Holography and AdS/CFT correspondence

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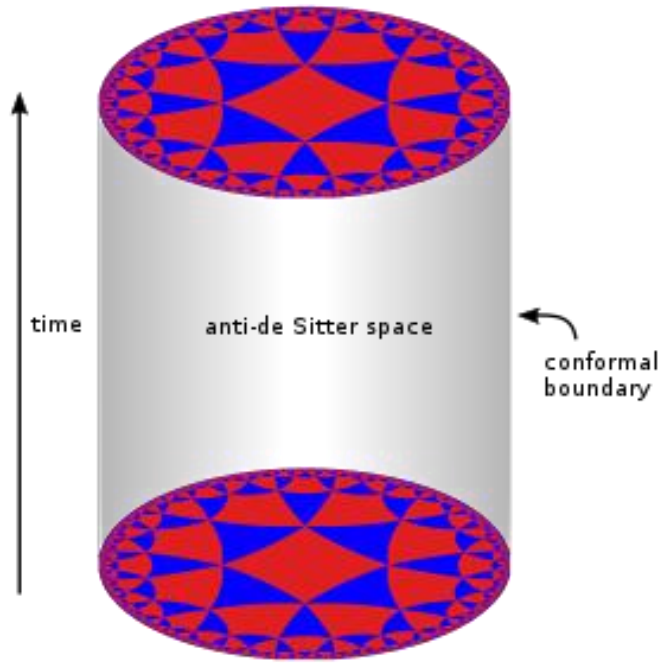
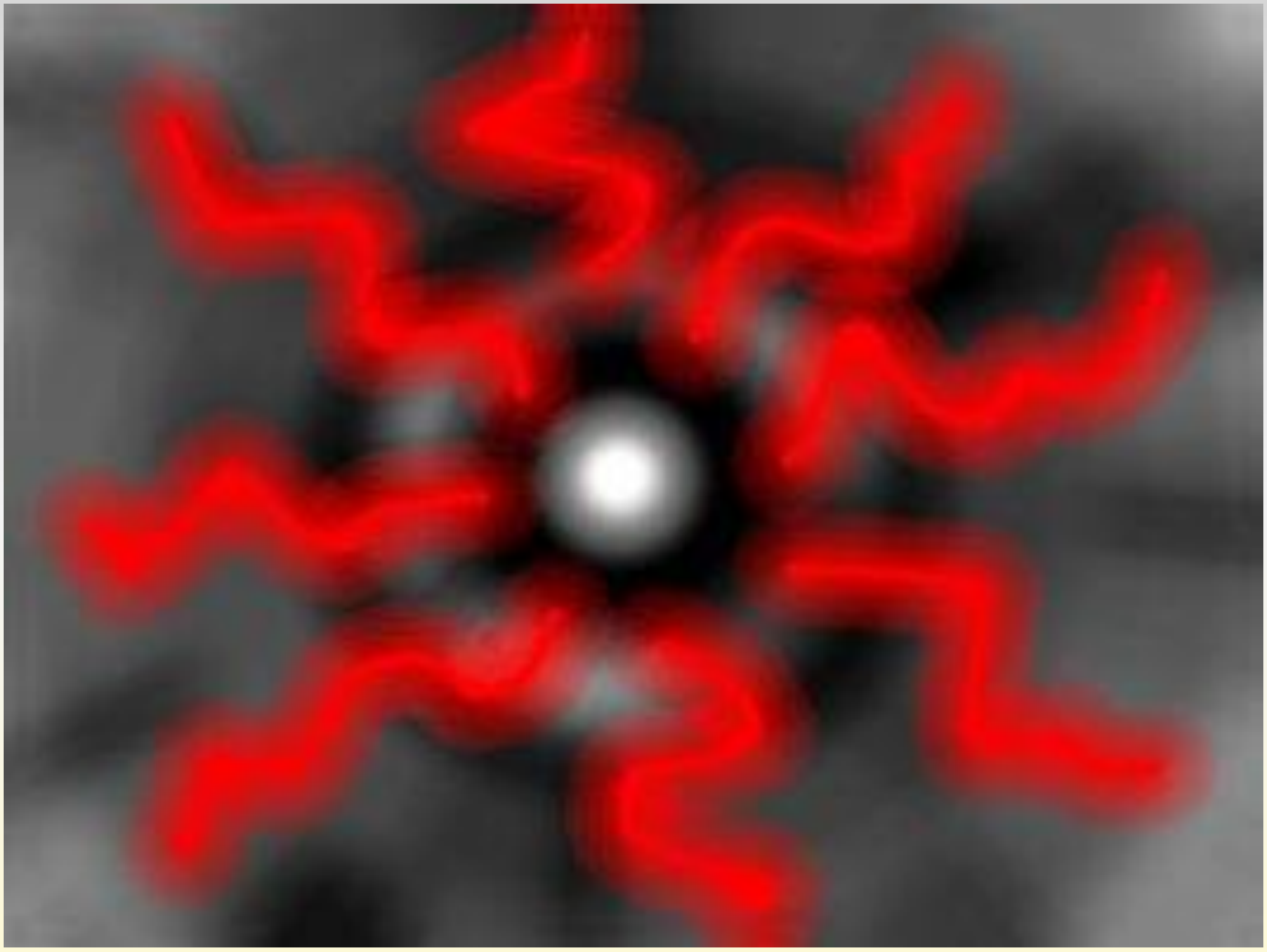


1) Black hole thermodynamics



Schwarzschild BH: basic properties

Horizon: $r = r_+ = 2M$

Mass: M

Surface gravity: $\kappa = \frac{f'(r_+)}{2} = \frac{M}{r_+^2} = \frac{M}{(2M)^2} = \frac{1}{4M}$

Horizon area: $A = \int \sqrt{\det \gamma} d\theta d\varphi = 4\pi r_+^2 \,.$

Good idea: $dM = \frac{dr_+}{2}, \quad dA = 8\pi r_+ dr_+$

1st law of black hole mechanics: $dM = \frac{\kappa}{2\pi}\frac{dA}{4}$

Black hole thermodynmics?

$$dM = \frac{\kappa}{2\pi}\frac{dA}{4}$$

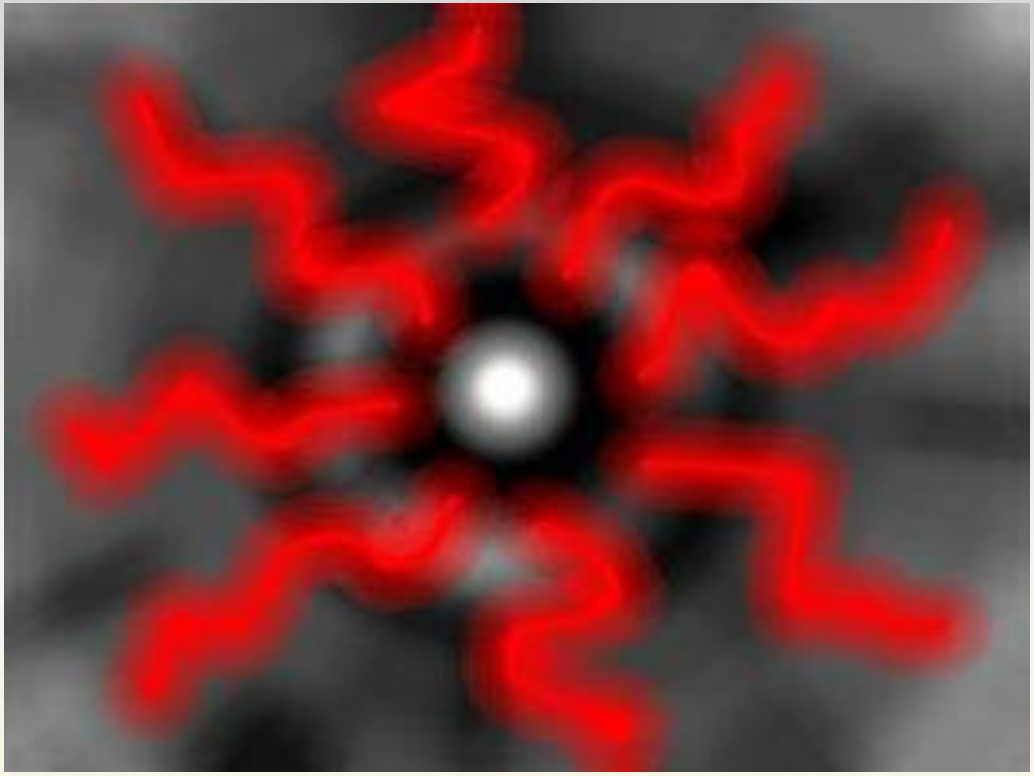
Bekenstein?

$$dE = TdS$$

Hawking (1974):

$$T = \frac{\kappa}{2\pi}$$

$$\Rightarrow \quad S = \frac{A}{4}$$



Derived using QFT in curved space (other derivations: LQG, ST, quantum tunnelling,….)

Black hole entropy

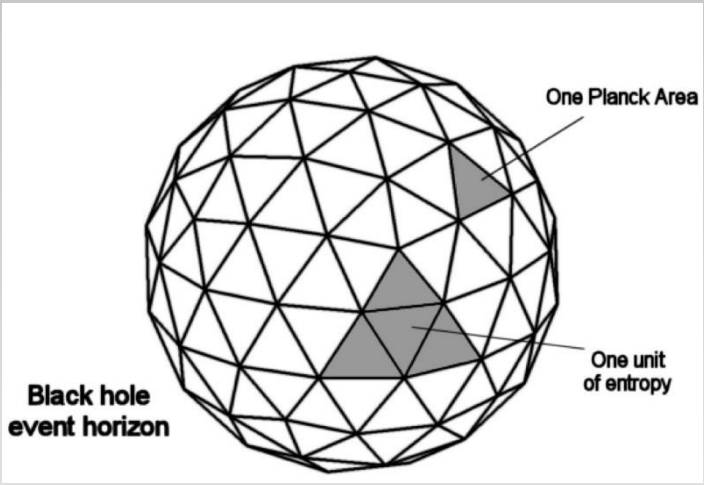
relativity

Stat. mech

$$S = \frac{A}{4}\frac{c^3 k_B}{\hbar G_N}$$

gravity

QM



- Is huge:

