\cdot\partial\!\!\!\!\!\;\overset{\leftrightarrow}{}_>\cdot\psi_> = \int[\partial_u\phi_>\,\psi_> - \phi_>\,\partial_u\psi_>]\,dz = \frac{1}{2}\int[\partial_u\phi_>\,\psi_> - \phi_>\,\partial_u\psi_>]\,du$$

$$\phi_<\cdot\partial\!\!\!\!\!\;\overset{\leftrightarrow}{}_<\cdot\psi_< = \int[\partial_v\phi_<\,\psi_< - \phi_<\,\partial_v\psi_<]\,dz = \frac{1}{2}\int[\partial_v\phi_<\,\psi_< - \phi_<\,\partial_v\psi_<]\,dv$$