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AdS/CFT T2

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Strings, throats, and D-branes

1 Nambu–Goto vs. Polyakov

In the lecture we have seen the Polyakov action for massless relativistic strings is given by:

$$S_P = -\frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{-h}h^{AB}\gamma_{AB}\,. \tag{1}$$

By 'integrating out' the auxiliary metric h_{AB} show that this is (classically) equivalent to the Nambu–Goto action:

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d^2\xi\sqrt{-\det(\gamma_{AB})}\,. \tag{2}$$

(Hint: write down the EOM for the metric h_{AB} and plug it back to the action.)

2 Extremal black holes & near horizon limit

There exists an interesting class of black hole solutions of the Einstein–Maxwell theory, given by the so called *Majumdar–Papapetrou solution*:

$$ds^2 = -\frac{1}{H^2}dt^2 + H^2 dx^i dx^i\,,\quad A_\mu dx^\mu = -\frac{1}{H}dt\,, \tag{3}$$

where H obeys $\Delta H = 0$, with Δ being the 3-dimensional Laplace operator in the flat space spanned by x^i, i.e., H is any harmonic function.

a) Argue, that by introducing spherical coordinates on the flat space, $dx^i dx^i = dr^2 + r^2 d\Omega^2$, we can have the following special solution:

$$ds^2 = -\frac{1}{H^2}dt^2 + H^2(dr^2 + r^2 d\Omega^2)\,, \quad A_\mu dx^\mu = -\frac{1}{H}dt\,, \quad H = 1 + \frac{M}{r}\,. \quad (4)$$

Show that this is nothing else than the extremal ($M = Q$) Reissner–Nordstrom solution. Where is the horizon located in these coordinates? Where is the infinity?

Can you guess from here what does the full solution (3) represent? (What does the general H look like?)

b) Consider now a near horizon limit of (4), characterized by

$$r \ll M\,. \quad (5)$$

Here we are deeply in an (infinite) throat of the extremal black hole. Introducing a new coordinate $z = M^2/r$, show that up to an overall scale M^2, the metric now becomes a direct product of $AdS_2 \times S_2$:

$$ds^2 = \underbrace{\frac{1}{z^2}\Big(-dt^2 + dz^2\Big)}_{AdS_2} + \underbrace{d\Omega^2}_{S_2}\,. \quad (6)$$

This is precisely how the $AdS_5 \times S_5$ emerges in the AdS/CFT correspondence. All we have to do is to replace the 'point-like' extremal Reissner–Nordstrom solution by an extended (extremal) $D3$-brane.

3 Feelings for D-branes

As we have discussed in the lecture, in string theory we can have closed and open strings. The first describe gravity, the latter can end on D-branes where they describe elementary charged particles. To get a feeling for D-branes, let us turn to classical open strings (for example those on a guitar) and study their transverse vibrations. These are governed by the following action:

$$S = \int_{t_1}^{t_2} L dt = \int_{t_1}^{t_2} dt \int_0^a dx \mathcal{L}(\psi, \dot\psi, , \psi_x, t, x)\,, \quad \mathcal{L} = \frac{1}{2}\mu\dot\psi^2 - \frac{1}{2}T\psi_x^2\,, \quad (7)$$

where μ is the mass density of the string, and T is its tension. We also denoted by $\psi_x = \partial\psi/\partial x$ and $\dot\psi = \partial\psi/\partial t$.

a) Denoting by

$$p^t = \frac{\partial \mathcal{L}}{\partial \dot{\psi}}, \quad p^x = \frac{\partial \mathcal{L}}{\partial \psi_x}, \tag{8}$$

show that the action variation reads

$$\delta S = -\int_{t_1}^{t_2} dt \int_0^a dx \big[\dot{p}^t + \partial_x p^x\big]\delta\psi + \int_0^a \big[p^t \delta\psi\big]_{t_1}^{t_2} dx + \int_{t_1}^{t_2} \big[p^x \delta\psi\big]_0^a dt\,. \tag{9}$$

In order to impose the action principle, $\delta S = 0$, the three terms that we got must vanish independently!

b) Turn first to the first (bulk) term which describes a motion of the string for $x \in (0, a)$ and $t \in (t_1, t_2)$. Show that this leads to the following wave equation:

$$\psi_{xx} - \frac{1}{v^2}\ddot{\psi} = 0\,. \tag{10}$$

Identify the speed of wave propagation v.

c) Turning to the second term, convince yourself that it is natural to set *initial data* (and final data):

$$\delta\psi(t_1, x) = 0\,, \quad \delta\psi(t_2, x) = 0\,. \tag{11}$$

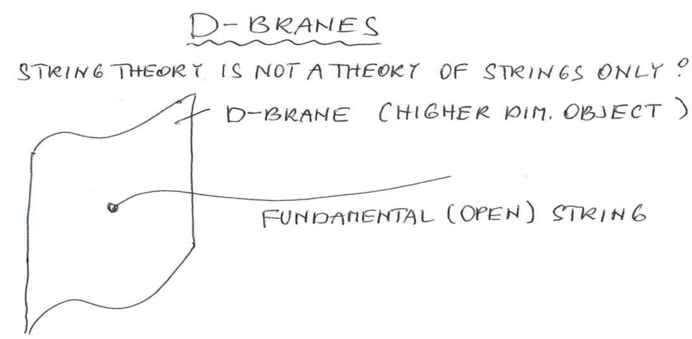
d) The last term is related to the evolution of endpoints $\psi(t, 0)$ and $\psi(t, a)$ and represents the *boundary conditions*. Two types of boundary conditions are often prescribed: Dirichlet and Neumann. Discuss these possibilities and show how they satisfy the variational principle.

e) Let us finally calculate the momentum/energy carried by the string:

$$P = \int_0^a p^t dx\,. \tag{12}$$

Show that for Neumann boundary conditions P is conserved, that is $\dot{P} = 0$. On the other hand, with Dirichlet conditions momentum can exchange with the 'wall" holding the fixed endpoints.

In string theory, the endpoints of open strings may be attached to *D-branes* that can exchange momentum with the string, see the following picture:



and [1] for more details.

References

[1] B. Zwiebach, *A first course in string theory.* Cambridge university press, 2004.