$$S = \frac{k_B}{4}\frac{A}{l_P^2}, \quad l_P = \sqrt{\frac{G\hbar}{c^3}}$$

- Is **holographic:**

$$\boxed{S \propto A}$$

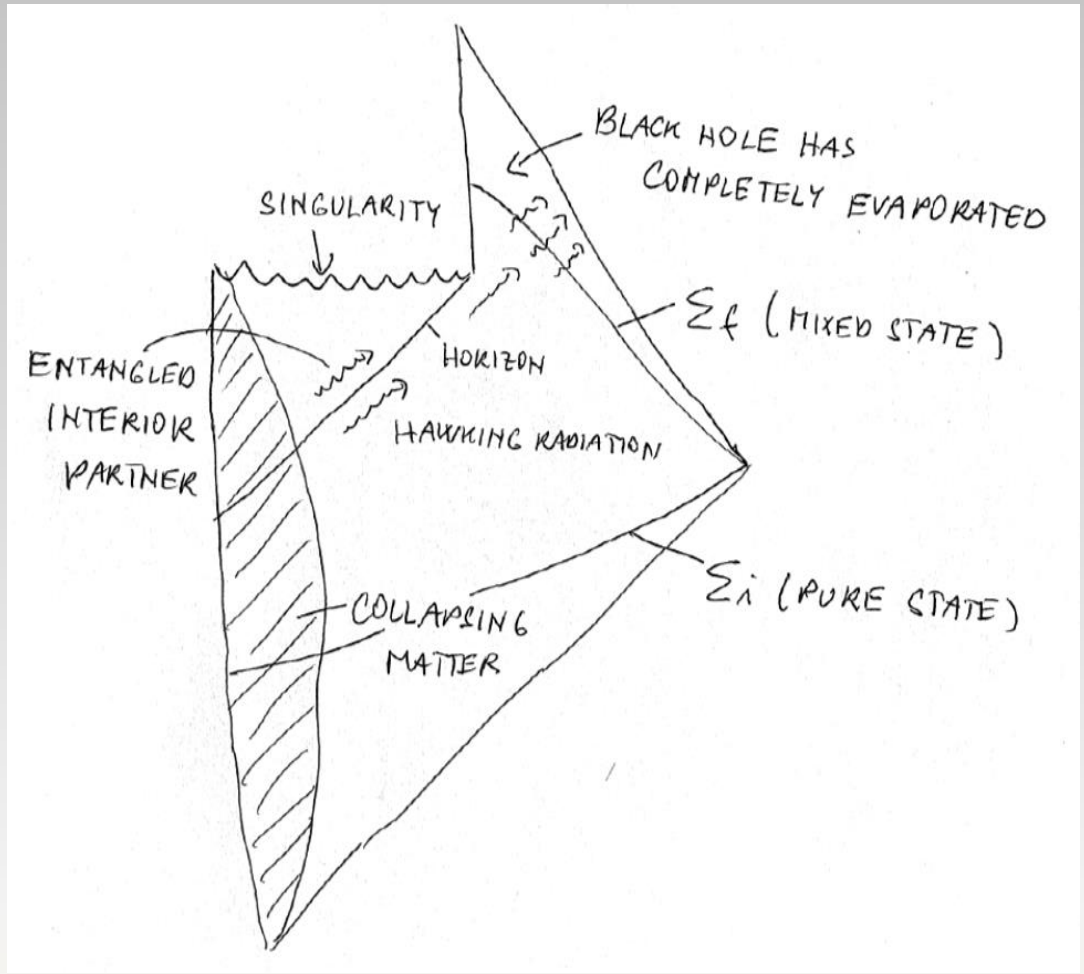
- Bekenstein's bound:

$$S \leq \frac{A}{4}$$

- **Generalized 2nd law:**

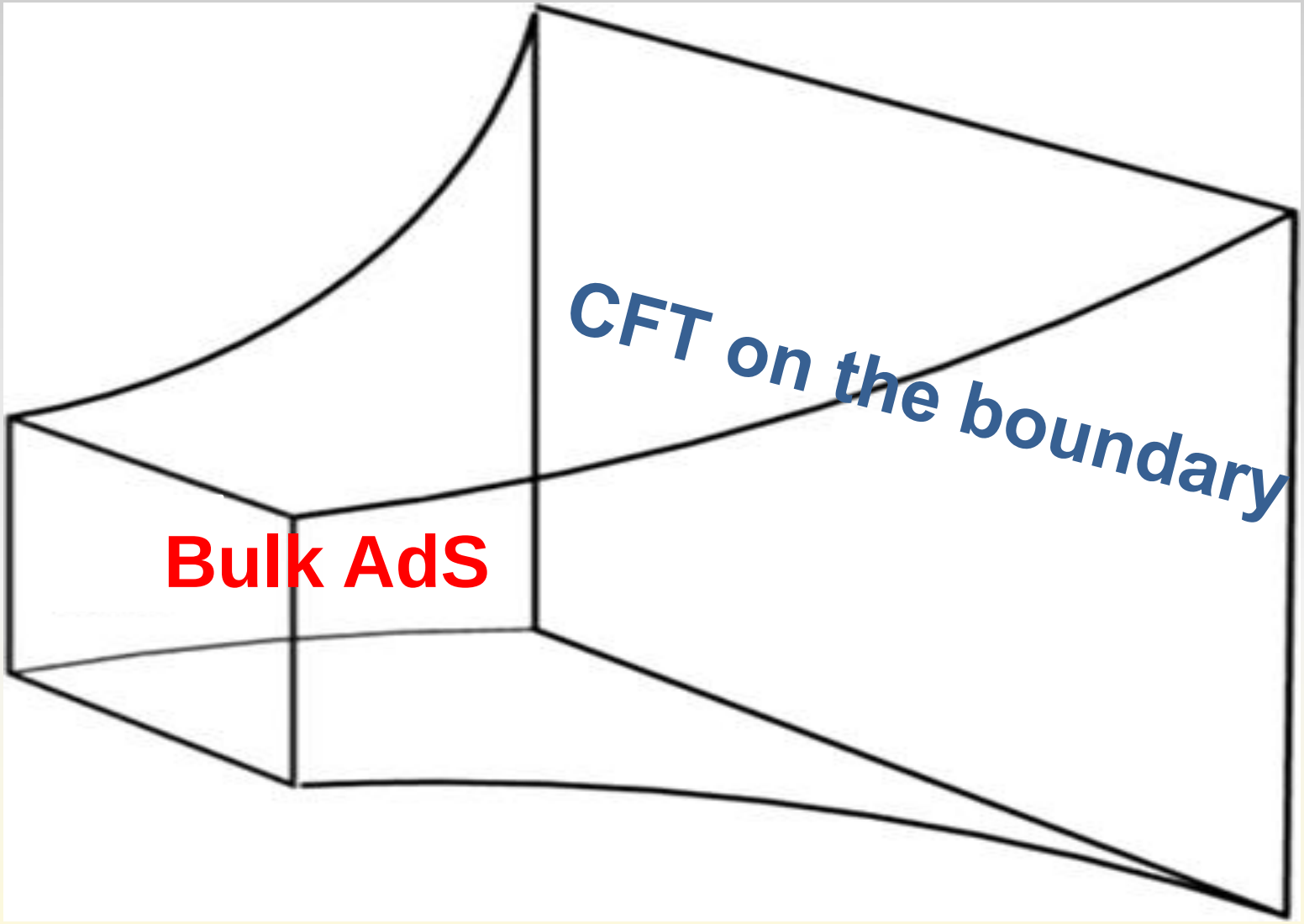
$$S_{\mathrm{TOT}} = S_{\mathrm{BH}} + S_{\mathrm{outside}} \geq 0$$

Black hole info paradox (Hawking 1976)



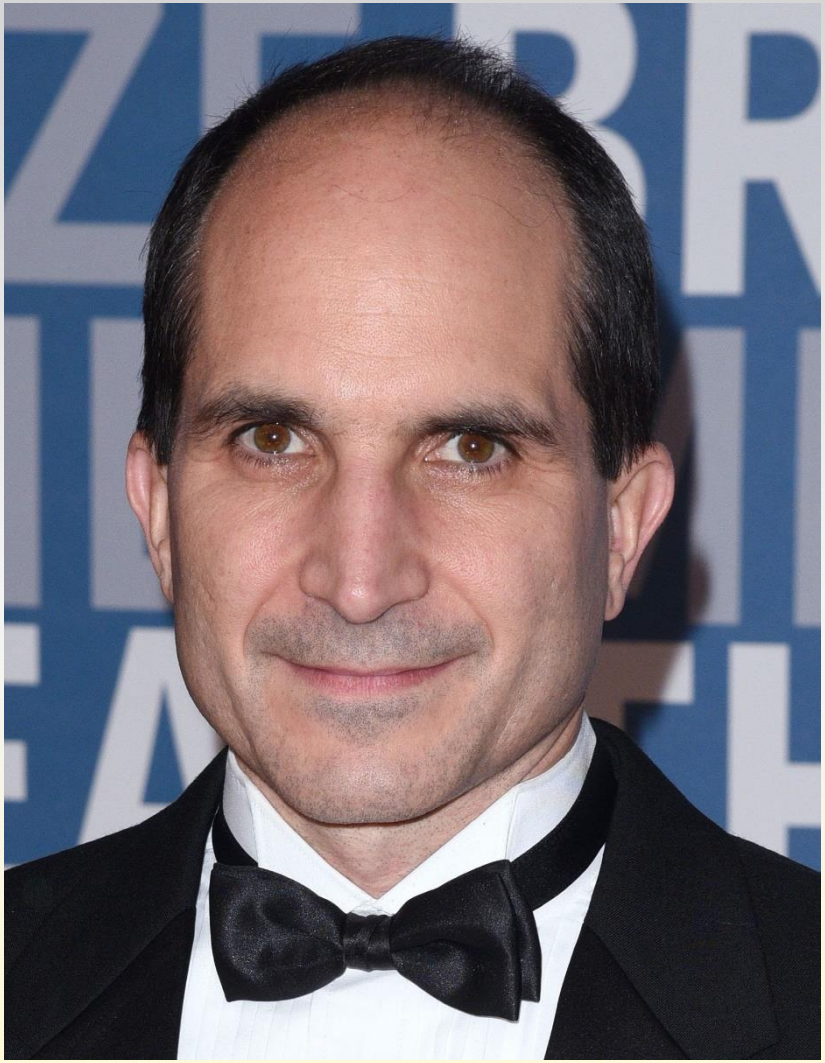
- Thermal Hawking radiation leads to black hole evaporation.
- If the BH completely evaporates, we violated unitary evolution of QM (evolved from the pure state in the beginning to a mixed state at the end) – info losss.
- Contradicts the **AdS/CFT correspondence**

2) AdS/CFT duality



Central conjecture

Conjecture (Maldacena 1997). Type IIB superstring theory on $AdS_5 \times S^5$ is *dual* to $\mathcal{N} = 4$ $SU(N_c)$ SYM in $d = (3+1)$ dimensions.



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String theory

$$g_s, \quad L/l_s$$

- A theory of QG

QFT

$$N_c\,, \quad \lambda = g_{YM}^2 N_c$$

- QFT without gravity

Duality = full (**holographic)** equivalence

$$2\pi g_s = g_{YM}^2 = \frac{\lambda}{N_c}\,, \qquad \frac{L^4}{l_s^4} = 4\pi g_s N_c = 2\lambda$$

Useful weaker version

$$2\pi g_s = g_{YM}^2 = \frac{\lambda}{N_c}\,, \qquad \frac{L^4}{l_s^4} = 4\pi g_s N_c = 2\lambda$$

Classical (super)gravity as a limit of ST:

- Strings almost point-like – standard geometry valid

$$L \gg l_s \quad \Rightarrow \quad \lambda \gg 1$$

- Quantum corrections (loops) are small $1 \gg g_s \sim \frac{\lambda}{N_c}$

Weak (weakly interacting) gravity is dual to strongly coupled QFT

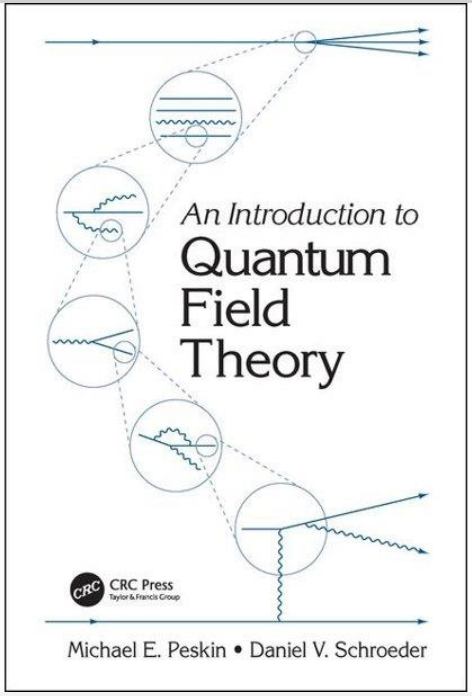
$$1 \ll \lambda \ll N_c$$

(**Strong-weak duality**)

(Simple gravitational calculations capture behavior of strongly coupled fully quantum systems.)

