Unruh-DeWitt detectors in cosmological spacetimes

UTF Special Topics

Lecture 3

Dr Aindriú Conroy

Institute of Theoretical Physics,

Faculty of Mathematics & Physics,

Charles University,

Prague.

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UNIVERZITA KARLOVA
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Outline

1. What is Quantum Field Theory on curved spacetime?

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2. What is the Unruh-DeWitt particle detector model?

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2. What is the Unruh-DeWitt particle detector model?
3. How do we model the temperature of the detector?
4. Analyse how the detector behaves in de Sitter space.

What is Quantum Field Theory in Curved Spacetime?

Quantum Field Theory in Curved Spacetime (QFTCS)

Introduction

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- QFTCS is a framework to understand how quantum fields behave as they propagate through a classical spacetime.
- In this sense, QFTCS is an approximation of a quantum theory of gravity where the quantum nature of fields and the effects of gravitation are important but the quantum nature of gravity itself is assumed to be negligible.

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QFTCS has enjoyed particular success in analysing quantum phenomena such as

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3. Unruh effect
   - This is the radiation experienced by uniformly accelerated observers.

Quantum Field Theory in Curved Spacetime (QFTCS)

Three Concepts

1. No unique vacuum
2. No well-defined notion of a particle
3. The Unruh-DeWitt detector model gives an *operational* meaning to the notion of a particle

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   (a) No natural choice which corresponds to the physical absence of ‘particles’
   (b) This is true even in flat space were the vacuum is chosen not because it is uniquely defined but because it agrees with all measuring devices passing along an inertial trajectory.
   (c) While a ‘particle’ may be observed by a detector tuned to one vacuum, it may not be observed by a detector tuned to another vacuum.

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[1] This is a consequence of the lack of global symmetries in curved space. In particular, one cannot identify a global time function to distinguish between positive frequency and negative frequency modes leading to an ambiguity in the particle concept

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2. No well-defined notion of a particle[1]
   (a) The canonical path of QFT is to treat fields rather than particles as the fundamental object of interest
   (b) However, we needn't entirely dispense with the particle concept as...

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  a particle is what a particle detector detects!

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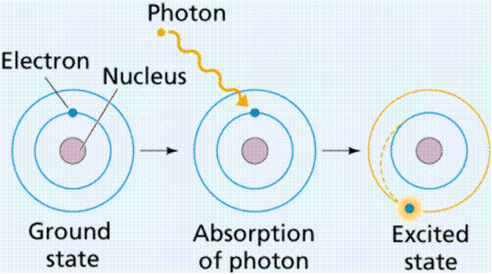

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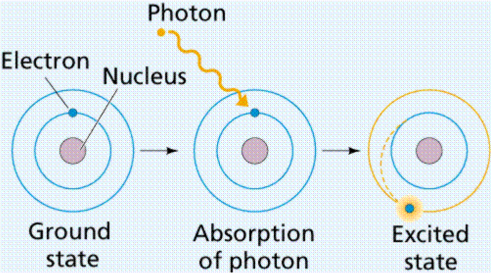

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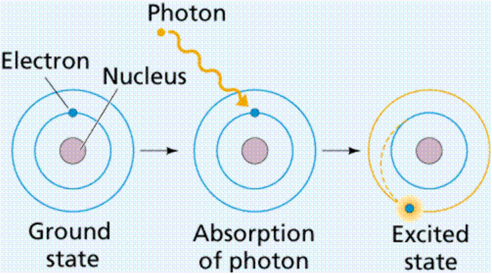

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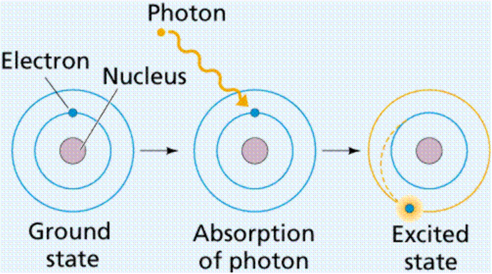

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- We interpret this atomic excitation as a detector registering a particle.
- Conversely, the detector can de-excite by emitting quanta.

