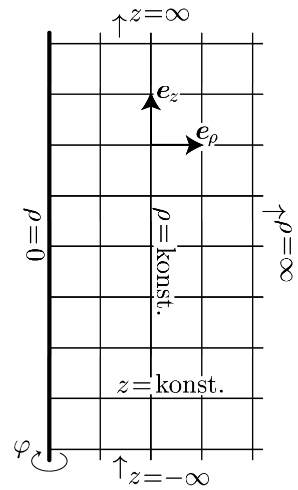
Cylindrické souřadnice (ρ, φ, z)



Vztah ke kartézským souřadnicím:

$$x = \rho\cos\varphi \qquad \rho = \sqrt{x^2+y^2}$$
$$y = \rho\sin\varphi \qquad \varphi = \arctan\frac{y}{x}$$
$$z = z \qquad z = z$$

Metrika a objemový element:

$$\boldsymbol{q} = \mathbf{d}\rho\,\mathbf{d}\rho + \rho^2\,\mathbf{d}\varphi\,\mathbf{d}\varphi + \mathbf{d}z\,\mathbf{d}z \qquad dV = \rho\,d\rho\,d\varphi\,dz$$

Laméovy koeficienty:

$$h_\rho = 1 \qquad h_\varphi = \rho \qquad h_z = 1$$

Triáda (normovaná báze):

$$\boldsymbol{e}_\rho = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\rho} \qquad \boldsymbol{e}_\varphi = \frac{1}{\rho}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial}\varphi} \qquad \boldsymbol{e}_z = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial}z}$$
$$\boldsymbol{e}^\rho = \mathbf{d}\rho \qquad \boldsymbol{e}^\varphi = \rho\,\mathbf{d}\varphi \qquad \boldsymbol{e}^z = \mathbf{d}z$$

Vztah cylindrických a kartézských bází vektorů

$$\boldsymbol{e}_x = \cos\varphi\,\boldsymbol{e}_\rho - \sin\varphi\,\boldsymbol{e}_\varphi = \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{e}_\rho - \frac{y}{\sqrt{x^2+y^2}}\boldsymbol{e}_\varphi$$
$$\boldsymbol{e}_y = \sin\varphi\,\boldsymbol{e}_\rho + \cos\varphi\,\boldsymbol{e}_\varphi = \frac{y}{\sqrt{x^2+y^2}}\boldsymbol{e}_\rho + \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{e}_\varphi$$
$$\boldsymbol{e}_z = \boldsymbol{e}_z$$

$$\boldsymbol{e}_\rho = \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{e}_x + \frac{y}{\sqrt{x^2+y^2}}\boldsymbol{e}_y = \cos\varphi\,\boldsymbol{e}_x + \sin\varphi\,\boldsymbol{e}_y$$
$$\boldsymbol{e}_\varphi = -\frac{y}{\sqrt{x^2+y^2}}\boldsymbol{e}_x + \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{e}_y = -\sin\varphi\,\boldsymbol{e}_x + \cos\varphi\,\boldsymbol{e}_y$$
$$\boldsymbol{e}_z = \boldsymbol{e}_z$$

verze 1.00 (2005-03-09)

Derivace bázových vektorů a forem

$$\nabla \boldsymbol{e}_\rho = \frac{1}{\rho}\,\boldsymbol{e}^\varphi\,\boldsymbol{e}_\varphi \qquad \nabla\cdot\boldsymbol{e}_\rho = \frac{1}{\rho} \qquad \nabla\times\boldsymbol{e}_\rho = 0$$

$$\nabla \boldsymbol{e}_\varphi = -\frac{1}{\rho}\,\boldsymbol{e}^\varphi\,\boldsymbol{e}_\rho \qquad \nabla\cdot\boldsymbol{e}_\varphi = 0 \qquad \nabla\times\boldsymbol{e}_\varphi = \frac{1}{\rho}\,\boldsymbol{e}_z$$

$$\nabla \boldsymbol{e}_z = 0 \qquad \nabla\cdot\boldsymbol{e}_z = 0 \qquad \nabla\times\boldsymbol{e}_z = 0$$

Operátory v cylindrických souřadnicích

gradient skaláru: $\boldsymbol{a} = \mathbf{d}f = \nabla f$

$$a_\rho = f_{,\rho} \qquad a_\varphi = \frac{1}{\rho}\,f_{,\varphi}a^\varphi \qquad a_z = f_{,z}$$

divergence: $f = \nabla\cdot\boldsymbol{a}$

$$f = \frac{1}{\rho}(\rho\,a^\rho)_{,\rho} + \frac{1}{\rho}\,a^\varphi{}_{,\varphi} + a^z{}_{,z}$$

rotace: $\boldsymbol{b} = \nabla\times\boldsymbol{a}$

$$b_\rho = \frac{1}{\rho}\,a_{z,\varphi} - a_{\varphi,z} \quad b_\varphi = a_{\rho,z} - a_{z,\rho} \quad b_z = \frac{1}{\rho}\,\left((\rho\,a_\varphi)_{,\rho} - a_{\rho,\varphi}\right)$$

laplace:

$$\nabla^2 f = \frac{1}{\rho}(\rho\,f_{,\rho})_{,\rho} + \frac{1}{\rho^2}f_{,\varphi\varphi} + f_{,zz} = f_{,\rho\rho} + \frac{1}{\rho^2}f_{,\varphi\varphi} + f_{,zz} + \frac{1}{\rho}f_{,\rho}$$