Is the information hidden in the correlations?

Let's start by the seminal postulation of this problem, where also nowadays most of the discussion takes place:

Page curve

It proposes to show that Hawking radiation has subtle hidden correlations and it is actually in a pure state.

We have a system consisting in a black hole and the Hawking radiation.

Assume that the black hole subsystem has dimension $n_H \sim e^{S_H}$ where $S_H = A/4$, and the radiation subsystem has dimension $n_R \sim e^{S_R}$ being S_R the thermodynamic radiation entropy.

The hypothesis is that there is no information lost. So, assume that these subsystems form a total system in a pure state, in a Hilbert space of dimension $n_H n_R$. See that if the black hole and the radiation are correlated, each of these subsystems would be in a mixed state.

Page uses a method of average subsystem entropies. Consider that total Hilbert space factorizes

$$\underbrace{\mathcal{H}_{RH} = \mathcal{H}_R}_{\text{radiation subsystem}} \otimes \underbrace{\mathcal{H}_H}_{\text{black hole subsystem}}$$

The total Hilbert space is in a pure state, so the normalized density matrix is given by $\rho = |\psi\rangle\langle\psi|$

We can define the subsystem density matrices as

$$\rho_H = tr_R\, \rho \quad \text{and} \quad \rho_R = tr_H\, \rho$$

Then, the entanglement entropy of each subsystem is given by

$$S_H = -tr\, \rho_H \ln \rho_H \quad \text{and} \quad S_R = -tr\, \rho_R \ln \rho_R$$

We have also seen that for a bipartite system in a pure state $S_{RH} = 0$ and the subsystem entropies $S_R = S_H$.

Page proposal works for every randomly chosen pure state, for that reason Page defines an average over all possible pure states that purify the subsystem.

There were some tries to exactly calculate these entropies but none of them was fruitful. In any case, all the methods were able to put some bounds on the entropies, and they found they are very close to its maximum value.

If one considers $n_H = \dim\{\mathcal{H}_H\}$, $n_R = \dim\{\mathcal{H}_R\}$ and

$m = \min[n_R, n_H]$

$$\langle S_H \rangle = \langle S_R \rangle \leq \ln m$$

The most accurate bound: $\langle S_H \rangle \in \left(\ln m - \frac{1}{2\ln 2}, \ln m \right)$ bits

(approx. 3/4 bits of maximal entropy)

In this scheme, Page defined an asymmetric information associated to each subsystem, given by the difference of the maximum entropy of the subsystem to the measured entropy: deviation of entanglement entropy from the maximum.

We already know that the maximum entropy for a subsystem is given by the logarithm of its number of microstates, that is, the dimension of its Hilbert space.

$$I_R = \ln n_R - S_R \quad , \quad I_H = \ln n_H - S_H$$

The information is maximum when the system is bigger because then the logarithm is much greater. When the system is smaller, the logarithm of its Hilbert space is the same then the logarithm of the minimum Hilbert space which appears in the calculation of von Neumann entropy.

We want to know how is the evolution of the entropy and information flow through the evaporation process.

Let's first check the initial state of the system. There is not yet any Hawking radiation, so $\mathcal{H}_R$ is trivial (1-dimensional). And the Hilbert space associated with the black hole is enormous. As the Page formula is determined by the minimum Hilbert space: $\langle S_H \rangle|_0 = 0$

As the black hole evaporates, the dimension of its Hilbert space reduces and the dimension of the radiation space increases.