What is conformal field theoy?

= special QFT with bigger spacetime symmetry



Invariant under conformal transformations:

$$x^\mu \rightarrow x'^\mu(x): \quad \eta_{\mu\nu} \rightarrow \Omega^2(x)\eta_{\mu\nu}$$

spec. $$t \rightarrow \lambda t\,, \quad \vec{x} \rightarrow \lambda \vec{x}$$

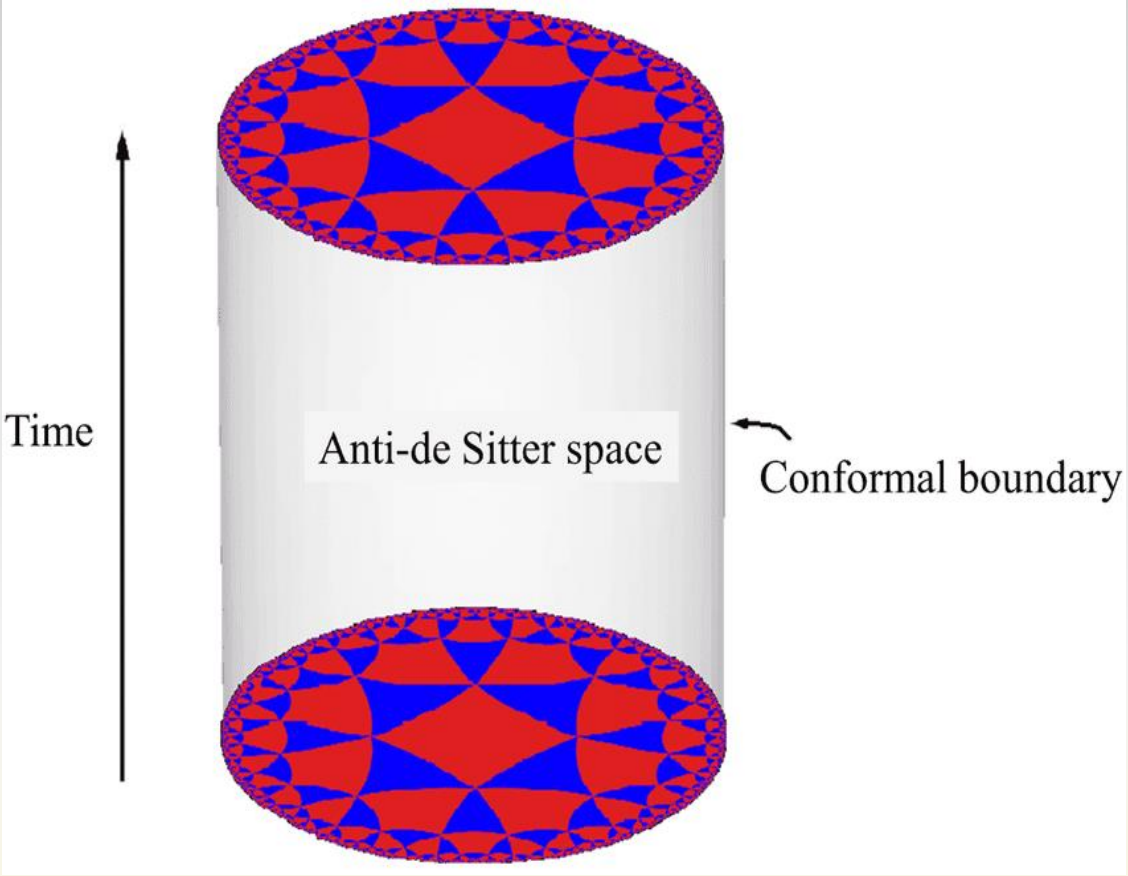
$$\phi(x) \rightarrow \phi'(x') = \lambda^{-\Delta}\phi(x)$$

Theorem: Conformal (superconformal symmetry is the largest one for non-trivial (interacting) QFTs.

Applications: phase transitions, string theory, mapping space of QFTs (“perturbations” of CFT)

What is AdS?

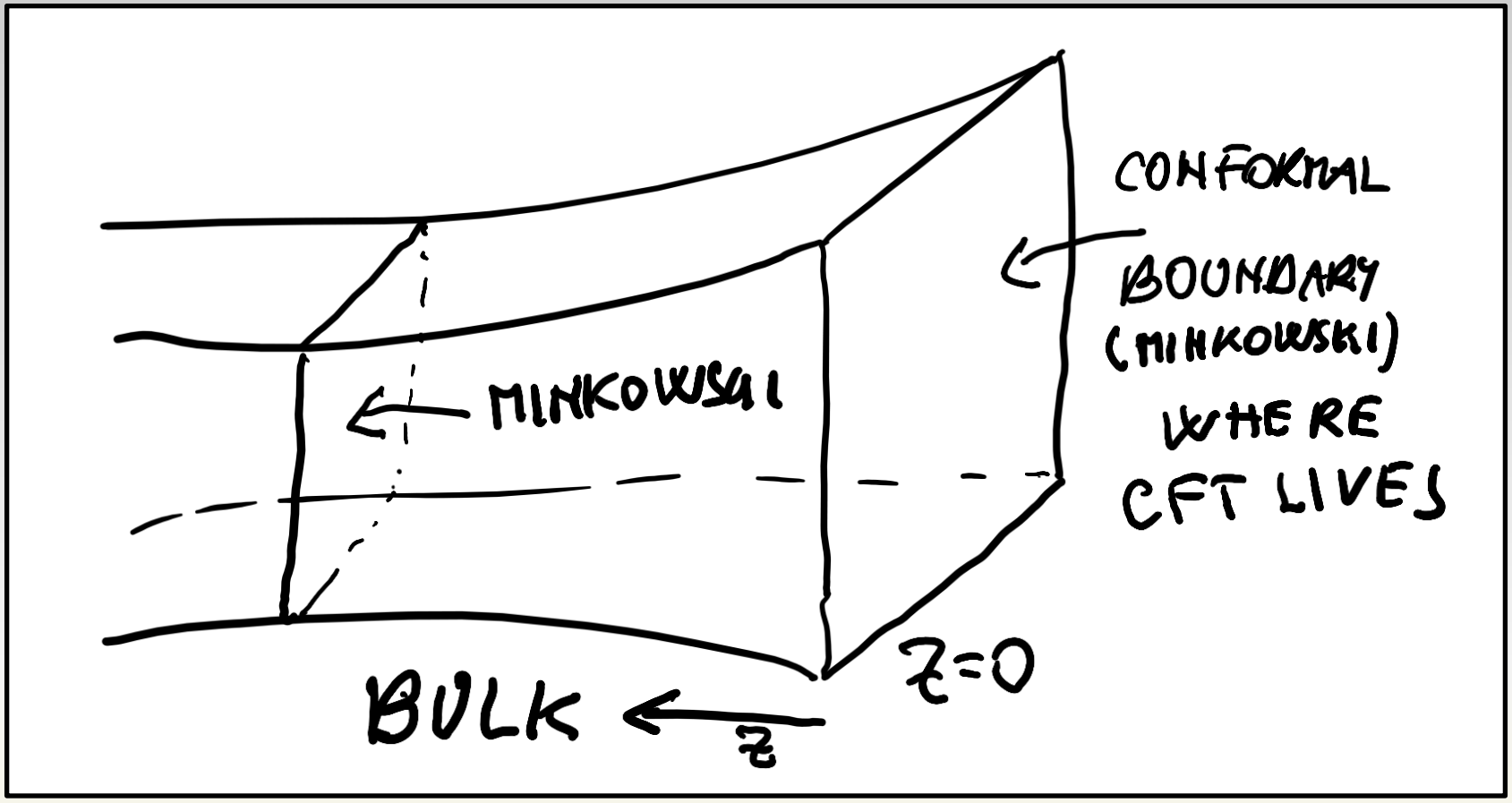
= maximally symmetric solution of EE with negative cosmological constant



$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}\,, \quad \Lambda = -\frac{3}{l^2}$$

