Cylindrické souřadnice (ρ, φ, z)

Vztah ke kartézským souřadnicím:

$$x = \rho\cos\varphi \qquad \rho = \sqrt{x^2+y^2}$$
$$y = \rho\sin\varphi \qquad \varphi = \arctan\frac{y}{x}$$
$$z = z \qquad z = z$$

Metrika a objemový element:

$$\boldsymbol{q} = \mathbf{d}\rho\,\mathbf{d}\rho + \rho^2\,\mathbf{d}\varphi\,\mathbf{d}\varphi + \mathbf{d}z\,\mathbf{d}z \qquad dV = \rho\,d\rho\,d\varphi\,dz$$

Laméovy koeficienty:

$$h_\rho = 1 \quad , \quad h_\varphi = \rho \quad , \quad h_z = 1$$

Triáda (normovaná báze):

$$\boldsymbol{e}_{\hat\rho} = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} \qquad \boldsymbol{e}_{\hat\varphi} = \frac{1}{\rho}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} \qquad \boldsymbol{e}_{\hat z} = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial}z}$$
$$\boldsymbol{e}^{\hat\rho} = \mathbf{d}\rho \qquad \boldsymbol{e}^{\hat\varphi} = \rho\,\mathbf{d}\varphi \qquad \boldsymbol{e}^{\hat z} = \mathbf{d}z$$

Vztah souřadnic tenzorů (různé druhy indexů):

$$a^\rho = a^{\hat\rho} = a_{\hat\rho} = a_\rho$$
$$\rho\,a^\varphi = a^{\hat\varphi} = a_{\hat\varphi} = \frac{1}{\rho}a_\varphi$$
$$a^z = a^{\hat z} = a_{\hat z} = a_z$$

Vektorové násobení a antisymetrický tenzor:

$$\boldsymbol{\epsilon} = \rho\;\mathbf{d}\rho\wedge\mathbf{d}\varphi\wedge\mathbf{d}z$$
$$\epsilon_{\rho\varphi z} = \rho \qquad \epsilon_{\hat\rho\hat\varphi\hat z} = 1 \qquad \epsilon^{\rho\varphi z} = \frac{1}{\rho}$$
$$\epsilon^{\rho}{}_{\varphi z} = \rho \qquad \epsilon^{\varphi}{}_{z\rho} = \frac{1}{\rho} \qquad \epsilon^{z}{}_{\rho\varphi} = \rho$$

$$\boldsymbol{c} = \boldsymbol{a}\times\boldsymbol{b}$$
$$c^\rho = \rho\,(a^\varphi b^z - a^z b^\varphi) \qquad c^\varphi = \frac{1}{\rho}\,(a^z b^\rho - a^\rho b^z) \qquad c^z = \rho\,(a^\rho b^\varphi - a^\varphi b^\rho)$$

Vztah cylindrických a kartézských bází vektorů a forem

$$\boldsymbol{e}_{\hat{x}} = \cos\varphi\,\boldsymbol{e}_{\hat{\rho}} - \sin\varphi\,\boldsymbol{e}_{\hat{\varphi}} = \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{e}_{\hat{\rho}} - \frac{y}{\sqrt{x^2+y^2}}\boldsymbol{e}_{\hat{\varphi}}$$
$$\boldsymbol{e}_{\hat{y}} = \sin\varphi\,\boldsymbol{e}_{\hat{\rho}} + \cos\varphi\,\boldsymbol{e}_{\hat{\varphi}} = \frac{y}{\sqrt{x^2+y^2}}\boldsymbol{e}_{\hat{\rho}} + \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{e}_{\hat{\varphi}}$$
$$\boldsymbol{e}_{\hat{z}} = \boldsymbol{e}_{\hat{z}}$$

$$\boldsymbol{e}_{\hat{\rho}} = \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{e}_{\hat{x}} + \frac{y}{\sqrt{x^2+y^2}}\boldsymbol{e}_{\hat{y}} = \cos\varphi\,\boldsymbol{e}_{\hat{x}} + \sin\varphi\,\boldsymbol{e}_{\hat{y}}$$
$$\boldsymbol{e}_{\hat{\varphi}} = -\frac{y}{\sqrt{x^2+y^2}}\boldsymbol{e}_{\hat{x}} + \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{e}_{\hat{y}} = -\sin\varphi\,\boldsymbol{e}_{\hat{x}} + \cos\varphi\,\boldsymbol{e}_{\hat{y}}$$
$$\boldsymbol{e}_{\hat{z}} = \boldsymbol{e}_{\hat{z}}$$