How do we interpret the response?

Particle Detector Theory & Response

Suppose that the particle detector travels along a world line $x^\mu(\tau)$. Interaction between the detector and the quantum field $\hat{\varphi}(x)$ is governed by the Hamiltonian[2]

$$H_{int} = c\chi(\tau)\hat{\mu}(\tau)\hat{\varphi}(x).$$

Interaction is turned on and off via the *switching function* $\chi(\tau)$. The probability that the detector will transition from ground state to excited state is described by

$$P(\omega) = c^2|\langle E|\mu(0)|E_0\rangle|^2\mathcal{F}(\omega),$$

where the *response function* is defined via

$$\mathcal{F}(\omega) = 2\lim_{\epsilon\to 0^+}\Re\int_{-\infty}^{\infty} du\,\chi(u)\int_0^{\infty} ds\,\chi(u-s)e^{-i\omega s}W_\epsilon(u,u-s).$$

[2] c is a coupling constant and $\hat{\mu}(\tau)$ the detector's monopole moment operator.

J. Louko, A. Satz, Classical and Quantum Gravity 25 055012 (2018).

Sharp-switching limit

$$\mathcal{F}(\omega) = 2 \lim_{\epsilon\to 0^+} \Re \int_{-\infty}^{\infty} du\, \chi(u) \int_0^{\infty} ds\, \chi(u-s) e^{-i\,\omega\, s} W_\epsilon(u, u-s).$$

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- We must initially assume that the switching function χ is smooth (and of compact support) so that the response function is regular
- We also assume that $W(x; x')$ follows the Hadamard singularity structure, i.e. singular behaviour at $x = x'$ which require the $i\epsilon-$regularisation.

Sharp-switching limit

$$\mathcal{F}(\omega) = \frac{1}{2\pi^2}\int_{\tau_0}^{\tau} du \int_0^{u-\tau_0} ds \left(\frac{\cos\omega s}{a(u)a(u-s)(\Delta x)^2} + \frac{1}{s^2}\right) + \frac{1}{2\pi^2}\ln\left(\frac{\Delta\tau}{\delta}\right) - \frac{\omega}{4\pi}\Delta\tau + C_1 + \mathcal{O}\left(\frac{\delta}{\Delta\tau}\right),$$

- The limit of sharp-switching ($\delta \to 0$) helps 'distil' transient effects associated with turning on the detector.

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- This will be our main object of interest and we interpret it as the *rate of particle detection* of quantum particles that are 'produced' as a result of interaction with the quantum field.
- We can use this to understand quantum phenomena associated with the curvature of the spacetime (*Hawking effect*) or the acceleration of the detector itself (*Unruh effect*)

How do we model the temperature of the detector?

A comoving detector in de Sitter space

Analytic example

Consider a transition rate of the form

$$\dot{\mathcal{F}}_\tau(\omega) = \frac{1}{2\pi^2}\int_0^{\Delta\tau} ds\left(\frac{\cos\omega s}{\sigma^2(\tau,s)} + \frac{1}{s^2}\right) + \frac{1}{2\pi^2\Delta\tau} - \frac{\omega}{4\pi},$$

where we have defined $\sigma^2(\tau,s) \equiv a(\tau)a(\tau-s)(\Delta x)^2$ s.t. $(\Delta x)^2 \equiv \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$. Equivalently, we may write

$$\dot{\mathcal{F}}_\tau(\omega) = \frac{1}{2\pi^2}\int_0^{\infty} ds\left(\frac{\cos\omega s}{\sigma^2(\tau,s)} + \frac{1}{s^2}\right) + J_\tau - \frac{\omega}{4\pi},$$

where we define the 'fluctuating tail'

$$J_\tau \equiv -\frac{1}{2\pi^2}\int_{\Delta\tau}^{\infty} \frac{\cos\omega s}{\sigma^2(\tau,s)} ds,$$

which vanishes at the limit $\Delta\tau \to \infty$.