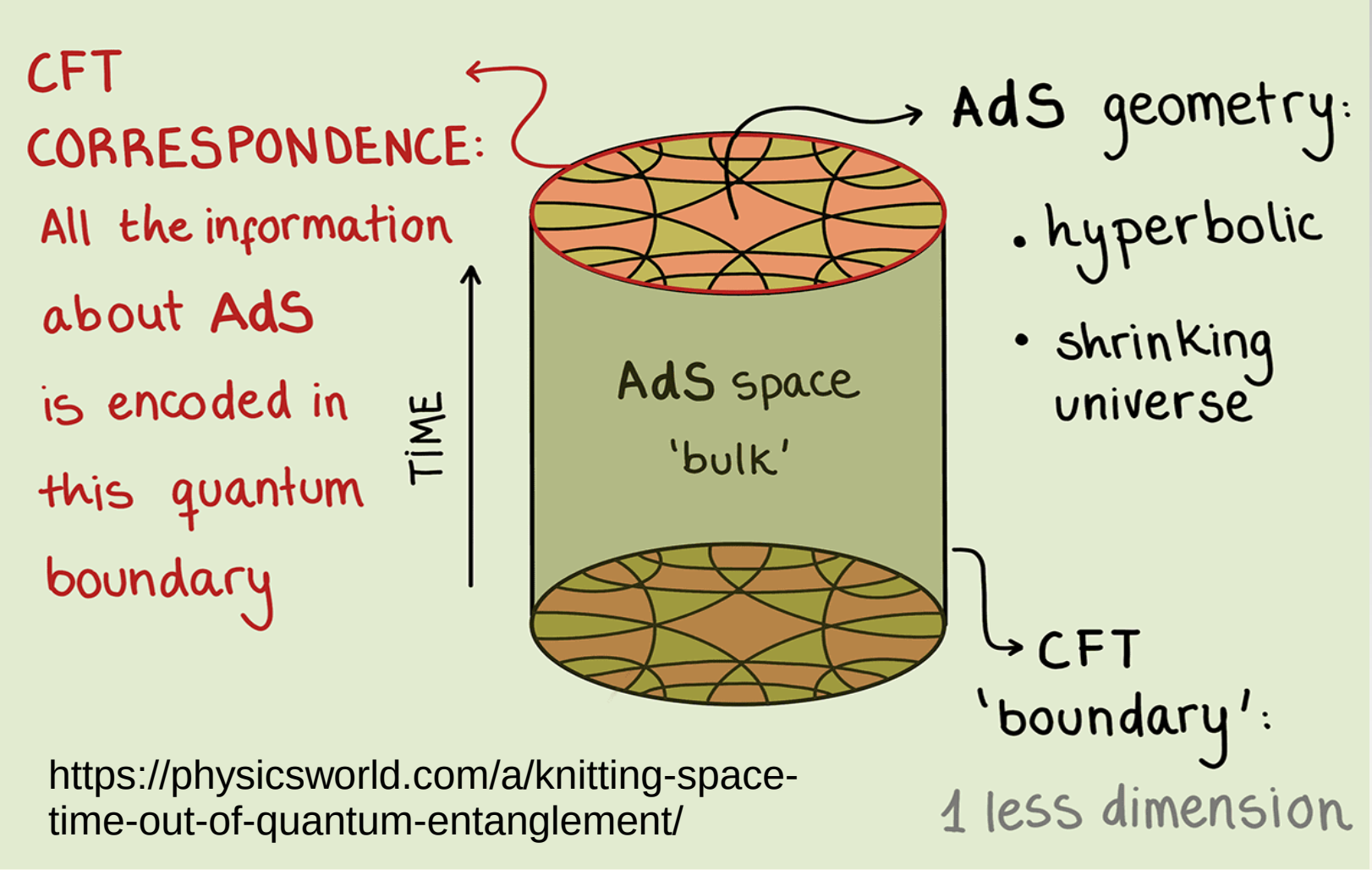
Poincaré AdS

= particularly sliced part of global AdS cylinder



$$ds^2 = \frac{\ell^2}{z^2}\Big(\underbrace{-dt^2 + d\vec{x}^2}_{\eta_{\mu\nu}dx^\mu dx^\nu} + dz^2\Big)$$

AdS/CFT correspondence



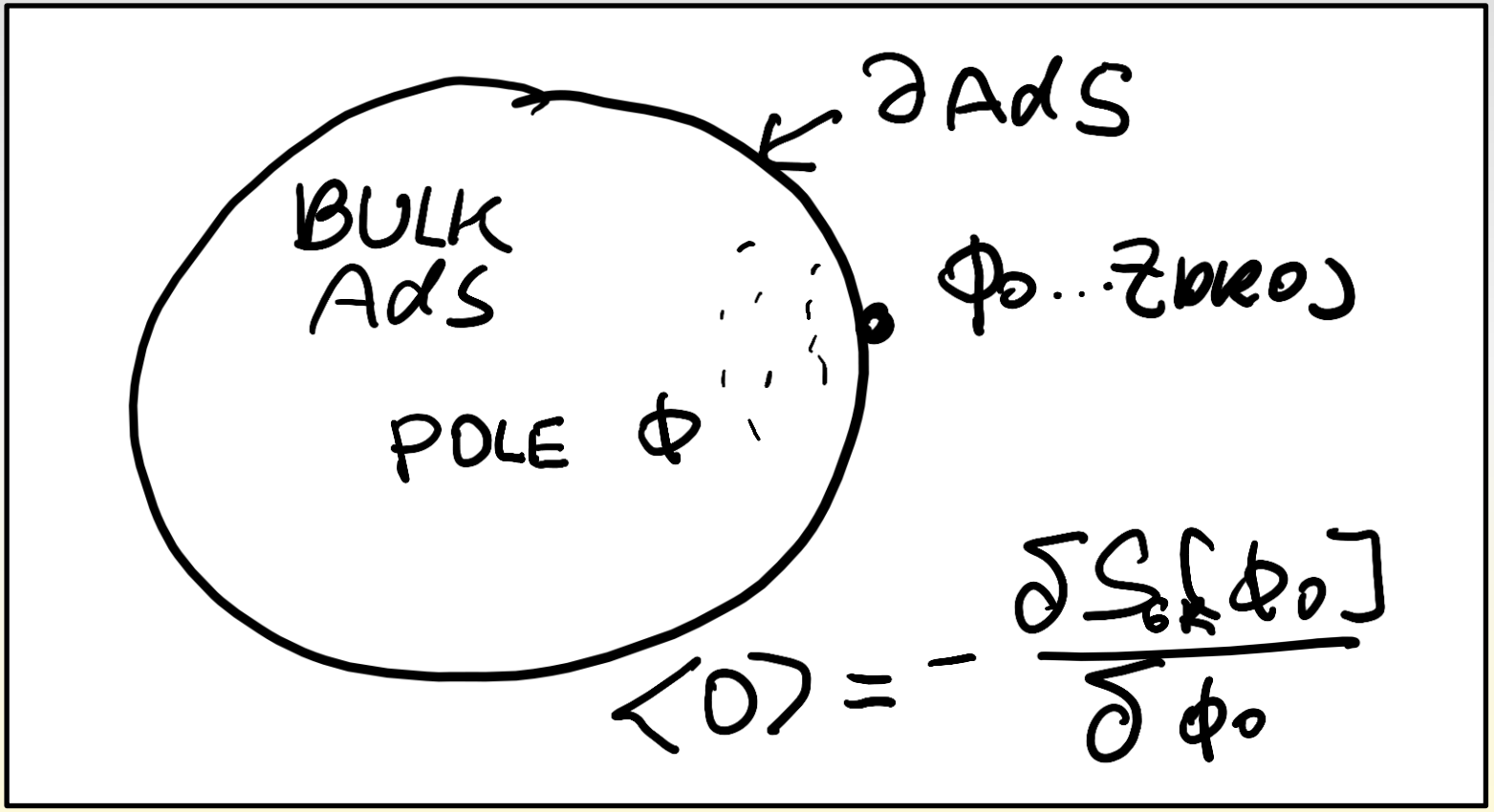
Contradicts info paradox: evolution of CFT on boundary is unitary!

