Úvodní kurz matematických metod fyziky

pro nastupující posluchače 1. ročníku MFF UK

Integrální počet

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Definice integrálu

Integrál reprezentuje “spojitou” sumu – součet velmi mnoha velmi malých hodnot.

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Integrál funkce $f(x)$ na intervalu $x \in \langle a, b \rangle$ je

$$\int_a^b f(x)\,dx \quad = \quad \sum_{i=1}^{N} f(x_i)\,dx \quad + \quad \text{malá chyba}$$

kde interval $\langle a, b \rangle$ je rozdělen na dostatečně velký počet N malých intervalů délky $dx = \frac{b-a}{N}$

a x_i jsou zvolené hodnoty v těchto intervalech (např. koncové body intervalů $x_i = a + i\,dx$).

Newtonův vzorec

Integrování je "inverzní" operace k derivování

$$\int_a^b F'(x)\,dx \quad = \quad F(b) - F(a)$$

často používáme zkrácené označení $\big[\,F\,\big]_a^b = F(b) - F(a)$.

Primitivní funkce a neurčitý integrál

Primitivní funkce F k funkci f nazýváme "invertovanou" derivaci, tj. funkci splňující

$$F' = f$$

Primitivní funkci označujeme též pomocí neurčitého integrálu (integrálu bez mezí)

$$F = \int f \, dx$$

Newtonův vzorec nám dává vztah integrálu na intervalu a primitivní funkce

$$\int_a^b f \, dx = F(b) - F(a)$$

Primitivní funkce je určena až na konstantu, tj. F a $F + \text{konst.}$ jsou primitivní funkce téže funkce f.

Pravidla pro integrály

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