Tutorial 4: Feelings for D-branes

As we have discussed in the lecture, in string theory we can have closed and open strings. The first describe gravity, the latter can end on D-branes where they describe elementary charged particles. To get a feeling for D-branes, let us turn to classical open strings (for example those on a guitar) and study their transverse vibrations. These are governed by the following action:

$$S = \int_{t_1}^{t_2} L dt = \int_{t_1}^{t_2} dt \int_0^a dx \mathcal{L}(\psi, \dot{\psi}, , \psi_x, t, x)\,, \quad \mathcal{L} = \frac{1}{2}\mu\dot{\psi}^2 - \frac{1}{2}T\psi_x^2\,, \tag{1}$$

where μ is the mass density of the string, and T is its tension. We also denoted by $\psi_x = \partial\psi/\partial x$ and $\dot{\psi} = \partial\psi/\partial t$.

a) Denoting by

$$p^t = \frac{\partial \mathcal{L}}{\partial \dot{\psi}}\,, \quad p^x = \frac{\partial \mathcal{L}}{\partial \psi_x}\,, \tag{2}$$

show that the action variation reads

$$\delta S = -\int_{t_1}^{t_2} dt \int_0^a dx \big[\dot{p}^t + \partial_x p^x\big]\delta\psi + \int_0^a \big[p^t\delta\psi\big]_{t_1}^{t_2} dx + \int_{t_1}^{t_2} \big[p^x\delta\psi\big]_0^a dt\,. \tag{3}$$

In order to impose the action principle, $\delta S = 0$, the three terms that we got must vanish independently!

b) Turn first to the first (bulk) term which describes a motion of the string for $x \in (0, a)$ and $t \in (t_1, t_2)$. Show that this leads to the following wave equation:

$$\psi_{xx} - \frac{1}{v^2}\ddot{\psi} = 0\,. \tag{4}$$

Identify the speed of wave propagation v.

c) Turning to the second term, convince yourself that it is natural to set *initial data* (and final data):

$$\delta\psi(t_1, x) = 0\,, \quad \delta\psi(t_2, x) = 0\,. \tag{5}$$

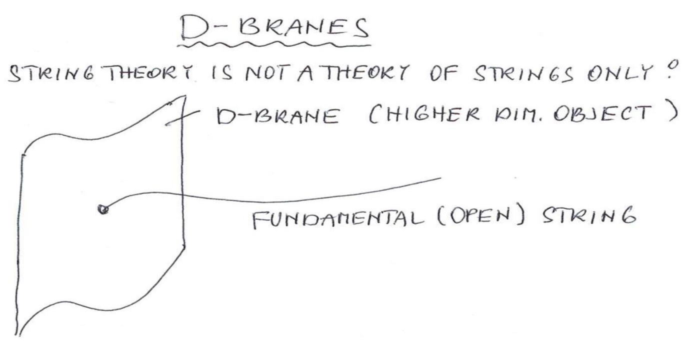
d) The last term is related to the evolution of endpoints $\psi(t, 0)$ and $\psi(t, a)$ and represents the *boundary conditions*. Two types of boundary conditions are often prescribed: Dirichlet and Neumann. Discuss these possibilities and show how they satisfy the variational principle.

e) Let us finally calculate the momentum/energy carried by the string:

$$P = \int_0^a p^t dx\,. \tag{6}$$

Show that for Neumann boundary conditions P is conserved, that is $\dot{P} = 0$. On the other hand, with Dirichlet conditions momentum can exchange with the 'wall" holding the fixed endpoints.

In string theory, the endpoints of open strings may be attached to *D-branes* that can exchange momentum with the string, see the following picture:



and [1] for more details.

References

[1] B. Zwiebach, *A first course in string theory.* Cambridge university press, 2004.