$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} x} = \cos\varphi\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} - \frac{\sin\varphi}{\rho}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} = \frac{x}{\sqrt{x^2+y^2}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} - \frac{y}{x^2+y^2}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi}$$
$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} y} = \sin\varphi\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} + \frac{\cos\varphi}{\rho}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} = \frac{y}{\sqrt{x^2+y^2}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} + \frac{x}{x^2+y^2}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi}$$
$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} z} = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial} z}$$

$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} = \frac{x}{\sqrt{x^2+y^2}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} x} + \frac{y}{\sqrt{x^2+y^2}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} y} = \cos\varphi\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} x} + \sin\varphi\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} y}$$
$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} = -y\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} x} + x\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} y} = -\rho\sin\varphi\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} x} + \rho\cos\varphi\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} y}$$
$$\frac{\boldsymbol{\partial}}{\boldsymbol{\partial} z} = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial} z}$$

$$\mathbf{d}x = \cos\varphi\,\mathbf{d}\rho - \rho\sin\varphi\,\mathbf{d}\varphi = \frac{x}{\sqrt{x^2+y^2}}\,\mathbf{d}\rho - y\,\mathbf{d}\varphi$$
$$\mathbf{d}y = \sin\varphi\,\mathbf{d}\rho + \rho\cos\varphi\,\mathbf{d}\varphi = \frac{y}{\sqrt{x^2+y^2}}\,\mathbf{d}\rho + x\,\mathbf{d}\varphi$$
$$\mathbf{d}z = \mathbf{d}z$$

$$\mathbf{d}\rho = \frac{1}{\sqrt{x^2+y^2}}(x\,\mathbf{d}x + y\,\mathbf{d}y) = \cos\varphi\,\mathbf{d}x + \sin\varphi\,\mathbf{d}y$$
$$\mathbf{d}\varphi = \frac{1}{x^2+y^2}(x\,\mathbf{d}y - y\,\mathbf{d}x) = \frac{\cos\varphi}{\rho}\,\mathbf{d}y - \frac{\sin\varphi}{\rho}\,\mathbf{d}x$$
$$\mathbf{d}z = \mathbf{d}z$$

Derivace bázových vektorů a forem

$$\nabla \boldsymbol{e}_{\hat\rho} = \frac{1}{\rho}\,\boldsymbol{e}^{\hat\varphi}\,\boldsymbol{e}_{\hat\varphi} \qquad \nabla\cdot\boldsymbol{e}_{\hat\rho} = \frac{1}{\rho} \qquad \nabla\times\boldsymbol{e}_{\hat\rho} = 0$$

$$\nabla \boldsymbol{e}_{\hat\varphi} = -\frac{1}{\rho}\,\boldsymbol{e}^{\hat\varphi}\,\boldsymbol{e}_{\hat\rho} \qquad \nabla\cdot\boldsymbol{e}_{\hat\varphi} = 0 \qquad \nabla\times\boldsymbol{e}_{\hat\varphi} = \frac{1}{\rho}\,\boldsymbol{e}_{\hat z}$$

$$\nabla \boldsymbol{e}_{\hat z} = 0 \qquad \nabla\cdot\boldsymbol{e}_{\hat z} = 0 \qquad \nabla\times\boldsymbol{e}_{\hat z} = 0$$

$$\nabla\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} = \frac{1}{\rho}\,\mathbf{d}\varphi\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} \qquad \nabla\cdot\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} = \frac{1}{\rho} \qquad \nabla\times\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} = 0$$

$$\nabla\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} = \frac{1}{\rho}\,\mathbf{d}\rho\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} - \rho\,\mathbf{d}\varphi\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} \qquad \nabla\cdot\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} = 0 \qquad \nabla\times\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} = 2\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}z}$$