Calculation of correlation functions

WE WANT: $\boxed{\langle O \ldots \rangle}$

CFT: $S \to S_0 + \int d^d x \, \phi_0(x) O(x)$

ϕ_0 ↑ ZEROS O ↑ OPERATOR

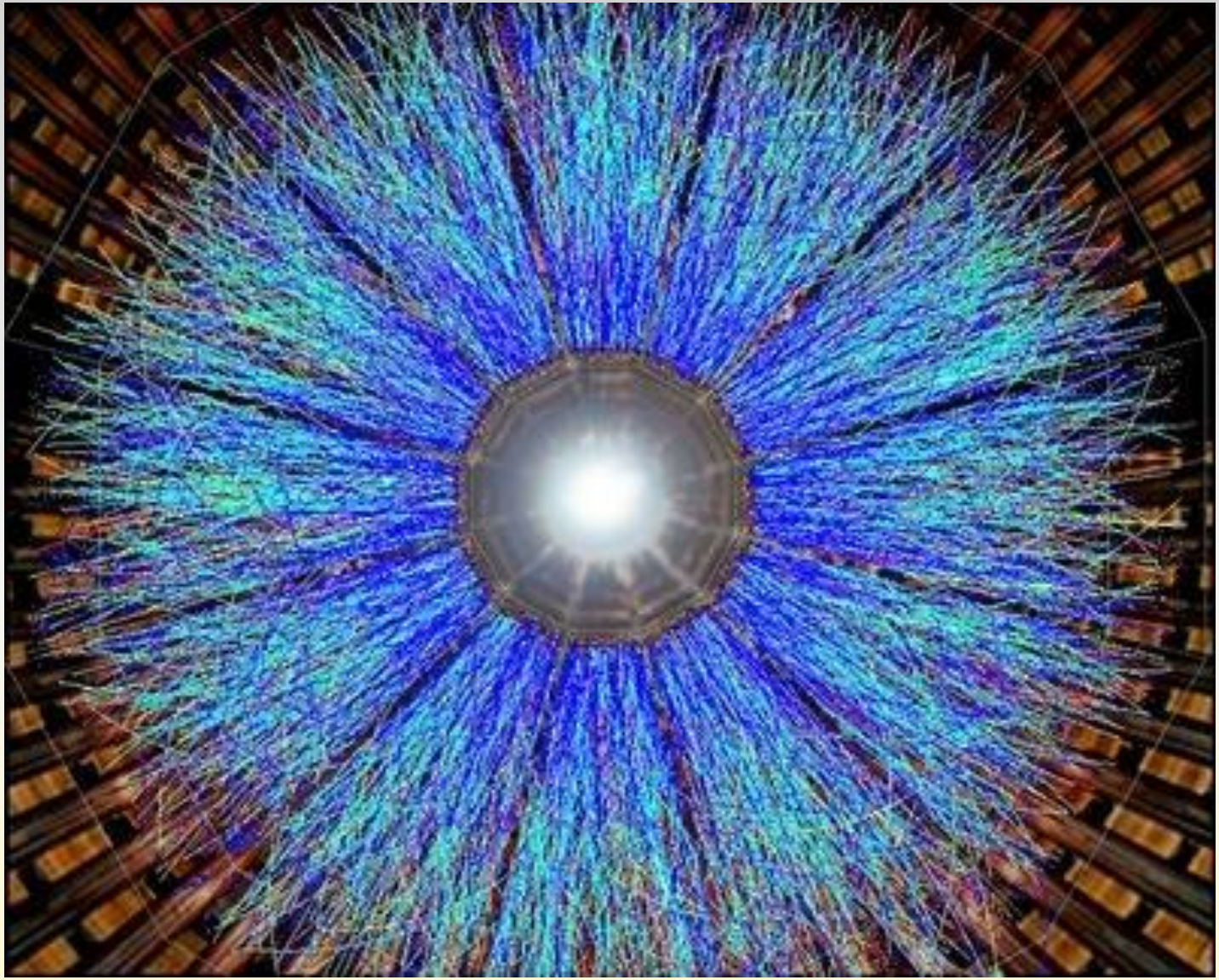


State-operator correspondence

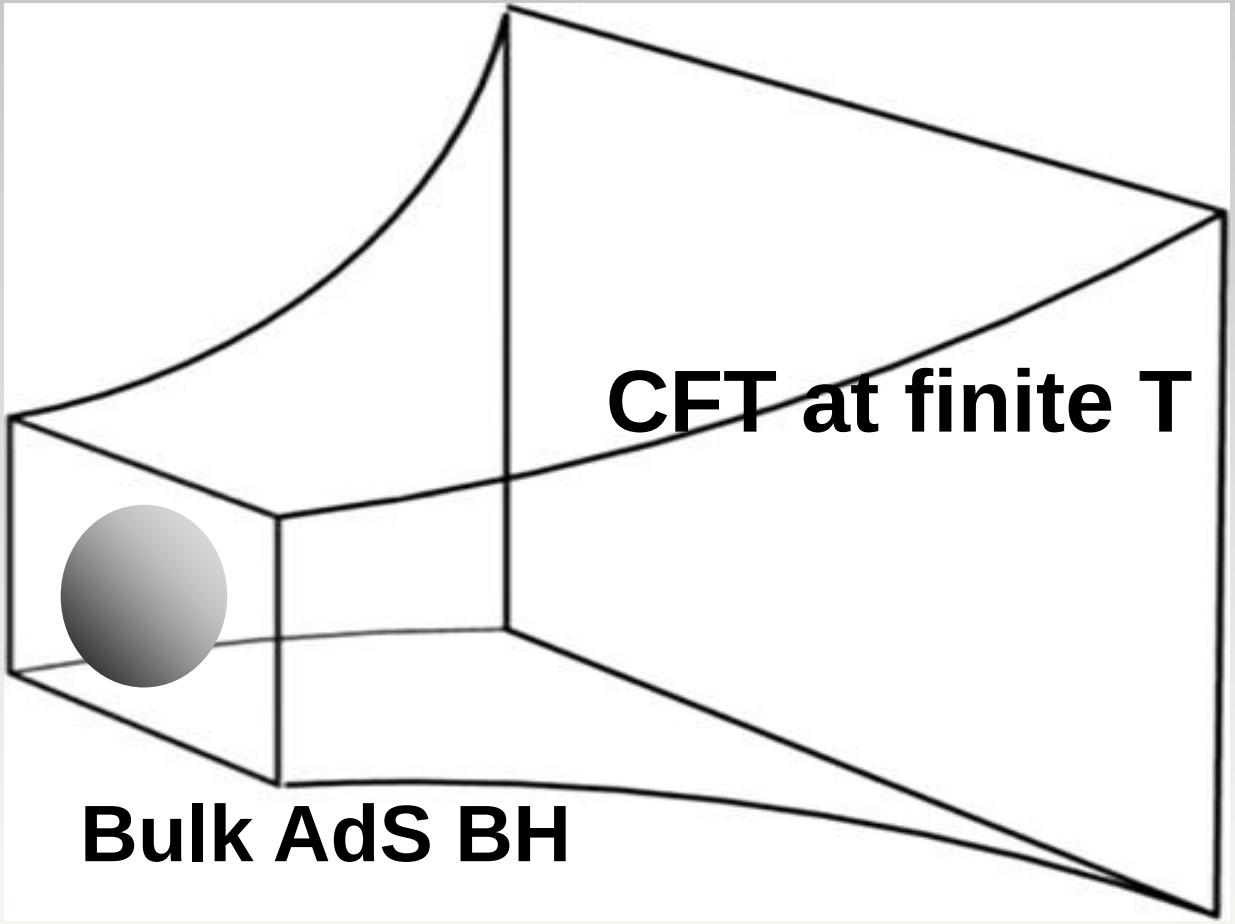
WE WANT; $\langle O \ldots \rangle$

operator (CFT)	field (AdS)
scalar O_Δ	ϕ
$T_{\mu\nu}$	$h^{\mu\nu}$
J_μ	A^μ

3) Generalization: gauge/gravity duality and its applications



Finite temperature

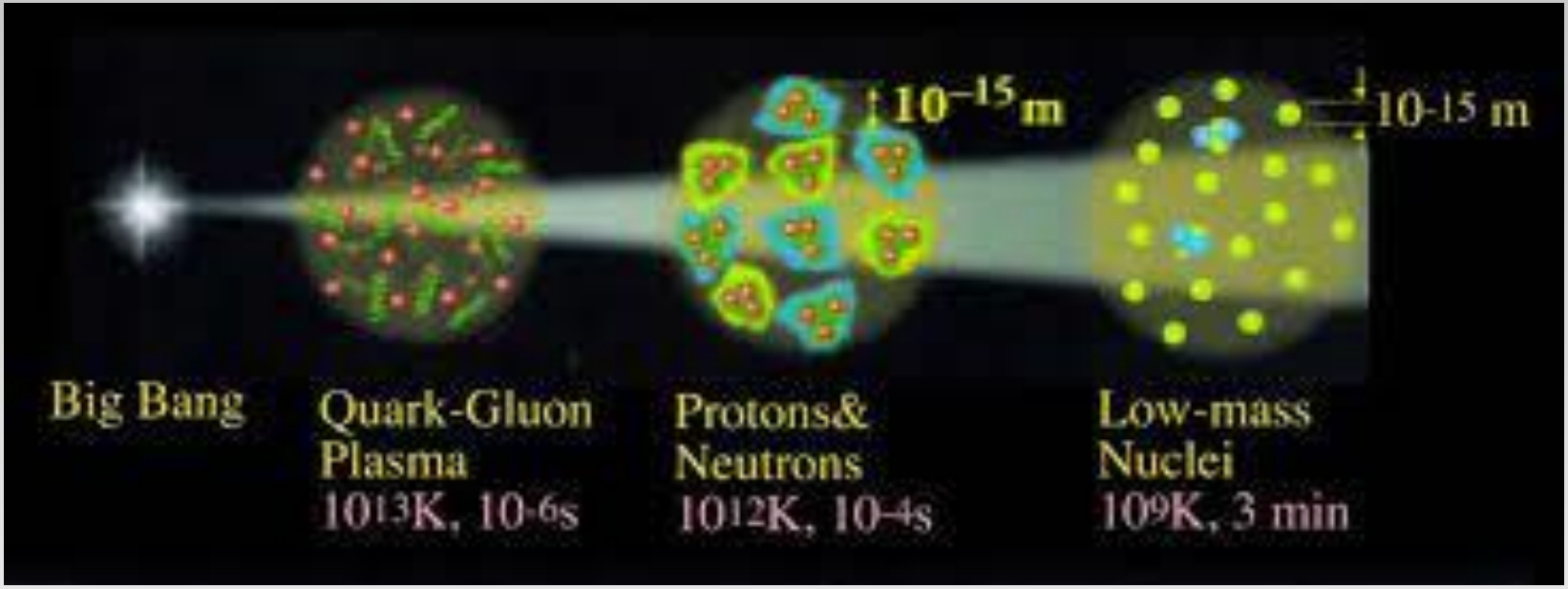


Bulk:
Hawking temp T
BH entropy S

⇔

Boundary:
CFT at finite T
CFT entropy S

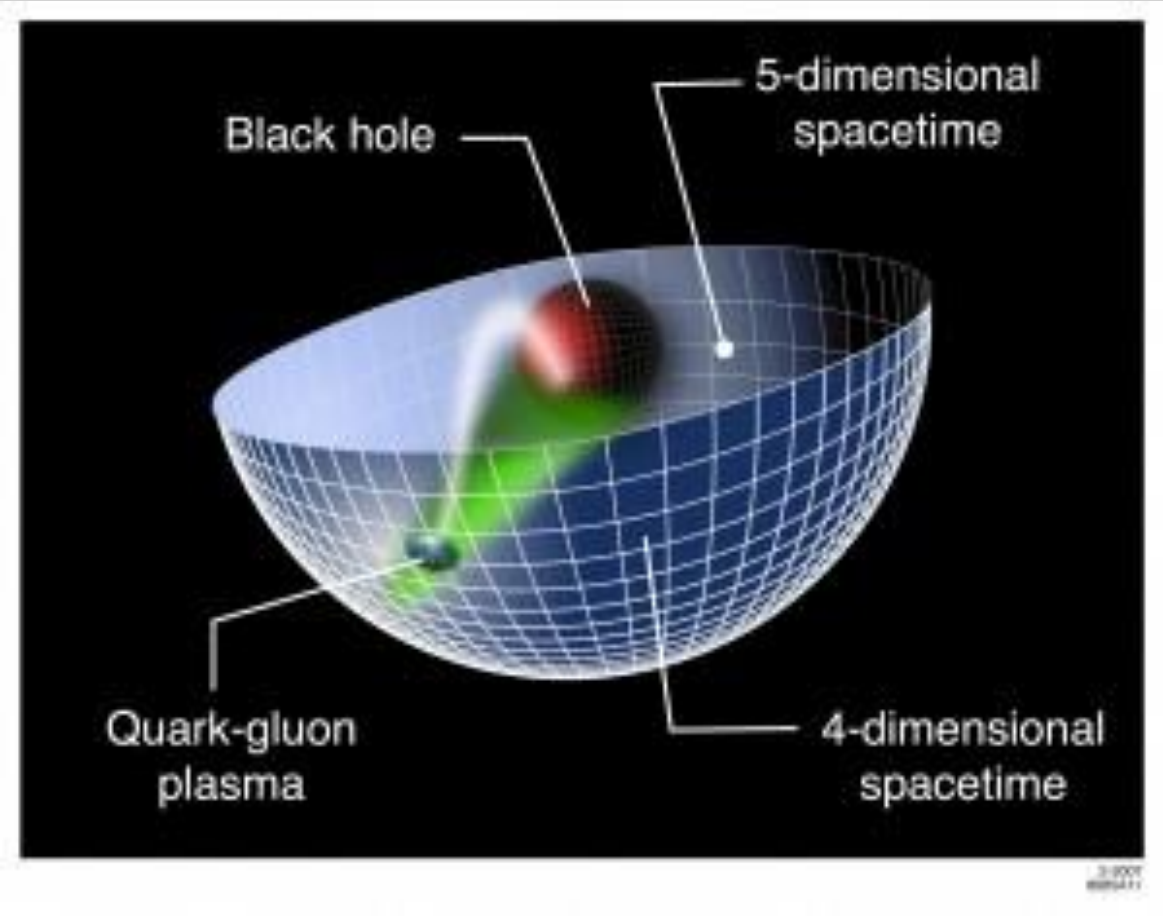
Allows to study quark-gluon plasma



- State of matter in Early Universe
- QGP has high temperature/density
- **Strongly correlated (not a CFT)**
- Recreated in LHC

Properties of QGP

- Well described by hydro regime
- Shear viscosity can be calculated using AdS/CFT