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Next, by adding and subtracting a $\cos(\omega s)/s^2$ term in the integrand, we can re-write this like so

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De Sitter space in FLRW coordinates is characterised by the scale factor $a(t) = e^{Ht}$, where H is now the Hubble constant.

This ensures that the 'world function' is independent of proper time, i.e. $\sigma^2(\tau,s) = \sigma^2(s)$, a simplification which is present also in the case of stationary (and static) black hole spacetimes.

A comoving detector in de Sitter space

Analytic result

For a comoving detector $t = \tau$, we have

$$\eta(\tau) = \int \frac{d\tau}{a(\tau)} = -H^{-1} e^{-H\tau}, \qquad \sigma^2(s) = -4H^{-2} \sinh^2\left(\frac{Hs}{2}\right),$$

and $r(\tau) = r_0$. After redefining the variable of integration s, we can write

$$\dot{\mathcal{F}}_\tau^{\text{corr}}(\omega) \equiv \frac{H}{8\pi^2} \int_{-\infty}^{\infty} ds\, e^{-2i\omega s/H} \left(-\frac{1}{\sinh^2 s} + \frac{1}{s^2}\right) + J_\tau,$$

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which, evaluated at $\Delta\tau \to \infty$, gives

$$\dot{\mathcal{F}}_\infty(\omega) = \frac{\omega}{2\pi} \left(\frac{1}{e^{\omega/T_{dS}} - 1}\right), \quad T_{dS} = H/2\pi,$$

where T_{dS} is the *de Sitter temperature*, i.e. the temperature an inertial observer in de Sitter space will read on their thermometer.

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We have also made some connection between the detector and temperature.

Next Question: How do we model the temperature of the detector for finite times?

A Kodama detector in de Sitter space

Analytic result

For a Kodama detector, we have a less simple relation beween cosmic time t and proper time τ, i.e. $\tau = \sqrt{V_{dS}}t$ where $V_{dS} \equiv 1 - H^2K^2$. We then have

$$a(\tau) = e^{\frac{H\tau}{\sqrt{V_{dS}}}}, \quad \eta(\tau) = -\frac{1}{H}e^{-\frac{H\tau}{\sqrt{V_{dS}}}}, \quad r(\tau) = \frac{\sqrt{1 - V_{dS}}}{H}e^{-\frac{H\tau}{\sqrt{V_{dS}}}},$$

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from which we find

$$\dot{\mathcal{F}}_\tau(\omega) \equiv \frac{\omega}{2\pi}\left[\frac{1}{e^{\omega/T^{\mathrm{loc}}_{dS}} - 1}\right], \quad T^{\mathrm{loc}}_{dS} \equiv \frac{H}{2\pi\sqrt{V_{dS}}},$$

as $\Delta\tau \to \infty$.

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as $\Delta\tau \to \infty$.

This is a very similar expression to the comoving case but where the temperature has been red-shifted away from the de Sitter temperature so that it is now the *locally-measured KMS temperature.*

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This corresponds to the local temperature of the field at the location of the detector and is red-shifted away from the de Sitter temperature.

From here, we reason that we can estimate the temperature of the detector using

$$T_{EDR} = -\frac{\omega}{\ln\mathcal{R}}, \qquad \mathcal{R} = \dot{\mathcal{F}}_\tau(\omega)/\dot{\mathcal{F}}_\tau(-\omega)$$

where T_{EDR} must be independent of ω.

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the detector has thermalised to the local field temperature T_{loc}.

