

AdS/CFT T4 David Kubiznak

## Wilson loops & finite temperature

## 1 Wilson loops: quark-antiquark potential



Let us consider the above quark-antiquark contour, where we have chosen the separation R in the x-direction, and parametrized z = z(x) in  $AdS_5$  (while fixing the string in  $S^5$ ).

a) Show that we have the following induced metric on the string worldsheet:

$$\gamma = \frac{\ell^2}{z^2} \left( d\tau^2 + (1 + z'^2) dx^2 \right),\tag{1}$$

where z' = dz/dx. Hence show that the Nambu–Goto action reads:

$$S_{\rm NG} = \frac{1}{2\pi\alpha'} \int_{\Sigma} d^2\sigma \sqrt{\det\gamma_{AB}} = \frac{T\ell^2}{2\pi\alpha'} \int dx \frac{\sqrt{1+z'^2}}{z^2} \,. \tag{2}$$

b) Think about the above action as a mechanical problem where x plays the role of time. Argue there is a conserved quantity, which can be (for example) written as

$$z_*^2 = z^2 \sqrt{1 + z'^2} = \text{const.}$$
 (3)

c) By integrating the latter equation, show that (taking into the boundary conditions) we find

$$z_* = kR, \quad k = \left(2\int_0^1 \frac{q^2 dq}{\sqrt{1-q^4}}\right)^{-1} \approx 0.83.$$
 (4)

d) Plug the first integral back to the action and show that it equals to

$$S_{\rm NG} = \frac{2}{z_*} \frac{T\ell^2}{2\pi\alpha'} k_\epsilon \,, \quad \tilde{k}_\epsilon \sim \int_\epsilon^1 \frac{dq}{q^2\sqrt{1-q^4}} \approx \frac{1}{\epsilon} - k_0 + O(\epsilon) \,, \quad k_0 \approx 0.599 \,. \tag{5}$$

e) Obviously, the previous result is infinite (contains infinite self-energy of the quarks) and has to be renormalized. This is, for example, done by subtracting  $S_{\text{NG}}^0$  of two parallel strings hanging between  $z = \epsilon z_*$  and  $z = \infty$ . Show that, we then find

$$S_{\rm NG} - S_{\rm NG}^0 = -\frac{2}{z_*} \frac{T\ell^2}{2\pi\alpha'} k_0 \,, \tag{6}$$

which, when compared to TV(R), yields:

$$V = -\frac{\ell^2}{\alpha'} \frac{k_0}{k\pi} \frac{1}{R} \approx -0.23 \frac{\sqrt{2\lambda}}{R} \,. \tag{7}$$

So you just derived the Coulomb law :)

## 2 CFT at finite temperature: dimensional analysis

a) Consider a d-dimensional CFT at finite temperature T. Argue that based on the dimensional analysis, the entropy density s must take the following form:

$$s = g(\lambda)T^{d-1} \tag{8}$$

where  $g(\lambda)$  is some function of the 't Hooft coupling  $\lambda$ .

- b) Use next the first law,  $d\epsilon = Tds$ , to calculate the internal energy.
- c) In the rest frame we have  $\langle T_{\mu\nu} \rangle = \text{diag}(\epsilon, P, P, P)$ . Use the property of CFT to calculate the pressure P, and the speed of sound:  $v_s^2 = \partial P / \partial \epsilon$ .
- d) Finally, calculate the free energy density f from the standard relation  $f = \epsilon Ts$  and show that we have the following Euler relation:

$$\epsilon = Ts - P \,. \tag{9}$$