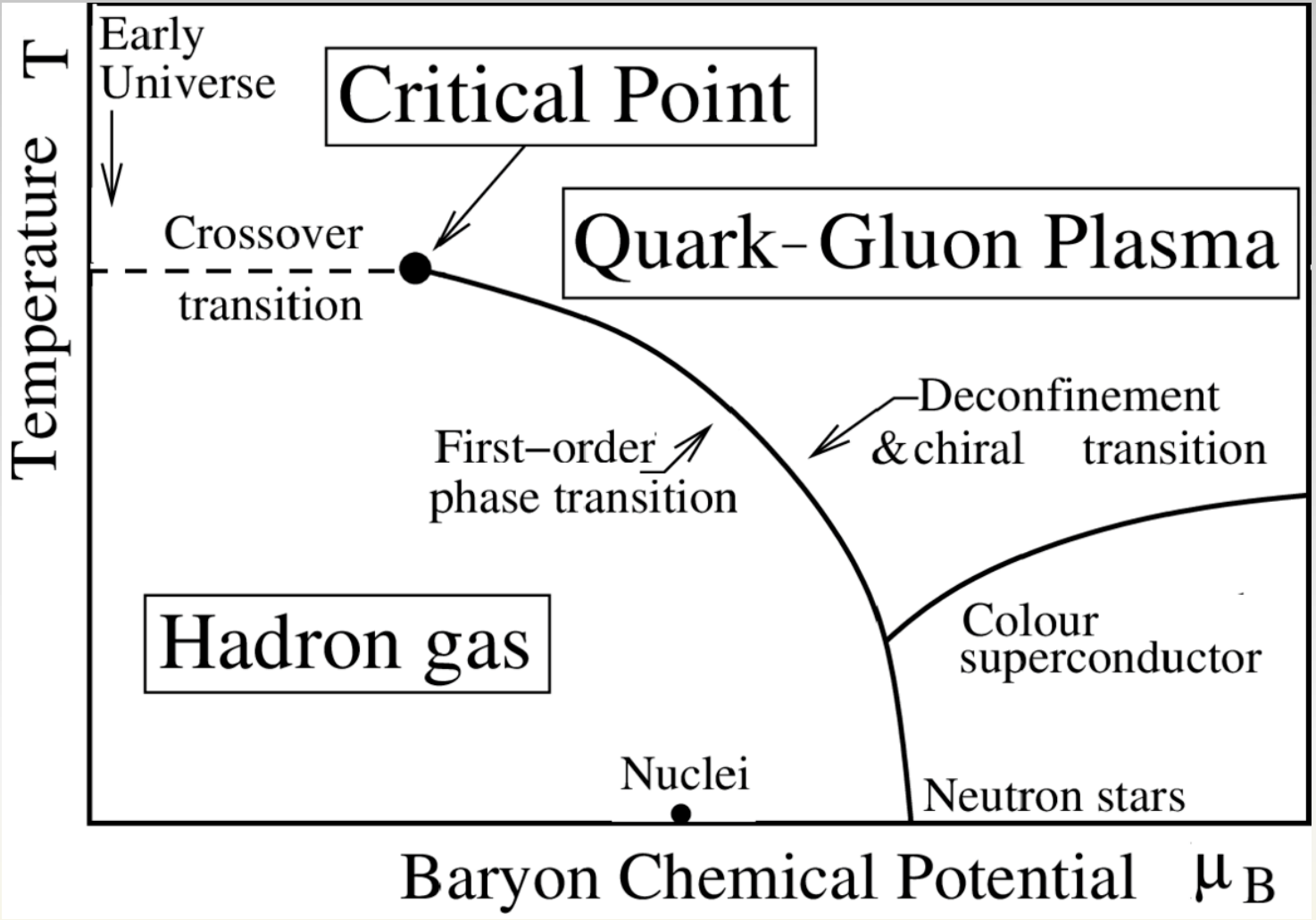


$$\frac{\eta}{s} = \frac{1}{4\pi}$$

$$\eta/s \in (1, 2.5)$$

- Almost ideal fluid (water has 1000x bigger)

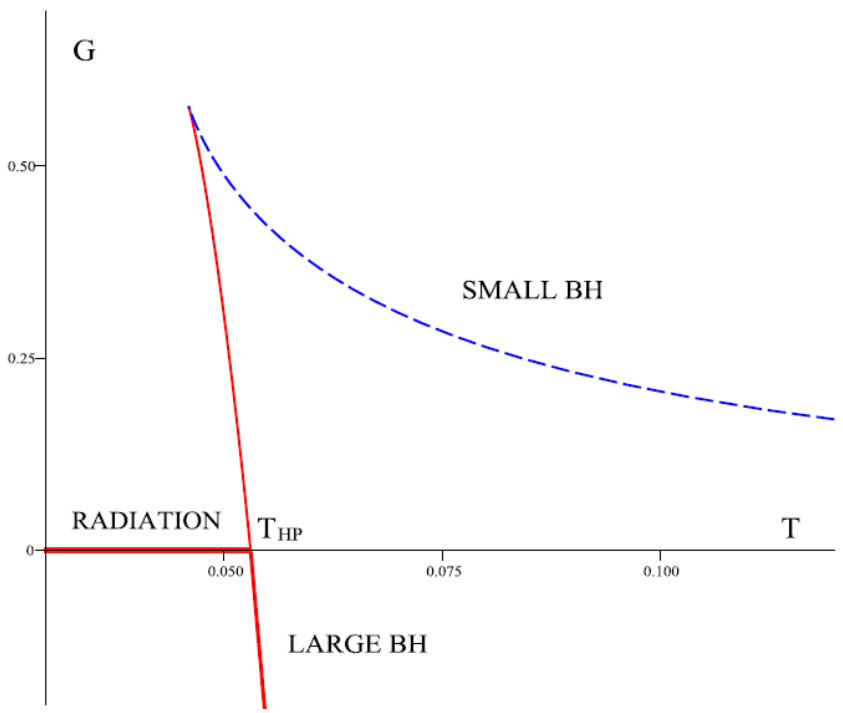
Phase transition

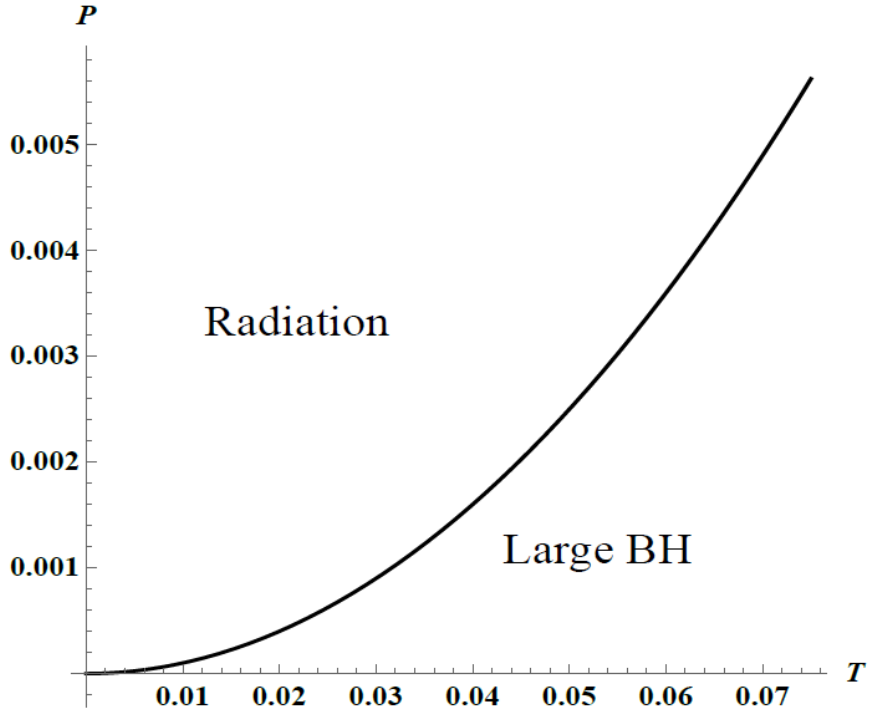


- **First-order phase transition** between hadro gas and QGP (confinement/deconfinement)

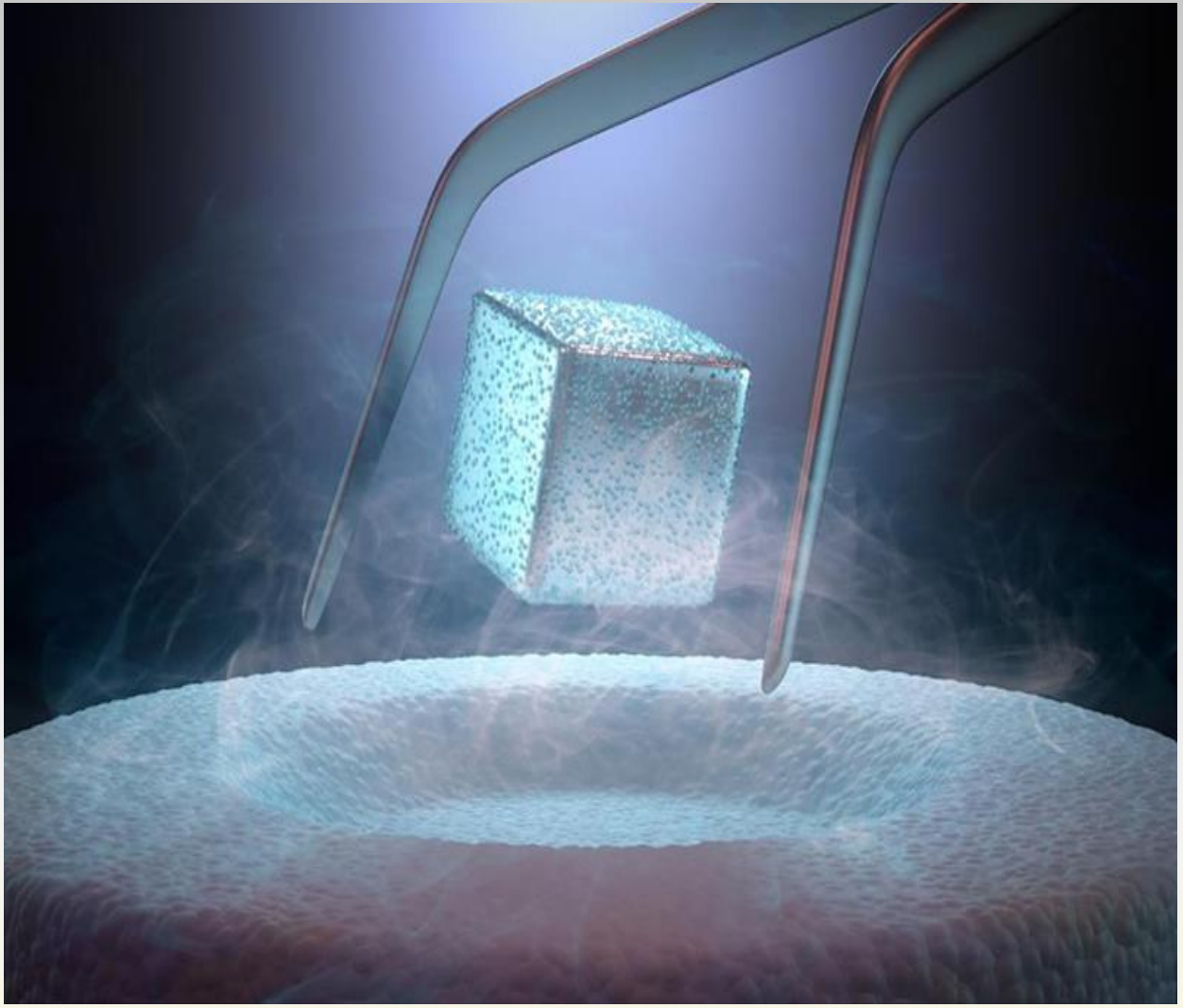
Simple gravitational dual: Hawking-Page phase transition

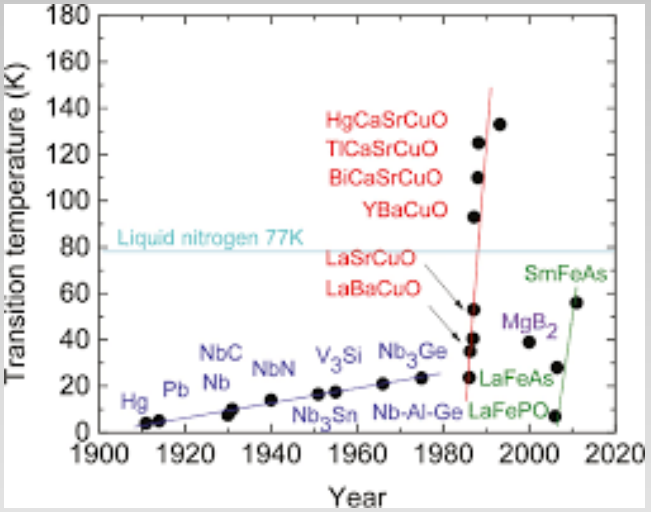
S.W. Hawking & D.N. Page, *Thermodynamics of black holes in anti-de-Sitter space*, Commun. Math. Phys. 87, 577 (1983).