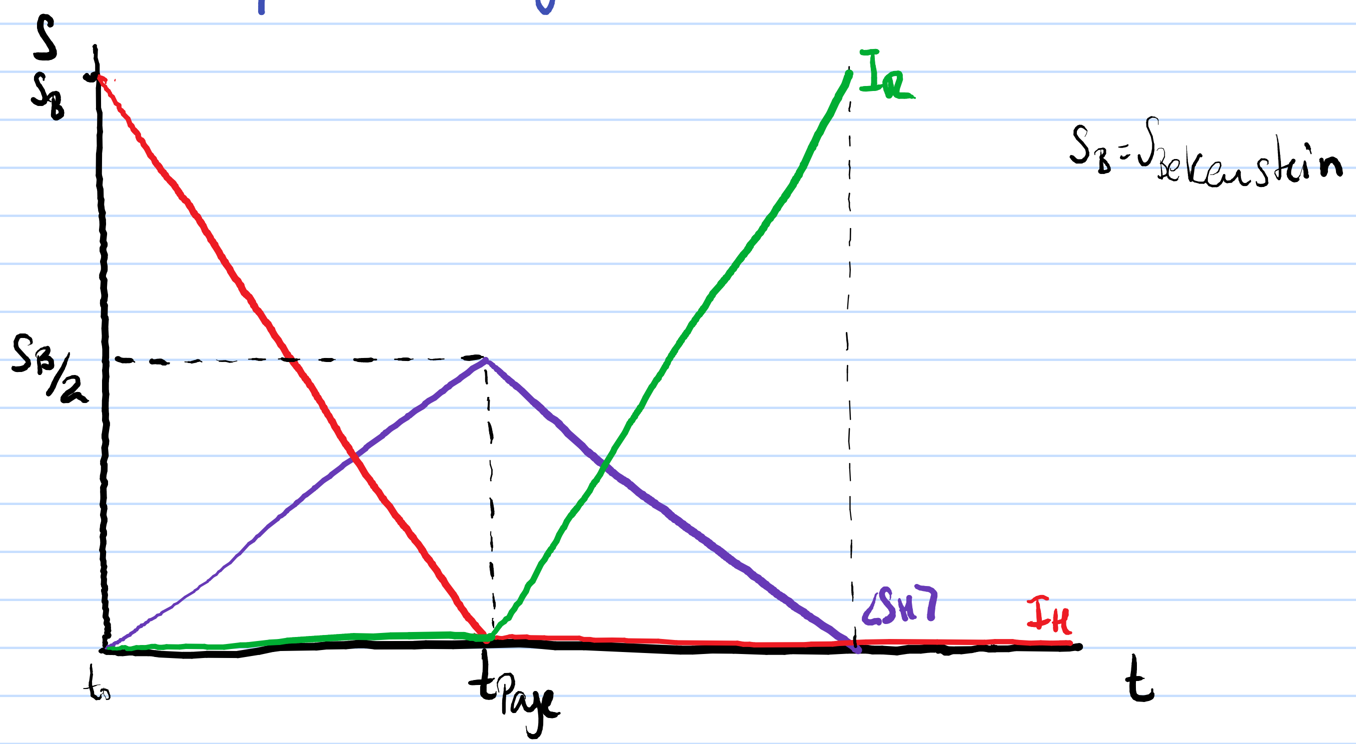
Then, in a similar way, the final state of the system is reached when the black hole has completely evaporated, so $\mathcal{H}_H$ is now trivial and $\mathcal{H}_R$ enormous. Identically, the relevant one is the minimum one, so symmetrically we have $\langle S_H \rangle|_\infty = 0$.

At intermediate times both dimensionalities are non trivial so $\langle S_H \rangle|_t \neq 0$. The way to qualitatively determine it is: the evolution is assumed to be unitary, so the total Hilbert space is constant: $n_H(t)\, n_R(t) = n_{H_0} = n_{R_\infty}$

Then, $\langle S_H \rangle|_t = \ln \min \left\{ n_H(t), \frac{n_{H_0}}{n_H(t)} \right\}$

The entropy is maximized when: $n_H(t) \approx \sqrt{n_{H_0}}$ reaching the value $\langle S_H \rangle (t = t_{Page}) \approx \frac{1}{2} \ln n_{H_0} = \frac{S_0}{2}$ and $t_{Page} \approx t_{evaporation}/2$

One can depict the Page curve as



All the three curves are of order of the Bekenstein entropy (GM^2)

Hawking : $0 \to GM^2$, Bekenstein : $GM^2 \to 0$

Page : $0 \to GM^2/2 \to 0$

➔ What would be the evolution of the mutual entropy and how would you interpret it?

Note that the smaller subsystem is the closer to be maximally mixed.

This curve is interpreted as the information coming out from the black hole gradually after the Page time, when the system purifies. This ends up by recovering all the information (and the black hole information vanishes). The information in the radiation is codified in correlations spread through it.

