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AdS/CFT T1

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Black hole thermodynamics

1 Unruh radiation

a) Let us consider a spacetime associated with a uniformly accelerated observer, characterized by the proper acceleration $a = \sqrt{|a_\mu a^\mu|}$, whose coordinates (T, X, y, z) are related to the Minkowski coordinates (t, x, y, z) by

$$t = \left(\frac{1}{a} + X\right) \sinh(aT)\,, \quad x = \left(\frac{1}{a} + X\right) \cosh(aT)\,. \tag{1}$$

Show that the corresponding (flat) metric takes the following form:

$$ds^2 = -(1 + aX)^2 dT^2 + dX^2 + dy^2 + dz^2\,. \tag{2}$$

b) By Wick-rotating to Euclidean time $\tau = iT$ (and repeating the argument from the lecture) show that this spacetime can be assigned the following *Unruh temperature*:

$$T = \frac{1}{\beta} = \frac{a}{2\pi}\,. \tag{3}$$

Accelerated observers thus see a thermal bath of particles – the famous *Unruh's effect*.

c) Let us introduce a boundary at $X = X_0$. Show that the corresponding extrinsic curvature, $K = \nabla_\mu n^\mu$, where n_μ is the normal to this boundary, reads

$$K = \nabla_\mu n^\mu = \frac{a}{1 + aX_0}\,. \tag{4}$$

Thence show that the Euclidean action (ignoring counter-terms) now reads

$$S_E = -\frac{a\beta A}{8\pi}\,, \tag{5}$$

where A is the 'perpendicular Rindler horizon area'.

d) Calculate the corresponding free energy and entropy. Thus show that the entropy of the Rindler horizon also obeys the *Bekenstein's law*:

$$S = \frac{A}{4}\,. \tag{6}$$

2 Bekenstein's universal bound

Bekenstein bound provides an upper limit on the thermodynamic entropy of classical and quantum systems:

$$S \leq \frac{2\pi k_B R E}{\hbar c}\,, \tag{7}$$

where E is the total energy of the system, and R is the radius of a sphere enclosing it.

a) Let us qualitatively outline proof of this statement for small E. To this purpose, consider a system with (small) energy E and (arbitrary) entropy S, contained in a box of radius R, and consider a Schwarzschild black hole with (large) mass M and the same radius R. Let's next lower the system to the black hole, obtaining a black hole of mass $M + E$. By using the generalized second law, argue that (omitting all factor) we have

$$S \leq RE\,. \tag{8}$$

b) Note that the above bound is saturated for the black hole. Thence show that we get a universal bound on the amount of information in a given spatial region with a boundary of area A:

$$\boxed{S \leq \frac{A}{4}\,,} \tag{9}$$

as measured in Planck units. This means that the upper bound is bounded holographically – by the area of the region instead of its volume.