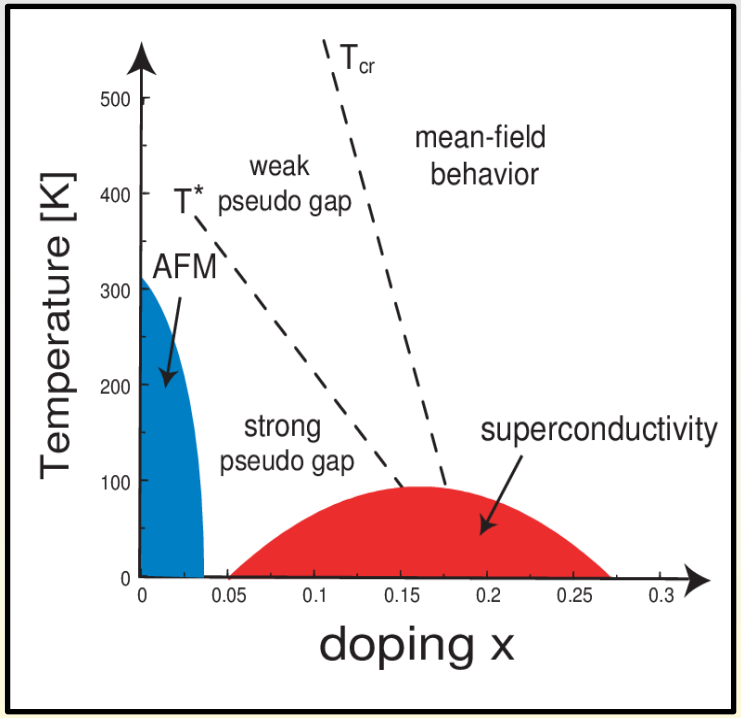




High temperature superconductors







- **Strongly correlated,** cannot e described by BCS theory

Holographic su[perconductors: scalar field condensate

No-hair theorems: “All BHs” described uniquely by 4 charges (M, J, Q, Qm).”

Wheeler 70’s **“Black holes have no hair”**

Possible:

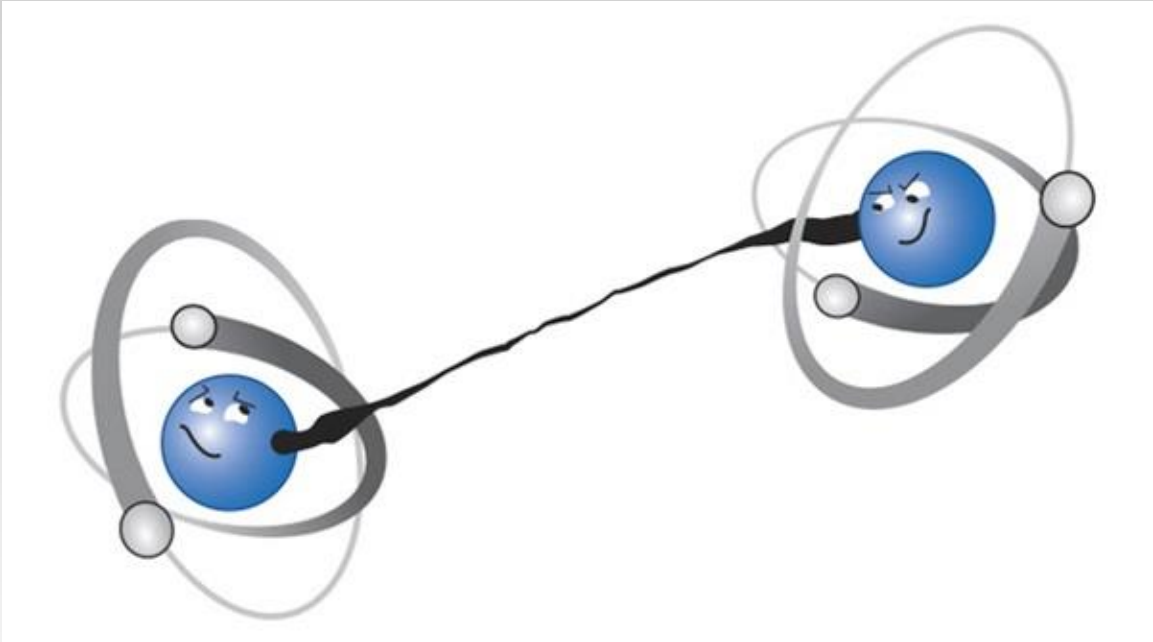
Not possible:

AdS: holographic superconductor: need BH with (scalar) hair at low temperatures and no hair at hight temperatures (Gubser 2008)

H^3 Theory (Hartnoll, Herzog, Horowitz, “Building a Holographic Superconductor,” Phys. Rev. Lett. 101, 031601, 2008)

Entanglement entropy

- **Entanglement** is one of the most important features of QM.



$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$\rho_A = \mathrm{Tr}_B \rho$$

$$S_A = -\mathrm{Tr}_A(\rho_A \log \rho_A)$$

Entanglement entropy

Properties:

- If the entire system is in pure state and B is a complement of A:

$$S_A = S_B$$

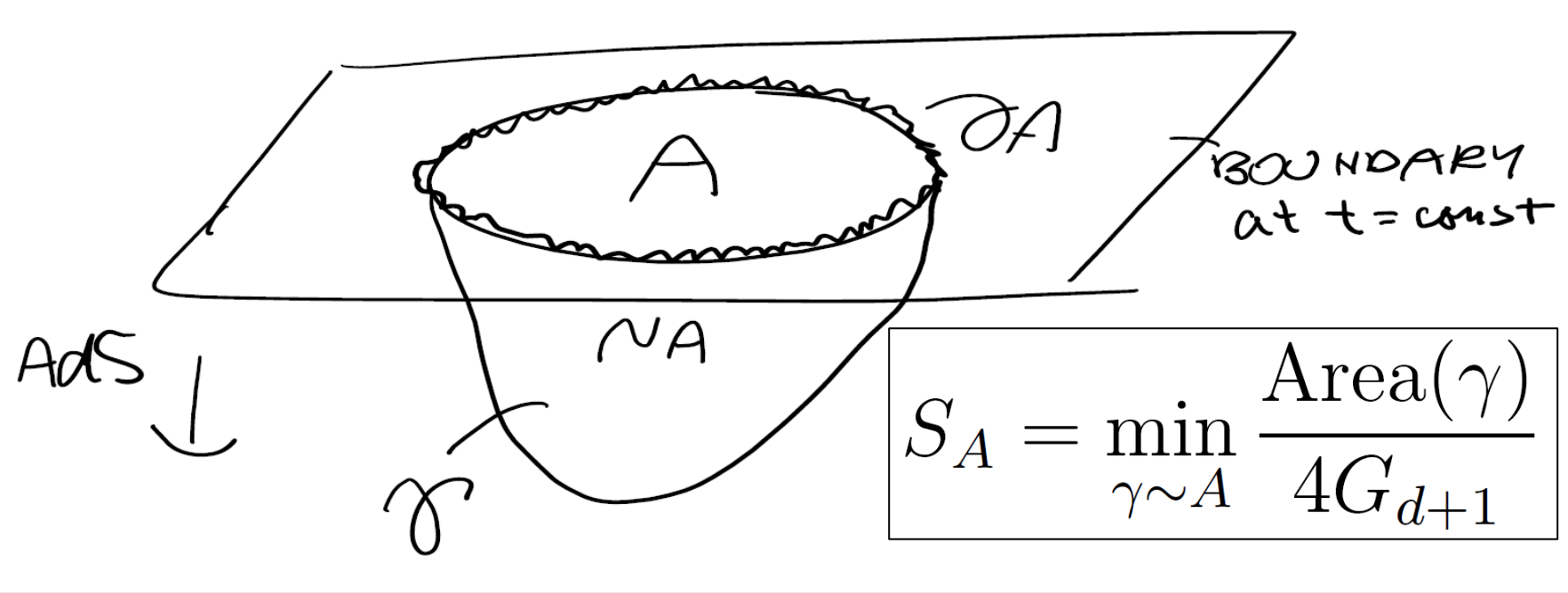
- Subaditivity:

$$S_A + S_B \geq S_{A \cup B} + S_{A \cap B}$$

In QFT (region A)

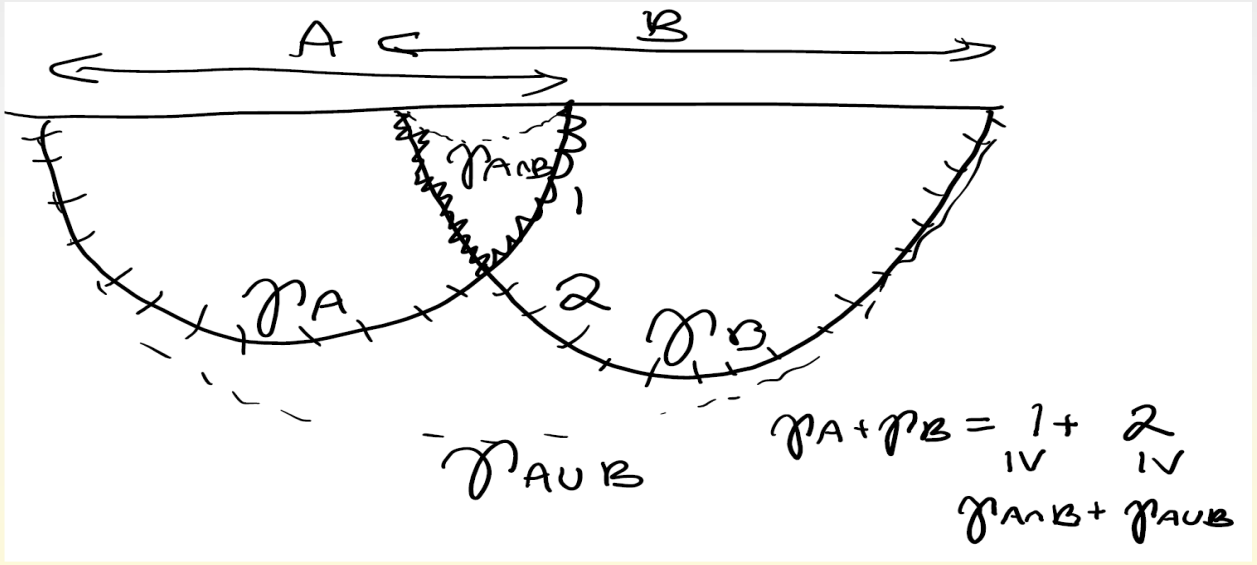
$$S_A \propto \frac{\text{Area}(\partial A)}{\epsilon^{d-2}} + \dots$$