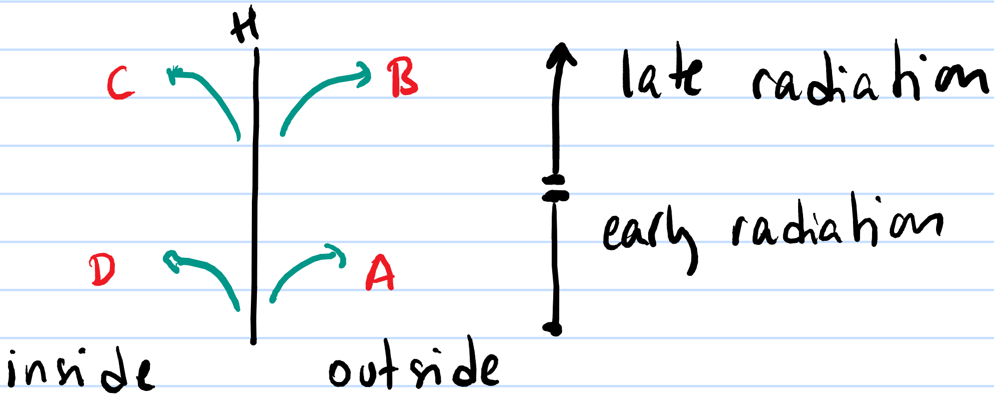
So, system starts purifying after Page time and it finishes in a pure state.

The radiation emitted before Page time is called: early radiation, and the radiation after Page time : late radiation.

Intuitively : At the beginning of the evaporation process, the radiation comes out entangled with the remaining black hole. But eventually it must start coming out entangled with the early radiation.

Soon, a problem was pointed out...

Consider two Hawking quanta, one early and one late



Each escaping particle is accompanied by a partner inside the horizon. In order for a free falling observer to see the standard vacuum near H, particle A must be entangled with its partner D. This correlation is part of the basic structure of the quantum vacuum; it can be violated only at the cost of introducing large vacuum expectation values of such quantities as the stress-energy tensor.

If the Hawking radiation is to be in a pure state in the end, particle A must be entangled with B (the other particle in the radiation). This violates monogamy of entanglement!

Let's suppose AB is in a pure state, so $S_{AB} = 0$. From the non-negativity of mutual information $I(D;AB) = S_D + S_{AB} - S_{ABD}$ and the triangle inequality $|S_D - S_{AB}| \leq S_{ABD}$, we see that

$$I(D;AB) = S_D + S_{AB} - S_{ABD} = 0$$

we know that the mutual information between two tensor factors vanishes only if the state is a product state, so we have then $\rho_{ABD} = \rho_D \otimes \rho_{AB}$, which is incompatible with any entanglement between A and D. Conversely we could have assumed that A and D are entangled, concluding that there is no possible correlation between A and B.

Mathur show it in the following way:

Consider three systems: a Hawking mode B emitted when the black hole is old, its interior partner C, and all prior Hawking radiation A. The total Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$

The von Neumann entropies satisfy strong subadditivity

$$S_{AB} + S_{BC} \geq S_B + S_{ABC}$$

B and C must be in a pure entangled state to give rise to vacuum (observed by an infalling observer) $S_{BC} = 0$. Therefore we have $S_{ABC} = S_A$

So previous inequality reads $S_{AB} - S_A \geq S_B$

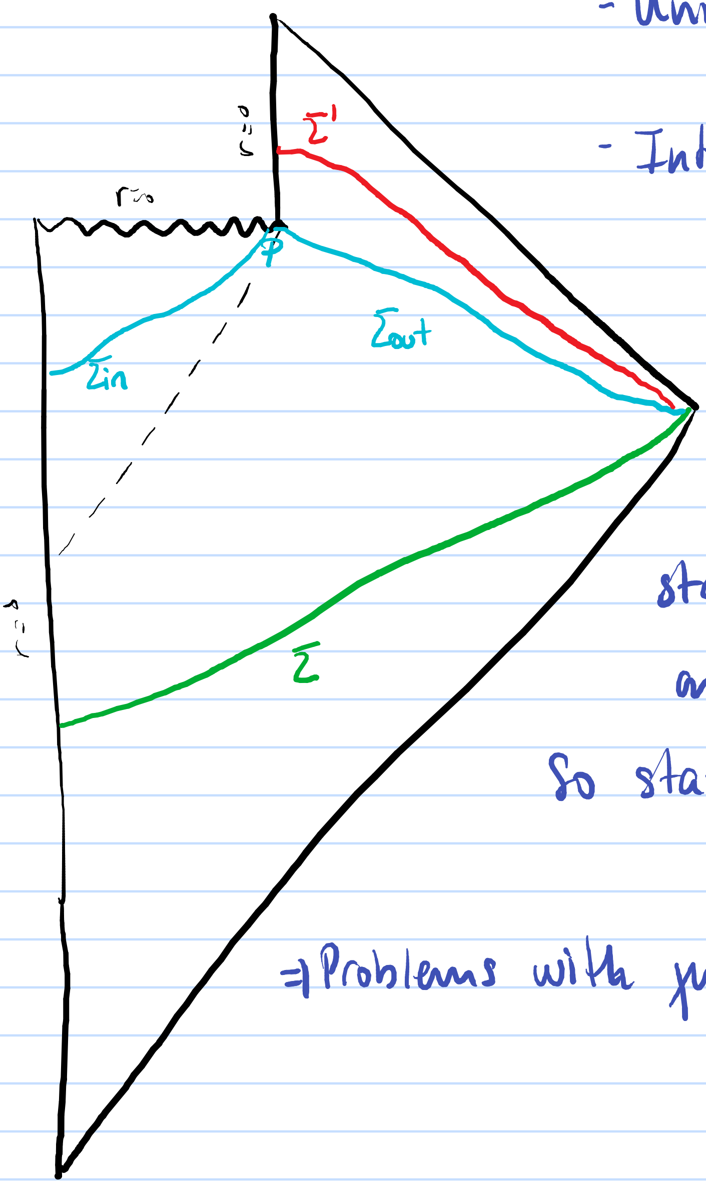
On the other hand, an old black hole begins purifying early radiation, so the entropy of AB is less than the entropy of A: $S_{AB} < S_A$

