Solutions T6

1 BF bound and spontaneous scalarization

When studying the holographic superconductor, we have set

$$m_\psi^2 = -\frac{2}{\ell^2} \tag{1}$$

for the mass of the scalar field. Going beyond the test field limit, let us consider planar *extremal* charged AdS_4 black hole as our gravity background. Recall that the BF bound in AdS_d discussed in the lecture is

$$m^2 \geq m_{\mathrm{BF}}^2 = -\frac{(d-1)^2}{4\ell^2}. \tag{2}$$

As long as this is satisfied the scalar field is stable. In particular in $d = 4$ we have

$$m_{\mathrm{BF4}}^2 = -\frac{9}{4\ell^2}\,. \tag{3}$$

From here it seems that our field should be stable, as asymptotically we have:

$$m_\psi^2 > m_{\mathrm{BF4}}^2\,. \tag{4}$$

However, this is not the case close to the horizon. Namely we already know that the near horizon limit of extremal Reissner–Nordstrom solution yields $AdS_2 \times R^2$ geometry (in the planar case). This is also the case with the extremal AdS_4 Reissner–Nordstrom solution. In the throat our field thus experiences AdS_2 rather than AdS_4. However, for $2d$ AdS space the BF bound is higher:

$$m_{\mathrm{BF2}}^2 = -\frac{1}{4\ell_2^2}\,, \tag{5}$$

This suggest that our field may be tachyonic close to the horizon – suggesting instability. Indeed, the near horizon limit yields $\ell_2^2 = \ell^2/6$, thus the bound is

$$m_{\rm BF2}^2 = -\frac{1}{4\ell_2^2} = -\frac{3}{2\ell^2} > m_\psi^2 \, . \tag{6}$$

Thence our field is indeed tachyonic. Can you now see the clever choice for the mass of the scalar above?