Seznam vzorců pro zápočtovou písemku.

Schrodinger a důležité komutátory: $i\hbar\frac{|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle$ $\quad [\hat{x}_\alpha, \hat{p}_\beta] = i\hbar\delta_{\alpha,\beta}$ $\quad \left[\hat{J}_\alpha, \hat{V}_\beta\right] = i\hbar\varepsilon_{\alpha,\beta,\gamma}\hat{V}_\gamma$

Pauliho matice: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$

Moment hybnosti a sférické harmoniky: $\hat{J}_\pm|jm\rangle = \hbar\sqrt{(j \mp m)(j \pm m + 1)}|jm \pm 1\rangle$

$$\hat{J}^2|jm\rangle = \hbar^2 j(j+1)|jm\rangle \qquad \hat{J}_z|jm\rangle = \hbar m|jm\rangle \qquad \hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$$

$Y_{00}=\frac{1}{\sqrt{4\pi}}$	$Y_{22}=\sqrt{\frac{15}{32\pi}}\frac{(x+iy)^2}{r^2}$	$Y_{33}=-\sqrt{\frac{35}{64\pi}}\frac{(x+iy)^3}{r^3}$
$=$	$Y_{21}=-\sqrt{\frac{15}{8\pi}}\frac{z(x+iy)}{r^2}$	$Y_{32}=\sqrt{\frac{105}{32\pi}}\frac{z(x+iy)^2}{r^3}$
$=$	$Y_{20}=\sqrt{\frac{5}{16\pi}}\frac{2z^2-x^2-y^2}{r^2}$	$Y_{31}=-\sqrt{\frac{21}{64\pi}}\frac{(4z^2-x^2-y^2)(x+iy)}{r^3}$
$Y_{11}=-\sqrt{\frac{3}{8\pi}}\frac{x+iy}{r}$	$Y_{2-1}=\sqrt{\frac{15}{8\pi}}\frac{z(x-iy)}{r^2}$	$Y_{30}=\sqrt{\frac{7}{16\pi}}\frac{z(2z^2-3x^2-3y^2)}{r^3}$
$Y_{10}=\sqrt{\frac{3}{4\pi}}\frac{z}{r}$	$Y_{2-2}=\sqrt{\frac{15}{32\pi}}\frac{(x-iy)^2}{r^2}$	$Y_{3-1}=\sqrt{\frac{21}{64\pi}}\frac{(4z^2-x^2-y^2)(x-iy)}{r^3}$
$Y_{1-1}=\sqrt{\frac{3}{8\pi}}\frac{x-iy}{r}$	$=$	$Y_{3-2}=\sqrt{\frac{105}{32\pi}}\frac{z(x-iy)^2}{r^3}$
$=$	$=$	$Y_{3-3}=\sqrt{\frac{35}{64\pi}}\frac{(x-iy)^3}{r^3}$

Harmonický oscilátor: $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2) \qquad \left[\hat{a}, \hat{a}^\dagger\right] = 1$

$$x_0 = \sqrt{\frac{\hbar}{m\omega}} \qquad p_0 = \frac{\hbar}{x_0} \qquad \hat{a} = \frac{\hat{x}/x_0 + i\hat{p}/p_0}{\sqrt{2}}$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\langle x|n\rangle = \frac{H_n(x/x_0)}{\sqrt{\sqrt{\pi}x_0 n! 2^n}}\exp(-\tfrac{1}{2}(\tfrac{x}{x_0})^2) \qquad \langle p|n\rangle = (-i)^n\frac{H_n(p/p_0)}{\sqrt{\sqrt{\pi}p_0 n! 2^n}}\exp(-\tfrac{1}{2}(\tfrac{p}{p_0})^2)$$

$$H_0(x) = 1,\ H_1(x) = 2x,\ H_2(x) = 4x^2 - 2,\ H_3(x) = 8x^3 - 12x.$$

Coulombický potenciál: $V(x) = \gamma/r, \qquad E_n = -\frac{m\gamma^2}{2\hbar^2 n^2}, \qquad a = \frac{\hbar^2}{m|\gamma|}$

$$R_{10}(r) = 2\sqrt{\frac{1}{a^3}}\exp(-r/a), \qquad R_{20}(r) = \sqrt{\frac{1}{2a^3}}(1-r/2a)\exp(-r/2a), \qquad R_{21}(r) = \frac{1}{2}\sqrt{\frac{1}{6a^3}}r/a\exp(-r/2a)$$

Sférické cylindrické funkce:

$$j_0(z) = \frac{\sin z}{z}, \qquad j_1(z) = \frac{\sin z}{z^2} - \frac{\cos z}{z}, \qquad j_2(r) = \left(\frac{3}{z^3} - \frac{1}{z}\right)\sin z - \frac{3}{z^2}\cos(z)$$

$$n_0(z) = -\frac{\cos z}{z}, \qquad n_1(z) = -\frac{\cos z}{z^2} - \frac{\sin z}{z}, \qquad n_2(r) = -\left(\frac{3}{z^3} - \frac{1}{z}\right)\cos z - \frac{3}{z^2}\sin(z)$$