But then one gets: $0 > S_B$ Contradiction!

Black hole complementarity

Assuming that unitarity is to be restored just by the evaporation, the infalling information should be found in the Hawking radiation. For that, it has remained above the black hole event horizon forming the stretched horizon. This is a kind of membrane hovering outside the event horizon. It is a timelike surface that contains all the degrees of freedom associated to the black hole. But as the information also falls into the black hole, it would violate the no cloning theorem of quantum mechanics. If the cloning does not happen, either the information is not recovered (then unitarity is violated) or no information can cross the horizon, which would violate the equivalence principle of General Relativity (which implies that nothing dramatic should happen at the event horizon).
Black hole complementarity assumes unitarity + equivalence principle at expenses of allowing cloning, but the cloning cannot be observed because each observer sees only one copy, so no observer is able to confirm both stories simultaneously and the different observers cannot meet to confirm it neither. According to an infalling observer, nothing special happens in the horizon and enters the black hole. According to an external observer infalling information hit the stretched horizon and re-radiates as Hawking radiation with all the information.

If we consider an entangled pair of bits A and B outside the black hole, and throw B in, then information of B codifies in the stretched horizon and a bit C emerges entangled with A and carrying all the information. Here is where previous problem of monogamy of entanglement shows up! But the claim is that any observer sees B and C. They would have to wait for the copy C to emerge, measure it, and then jump into the black hole and see the original B before it hits the singularity.



- Unitary evolution from Σ to Σ'

- Intermediate: Σ_p where

$$\mathcal{H} = \mathcal{H}_{in} \otimes \mathcal{H}_{out}$$

But state on Σ' evolved from Σ_{out}, and also states there depend linearly on states on Σ

So states in Σ_{in}?

$\Rightarrow$ Problems with quantum theory

Postulates of black hole complementarity:

- Postulate 1 (Purity): Formation and evaporation of black holes can be described in the context of standard quantum theory. In particular: unitary evolution
- Postulate 2 (Effective field theory): Outside the stretched horizon physics is accurately described by semiclassical field equations
- Postulate 3 (microscopic BH entropy): To a distant observer, a black hole appears to be a quantum system with discrete energy levels. The dimension of the subspace of states describing a black hole is the exponential of the Bekenstein entropy.
- Postulate 4 (no drama): A free falling observer experiences nothing out of ordinary when crossing the horizon. This is equivalent to equivalence principle.

Firewalls (AMPS)

Based in the paradox of monogamy of entanglement they claim P1, P2, P3 and P4 are inconsisten with each other.

P1: old black hole is almost maximally entangled with the Hawking radiation it has radiated so far (early radiation)

P4: Requires that the infalling observer sees a quantum vacuum at the event horizon.

This is what we have already seen in the analysis of Page curve.

If one breaks the entanglement accross the horizon to avoid monogamy of entanglement, the quantum state around the horizon cannot be vacuum. The resulting large expectation values of the stress-energy tensor would then form a "firewall" for any infalling observer. Infalling observers detect particles of high energy at the horizon, for that reason it's so called firewall.

The equivalence principle requires that the event horizon cannot be determined locally, so the region of spacetime around the horizon is not locally different from any other region and it's in Minkowski vacuum state. But if an observers encounters a firewall, horizon is a special place ⇒ This proposal implies the breakdown of equivalence principle.

→ Islands of entanglement proposal (Almheiri, Maldacena, etc...)