Some results

Black hole vs. Cosmological spacetimes

Some things to bear in mind

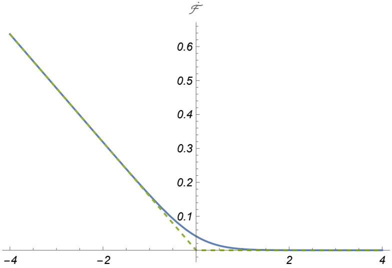
Black hole

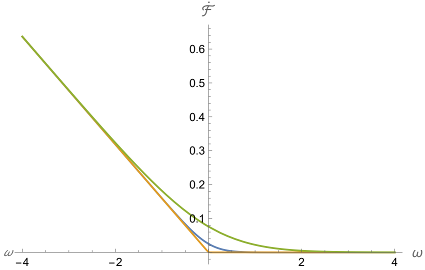
- – In general, the propagator is not known in closed form and requires significant numerical work.
- + In stationary black hole spacetimes, the existence of a Killing vector allows us to define physical quantities such as surface gravity and temperature on the event horizon.

Cosmology

- + FRW spacetime is conformally-flat and so we can use the conformal mode decomposition, easing numerical burden.
- – Due to the dynamical nature of the cosmological apparent horizon, surface gravity and temperature become ambiguous.

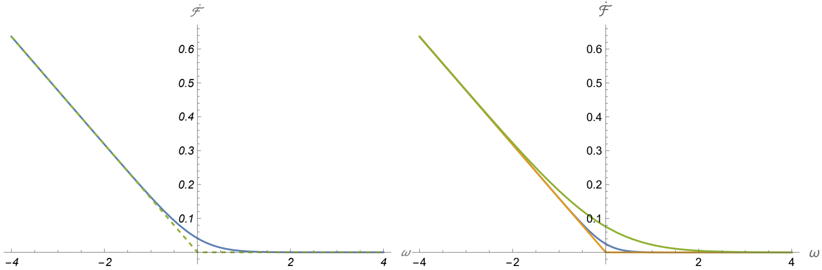
Static vs. Comoving detector





- Here, we plot the transition rate as a function of the energy gap.

Static vs. Comoving detector



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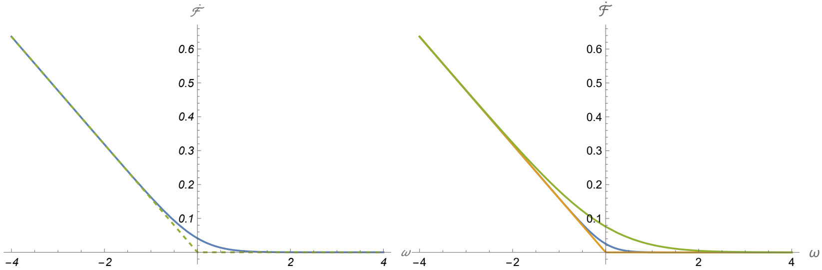

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- Here we see that in both cases, we have the expected behaviour of a detector being generically more likely to emit quanta ($\omega < 0$) than it is to absorb ($\omega > 0$).

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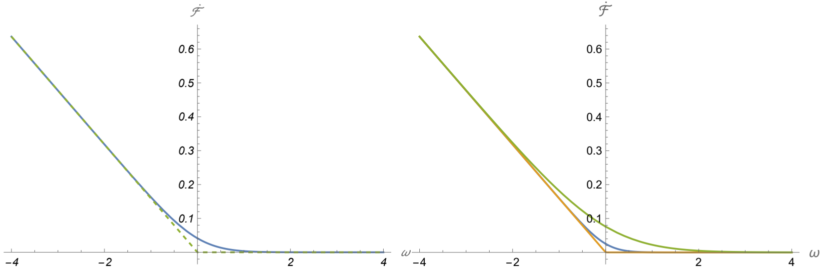

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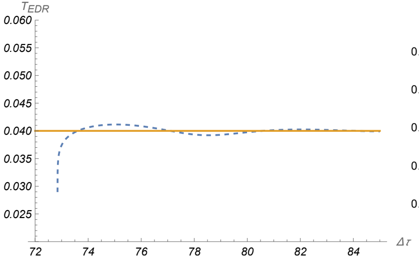
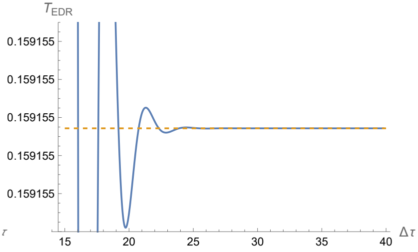
- In the comoving case (right), the detector more closely resembles the static case when the scale factor approaches flat space (yellow).

