FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

Selected Topics in AdS/CFT correspondence: List of Exam Questions

What to expect

You will be asked one theoretical and one computational question out of the following two sets. You will have 30 minutes to prepare your answers. After that, we will discuss your answers together with some (un)related problems. The discussion should last about 30 minutes, after which I will provide my feedback to you.

1 Computational questions

1) Given the Rindler spacetime

$$ds^2 = -(1+aX)^2 dT^2 + dX^2 + dy^2 + dz^2\,, \tag{1}$$

discuss the Unruh effect – derive the temperature and entropy associated with the Rindler horizon.

2) Show the classical equivalence of the Polyakov, $S_{\rm P}$, and the Nambu–Goto, $S_{\rm NG}$, actions:

$$S_{\rm P} = -\frac{1}{4\pi\alpha'}\int d^2\xi\sqrt{-h}h^{AB}\gamma_{AB}\,, \quad S_{\rm NG} = -\frac{1}{2\pi\alpha'}\int d^2\xi\sqrt{-\det(\gamma_{AB})}\,. \tag{2}$$

3) Derive the near horizon limit of the non-extremal $D3$-brane:

$$\begin{aligned} ds_{10}^2 &= H^{-1/2}(-f dt^2 + dx^2 + dy^2 + dz^2) + H^{1/2}(\frac{dr^2}{f} + r^2 d\Omega_5^2)\,, \\ f &= 1 - \left(\frac{r_0}{r}\right)^4, \quad H = 1 + \frac{\ell^4}{r^4}\,, \end{aligned} \tag{3}$$

and calculate its temperature. How is this related to the temperature of the dual CFT?

4) By performing the dimensional analysis, derive the temperature dependence of various thermodynamic quantities (s, P, ρ, v_s) of the CFT at finite temperature. Write down the corresponding fluid energy momentum tensor, assuming the perfect fluid approximation.

5) Show that, provided the matter action S_m is diffeomorphism invariant and the equation of motion for the matter are satisfied, $\delta S_m/\delta\phi = 0$, the matter energy momentum tensor is conserved: $\nabla_\mu T^{\mu\nu} = 0$. Similarly show, that CFTs have traceless energy momentum tensor, $T^\mu{}_\mu = 0$.

6) Consider infinitesimal conformal transformation $x^\mu \to x'^\mu = x^\mu - \xi^\mu$, obeying $\eta_{\mu\nu} \to (1 + \omega(x))^2 \eta_{\mu\nu}$. Show that ξ must satisfy the conformal Killing vector equation $\partial_\mu \xi_\nu + \partial_\nu \xi_\mu \propto \eta_{\mu\nu}$ and can be found in the following form:

$$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + (b^\mu x^2 - 2x^\mu b \cdot x)\,. \tag{4}$$

Can you identify the various pieces and the symmetry group?

7) Show strong subaditivity for the holographic entanglement entropy in the static case.

2 Theoretical questions

1) Sketch the Penrose diagram of a completely evaporating black hole and formulate the black hole information paradox. What kind of resolution follows from the AdS/CFT correspondence?

2) Describe the Hawking–Page transition and discuss its dual interpretation.

3) Motivate the AdS/CFT correspondence by sketching the open and closed string pictures of the duality.

4) Discuss the basic characteristics of the AdS/CFT correspondence. Can you formulate its (practical) weak version?

5) Introduce AdS as a maximally symmetric space – why is it maximally symmetric?

6) Formulate the state–operator correspondence. How to calculate the CFT correlation functions via AdS/CFT?

7) Introduce Wilson loops and motivate how to calculate their expectation value on the gravity side.

8) Discuss the basic idea of hydrodynamics – introduce concepts as local equilibrium, constitutive relations, derivative expansion, transport coefficients. What remarkable prediction AdS/CFT gives for the shear viscosity?

9) Introduce the concept of entanglement entropy and provide a prescription for calculating it holographically.

10) Introduce the Page curve, and tell us a fairy tale as to how it can be recovered via the quantum island construction.