$$\nabla\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}z} = 0 \qquad \nabla\cdot\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}z} = 0 \qquad \nabla\times\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}z} = 0$$

$$\nabla\,\mathbf{d}\rho = \rho\;\mathbf{d}\varphi\,\mathbf{d}\varphi \qquad \nabla^2\,\rho = \frac{1}{\rho}$$

$$\nabla\,\mathbf{d}\varphi = -\frac{1}{\rho}(\,\mathbf{d}\rho\,\mathbf{d}\varphi + \,\mathbf{d}\varphi\,\mathbf{d}\rho) \qquad \nabla^2\,\varphi = 0$$

$$\nabla\,\mathbf{d}z = 0 \qquad \nabla^2\,z = 0$$

Souřadnice kovariantní derivace

$$a^k{}_{;m} = a^k{}_{,m} + \Gamma^{\,k}_{mn} a^n \quad , \quad a_{n;m} = a_{n,m} - \Gamma^{\,k}_{mn} a_k$$

$$\Gamma^{\,\rho}_{\varphi\varphi} = -\rho \quad , \quad \Gamma^{\,\varphi}_{\rho\varphi} = \Gamma^{\,\varphi}_{\varphi\rho} = \frac{1}{\rho} \quad , \quad \text{ostatní } \Gamma\text{-koeficienty jsou nulové}$$

Operátory v cylindrických souřadnicích

gradient skaláru: $\boldsymbol{a} = \,\mathbf{d}f = \nabla f$

$$a_\rho = f_{,\rho} \qquad a_{\hat\rho} = f_{,\rho} \qquad a^\rho = f_{,\rho}$$

$$a_\varphi = f_{,\varphi} \qquad a_{\hat\varphi} = \frac{1}{\rho}\,f_{,\varphi} \qquad a^\varphi = \frac{1}{\rho^2}\,f_{,\varphi}$$

$$a_z = f_{,z} \qquad a_{\hat z} = f_{,z} \qquad a^z = f_{,z}$$

divergence: $f = \nabla\cdot\boldsymbol{a}$

$$f = \frac{1}{\rho}(\rho\,a^\rho)_{,\rho} + a^\varphi{}_{,\varphi} + a^z{}_{,z} = \frac{1}{\rho}(\rho\,a^{\hat\rho})_{,\rho} + \frac{1}{\rho}\,a^{\hat\varphi}{}_{,\varphi} + a^{\hat z}{}_{,z} = \frac{1}{\rho}(\rho\,a_\rho)_{,\rho} + \frac{1}{\rho^2}\,a_{\varphi,\varphi} + a_{z,z}$$

rotace: $\boldsymbol{b} = \nabla\times\boldsymbol{a}$

$$b_\rho = \frac{1}{\rho}\,(a_{z,\varphi} - a_{\varphi,z}) \qquad b_{\hat\rho} = \frac{1}{\rho}\,a_{\hat z,\varphi} - a_{\hat\varphi,z} \qquad b^\rho = \frac{1}{\rho}\,a^z{}_{,\varphi} - \rho\,a^\varphi{}_{,z}$$

$$b_\varphi = \rho\,(a_{\rho,z} - a_{z,\rho}) \qquad b_{\hat\varphi} = a_{\hat\rho,z} - a_{\hat z,\rho} \qquad b^\varphi = \frac{1}{\rho}\,(a^\rho{}_{,z} - a^z{}_{,\rho})$$

$$b_z = \frac{1}{\rho}\,(a_{\varphi,\rho} - a_{\rho,\varphi}) \qquad b_{\hat z} = \frac{1}{\rho}\,((\rho\,a_{\hat\varphi})_{,\rho} - a_{\hat\rho,\varphi}) \qquad b^z = \frac{1}{\rho}\,\big((\rho^2\,a^\varphi)_{,\rho} - a^\rho{}_{,\varphi}\big)$$

laplace:

$$\nabla^2 f = \frac{1}{\rho}(\rho\,f_{,\rho})_{,\rho} + \frac{1}{\rho^2}f_{,\varphi\varphi} + f_{,zz} = f_{,\rho\rho} + \frac{1}{\rho^2}f_{,\varphi\varphi} + f_{,zz} + \frac{1}{\rho}f_{,\rho}$$