Table of Clebsch–Gordan coefficients

From Wikipedia, the free encyclopedia

This is a **table of Clebsch–Gordan coefficients** used for adding angular momentum values in quantum mechanics. The overall sign of the coefficients for each set of constant j_1, j_2, j is arbitrary to some degree and has been fixed according to the Condon–Shortley and Wigner sign convention as discussed by Baird and Biedenharn.[1] Tables with the same sign convention may be found in the Particle Data Group's *Review of Particle Properties*[2] and in online tables.[3]

Formulation [edit]

The Clebsch–Gordan coefficients are the solutions to

$$|j_1, j_2; J, M\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} |j_1, m_1; j_2, m_2\rangle \langle j_1, j_2; m_1, m_2 \mid j_1, j_2; J, M\rangle$$

Explicitly:

$$\langle j_1, j_2; m_1, m_2 \mid j_1, j_2; J, M\rangle$$

$$= \delta_{M,m_1+m_2} \sqrt{\frac{(2J+1)(J+j_1-j_2)!(J-j_1+j_2)!(j_1+j_2-J)!}{(j_1+j_2+J+1)!}} \times$$

$$\sqrt{(J+M)!(J-M)!(j_1-m_1)!(j_1+m_1)!(j_2-m_2)!(j_2+m_2)!} \times$$

$$\sum_k \frac{(-1)^k}{k!(j_1+j_2-J-k)!(j_1-m_1-k)!(j_2+m_2-k)!(J-j_2+m_1+k)!(J-j_1-m_2+k)!}.$$

The summation is extended over all integer k for which the argument of every factorial is nonnegative.[4]

For brevity, solutions with $M < 0$ and $j_1 < j_2$ are omitted. They may be calculated using the simple relations

$$\langle j_1, j_2; m_1, m_2 \mid j_1, j_2; J, M\rangle = (-1)^{J-j_1-j_2} \langle j_1, j_2; -m_1, -m_2 \mid j_1, j_2; J, -M\rangle.$$

and

$$\langle j_1, j_2; m_1, m_2 \mid j_1, j_2; J, M\rangle = (-1)^{J-j_1-j_2} \langle j_2, j_1; m_2, m_1 \mid j_2, j_1; J, M\rangle.$$

$j_2 = 0$ [edit]

When $j_2 = 0$, the Clebsch–Gordan coefficients are given by $\delta_{j,j_1}\delta_{m,m_1}$.

$j_1 = \frac{1}{2}, j_2 = \frac{1}{2}$ [edit]

$m = 1$

m_1, m_2 \ j	1
$\frac{1}{2}, \frac{1}{2}$	1

$m = -1$

m_1, m_2 \ j	1
$-\frac{1}{2}, -\frac{1}{2}$	1

$m = 0$

m_1, m_2 \ j	1	0
$\frac{1}{2}, -\frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$-\frac{1}{2}, \frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$j_1 = 1, j_2 = \frac{1}{2}$ [edit]

$m = \frac{3}{2}$

m_1, m_2 \ j	$\frac{3}{2}$
$1, \frac{1}{2}$	1

$m = \frac{1}{2}$

m_1, m_2 \ j	$\frac{3}{2}$	$\frac{1}{2}$
$1, -\frac{1}{2}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$
$0, \frac{1}{2}$	$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$

$j_1 = \frac{3}{2}, j_2 = \frac{1}{2}$ [edit]

$m = 2$

m_1, m_2 \ j	2
$\frac{3}{2}, \frac{1}{2}$	1

$m = 1$

m_1, m_2 \ j	2	1
$\frac{3}{2}, -\frac{1}{2}$	$\frac{1}{2}$	$\sqrt{\frac{3}{4}}$
$\frac{1}{2}, \frac{1}{2}$	$\sqrt{\frac{3}{4}}$	$-\frac{1}{2}$

$m = 0$

m_1, m_2 \ j	2	1
$\frac{1}{2}, -\frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$-\frac{1}{2}, \frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$j_1 = 2, j_2 = \frac{1}{2}$ [edit]

$m = \frac{5}{2}$

m_1, m_2 \ j	$\frac{5}{2}$
$2, \frac{1}{2}$	1

$m = \frac{3}{2}$

m_1, m_2 \ j	$\frac{5}{2}$	$\frac{3}{2}$
$2, -\frac{1}{2}$	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$
$1, \frac{1}{2}$	$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$

$m = \frac{1}{2}$

m_1, m_2 \ j	$\frac{5}{2}$	$\frac{3}{2}$
$1, -\frac{1}{2}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$
$0, \frac{1}{2}$	$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$