Ryu-Takayanagi prescription



$$S_A = \min_{\gamma \sim A} \frac{\text{Area}(\gamma)}{4G_{d+1}}$$

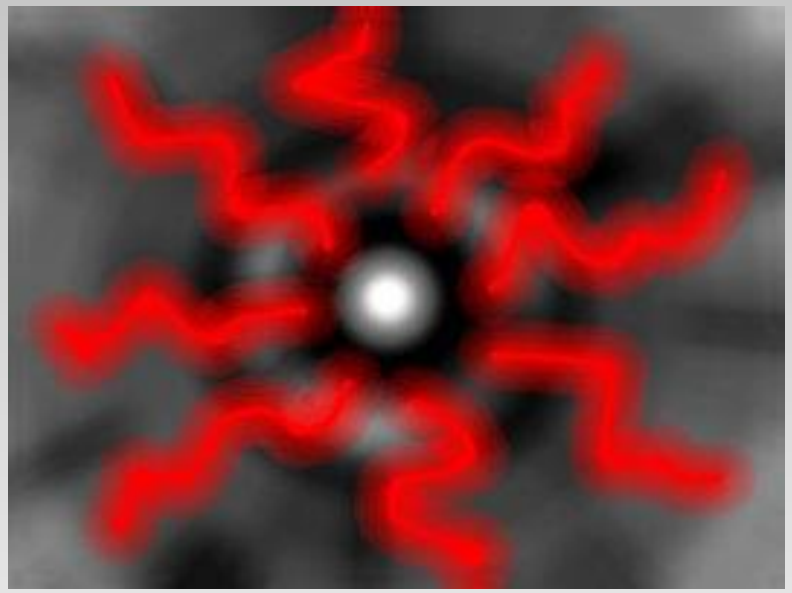
- Subaditivity:



“It from Qubit”

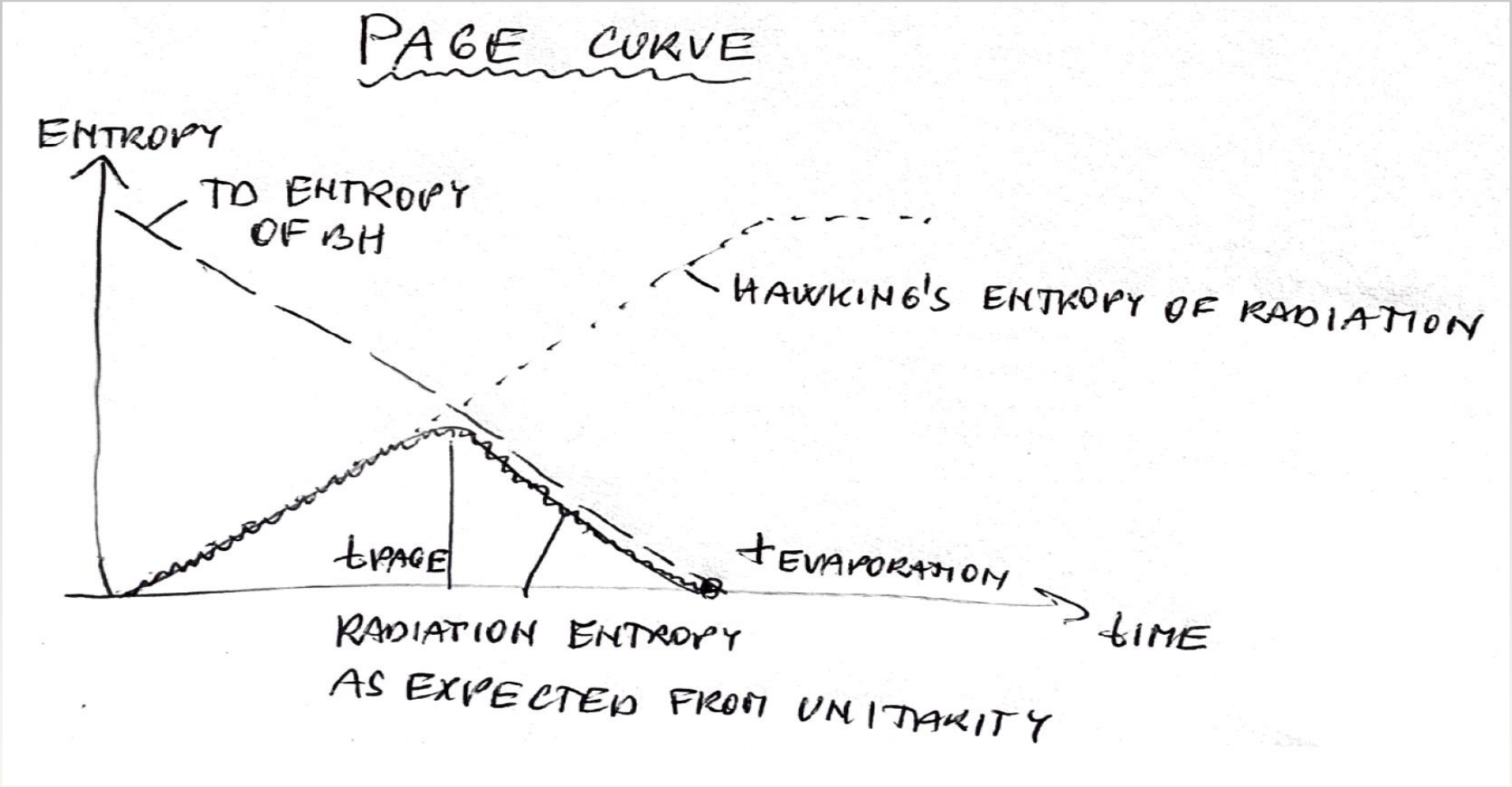
= how spacetime emerges from quantum information concepts

Back to BH info paradox



- AdS/CFT: bulk physics has to evolve unitarily
- To preserve unitarity: Hawking radiation cannot be perfectly thermal – info has to start coming out of the BH.
- Quantum entropy of Hawking radiation has to follow **Page curve** (so that we have pure state at the end)

Page curve


PAGE CURVE
ENTROPY
TO ENTROPY OF BH
HAWKING'S ENTROPY OF RADIATION
t PAGE
t EVAPORATION
RADIATION ENTROPY AS EXPECTED FROM UNITARITY
time

Gravitational entropy

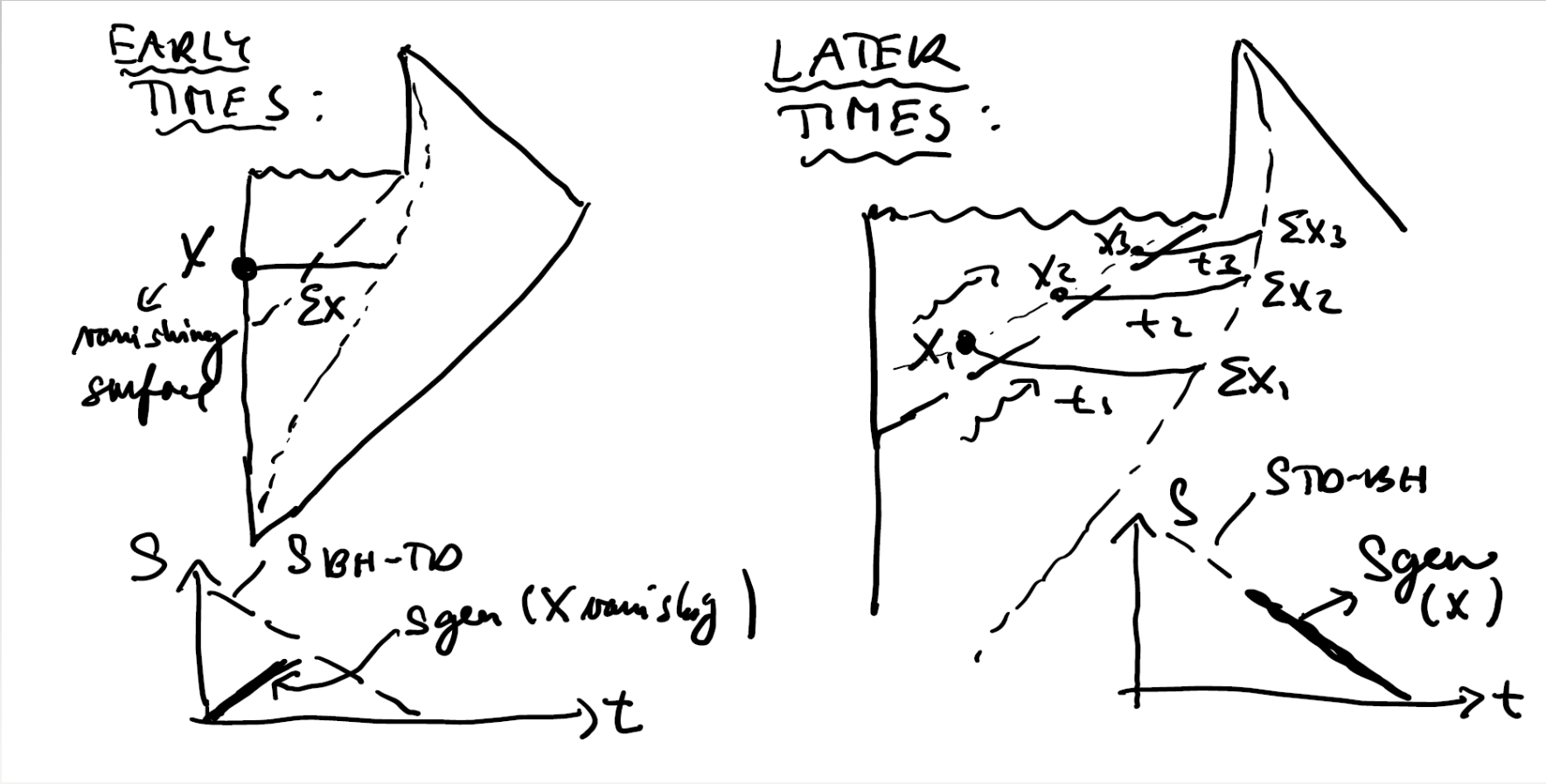
- "Generalization" of Ryu-Takayanagi formula for **quantum entropy of gravitating systems**:

$$S = \min_X \left[\mathrm{ext}_X \underbrace{\left(\frac{\mathrm{Area}(X)}{4G} + S_{\mathrm{semicl}}(\Sigma_X) \right)}_{S_{\mathrm{gen}(X)}} \right]$$

  - X… quantum extremal surface (generalization of a BH horizon)

- Reproduces Page curve:

Fairy tale



- It is not clear how the information starts coming out of the BH!

Summary

1) **AdS/CFT correspondence** was originally formulated as suality (equivalence) between certain type of string theory (Typ IIB) and maximally symmetric (conformal) field theory.
   - QG/QFT duality
   - holographic
   - Strong/weak coupling duality

2) More generally, people believe in **gauge/gravity duality.** Used for:
   - QGP
   - Superconductivity
   - Hydrodynamics and turbulence
   - Quantum properties as entanglement, complexity,…
   - Better understanding gravity: BH info paradox, dynamical spacetimes, emergence of spacetime from QI concepts