

## Seznam vzorců pro zápočtovou písemku.

$$i\hbar \frac{|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle$$

### Důležité komutátory

$$[\hat{x}_\alpha, \hat{p}_\beta] = i\hbar\delta_{\alpha,\beta} \quad [\hat{J}_\alpha, \hat{V}_\beta] = i\hbar\varepsilon_{\alpha,\beta,\gamma}\hat{V}_\gamma$$

### Moment hybnosti a sférické harmoniky

$$\hat{J}^2|jm\rangle = \hbar^2 j(j+1)|jm\rangle \quad \hat{J}_z|jm\rangle = \hbar m|jm\rangle \quad \hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$$

$$\hat{J}_\pm|jm\rangle = \hbar\sqrt{(j \mp m)(j \pm m + 1)}|jm \pm 1\rangle$$

### Pauliho matice

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

### Několik prvních sférických harmonik:

$$\begin{aligned} Y_{00} &= \frac{1}{\sqrt{4\pi}} & Y_{22} &= \sqrt{\frac{15}{32\pi}} \frac{(x+iy)^2}{r^2} & Y_{33} &= -\sqrt{\frac{35}{64\pi}} \frac{(x+iy)^3}{r^3} \\ &= & Y_{21} &= -\sqrt{\frac{15}{8\pi}} \frac{z(x+iy)}{r^2} & Y_{32} &= \sqrt{\frac{105}{32\pi}} \frac{z(x+iy)^2}{r^3} \\ &= & Y_{20} &= \sqrt{\frac{5}{16\pi}} \frac{2z^2 - x^2 - y^2}{r^2} & Y_{31} &= -\sqrt{\frac{21}{64\pi}} \frac{(4z^2 - x^2 - y^2)(x+iy)}{r^3} \\ Y_{11} &= -\sqrt{\frac{3}{8\pi}} \frac{x+iy}{r} & Y_{2-1} &= \sqrt{\frac{15}{8\pi}} \frac{z(x-iy)}{r^2} & Y_{30} &= \sqrt{\frac{7}{16\pi}} \frac{z(2z^2 - 3x^2 - 3y^2)}{r^3} \\ Y_{10} &= \sqrt{\frac{3}{4\pi}} \frac{z}{r} & Y_{2-2} &= \sqrt{\frac{15}{32\pi}} \frac{(x-iy)^2}{r^2} & Y_{3-1} &= \sqrt{\frac{21}{64\pi}} \frac{(4z^2 - x^2 - y^2)(x-iy)}{r^3} \\ Y_{1-1} &= \sqrt{\frac{3}{8\pi}} \frac{x-iy}{r} & &= & Y_{3-2} &= \sqrt{\frac{105}{32\pi}} \frac{z(x-iy)^2}{r^3} \\ &= & &= & Y_{3-3} &= \sqrt{\frac{35}{64\pi}} \frac{(x-iy)^3}{r^3} \end{aligned}$$

### Coulombický potenciál

$$V(x) = \gamma/r, \quad E_n = -\frac{m\gamma^2}{2\hbar^2 n^2}, \quad a = \frac{\hbar^2}{m|\gamma|}$$

### Radiální vlnová $R_{nl}(r)$ funkce pro prvních pár stavů:

$$\begin{aligned} R_{10}(r) &= 2\sqrt{\frac{1}{a^3}} \exp(-r/a) \\ R_{20}(r) &= \sqrt{\frac{1}{2a^3}} (1 - r/2a) \exp(-r/2a) \\ R_{21}(r) &= \frac{1}{2} \sqrt{\frac{1}{6a^3}} r/a \exp(-r/2a) \end{aligned}$$

### Gauntova formule

$$\int Y_{lm}(\theta, \varphi)^* Y_{l_1 m_1}(\theta, \varphi) Y_{l_2 m_2}(\theta, \varphi) d\Omega = \sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2l+1)}} \langle l_1 l_2 00 | l 0 \rangle \langle l_1 l_2 m_1 m_2 | l m \rangle$$

### Wignerova-Eckartova věta

$$\langle \alpha j m | T_M^{(J)} | \alpha' j' m' \rangle = \langle J j' M m' | j m \rangle \frac{\langle \alpha j || T^{(J)} || \alpha' j' \rangle}{\sqrt{2j+1}}$$

*Harmonický oscilátor*

$$x_0 = \sqrt{\frac{\hbar}{m\omega}} \quad p_0 = \frac{\hbar}{x_0} \quad \hat{a} = \frac{\hat{x}/x_0 + i\hat{p}/p_0}{\sqrt{2}}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 = \hbar\omega(\hat{a}^\dagger\hat{a} + 1/2) \quad [\hat{a}, \hat{a}^\dagger] = 1$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \quad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$\langle x|n\rangle = \frac{H_n(x/x_0)}{\sqrt{\sqrt{\pi}x_0n!2^n}} \exp(-\frac{1}{2}(\frac{x}{x_0})^2) \quad \langle p|n\rangle = (-i)^n \frac{H_n(p/p_0)}{\sqrt{\sqrt{\pi}p_0n!2^n}} \exp(-\frac{1}{2}(\frac{p}{p_0})^2)$$

Hermitovy polynomy

$$H_0(x) = 1, H_1(x) = 2x, H_2(x) = 4x^2 - 2, H_3(x) = 8x^3 - 12x.$$