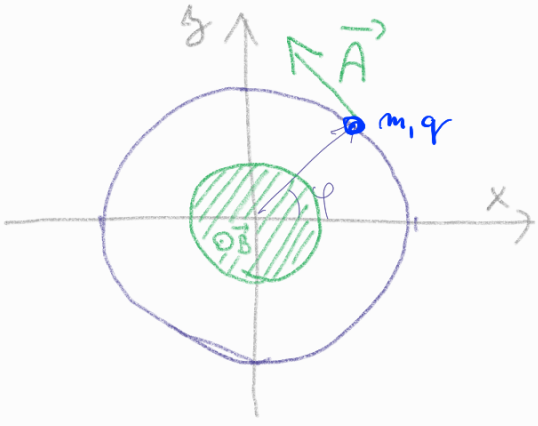
Homework 2a

Consider particle fixed on a circle in xy-plane:



R ... circle radius
m ... particle mass
q ... its charge
$\vec{A} = \frac{\Phi}{2\pi R} \cdot \vec{e}_\varphi$ vector potential
Φ = mag. flux $\vec{B}$.$\vec{S}$ through green area S
$\vec{e}_\varphi = (-\sin\varphi, \cos\varphi)$ unit vector

Its state is described by periodic quadratically integrable function $\psi(\varphi) \in L^2\langle 0, 2\pi \rangle$.

Hamiltonian in magnetic field is

$$\hat{H} = \frac{1}{2m}(\hat{\vec{p}} - q\vec{A})^2 = \frac{1}{2m}\left(-\frac{i\hbar}{R}\partial_\varphi - \frac{\hbar\Phi}{R\Phi_0}\right)^2 = \frac{\hbar^2}{2mR^2}\left(-i\partial_\varphi - \frac{\Phi}{\Phi_0}\right)^2$$

with $\Phi_0 \equiv 2\pi\hbar/q$ is called quantum of magnetic flux.

Find the eigenfunctions and eigenenergies.

Discuss its behavior with changing value of B and its angular momentum.