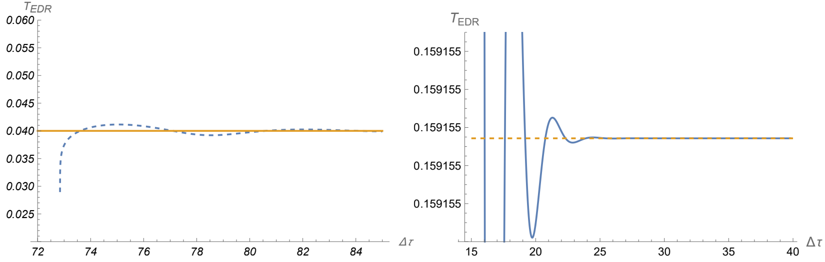
Static vs. Comoving detector





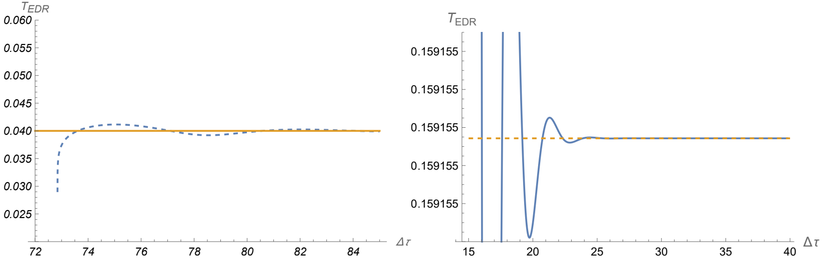
- Here, we plot the temperature of the detector over time.

Static vs. Comoving detector



- Here, we plot the temperature of the detector over time.
- In the static case (left), we observe that the once transient effects associated with turning on the detector are distilled by a suitably long detection time, the temperature of the detector thermalises to T_{loc}

Static vs. Comoving detector



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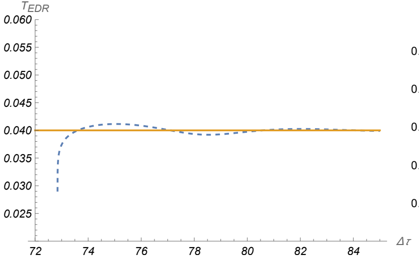
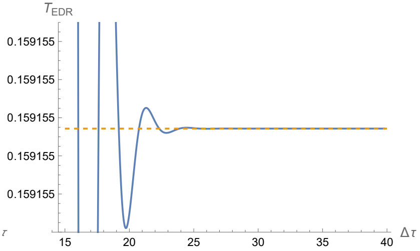
- Similarly, in the case of a comoving detector coupled to a field in the de Sitter Universe, the detector thermalises to the de Sitter temperature T_{dS}.

Static vs. Comoving detector





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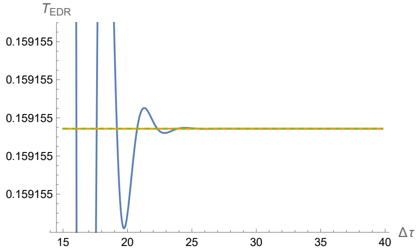
- Similarly, in the case of a comoving detector coupled to a field in the de Sitter Universe, the detector thermalises to the de Sitter temperature T_{dS}.
- This is the temperature an inertial observer in de Sitter space will read on their thermometer.

Thermalisation of the detector



- In Phys.Rev.D 105 (2022) 12, 123513 arXiv:2204.00359, we show that for a broader class of asymptotically-de Sitter spacetimes, that the temperature of the detector thermalises to the Hayward-Kodama temperature, discussed in Lecture 1.