Homework 3a

Verify the commutation relations:

$$[\hat{J}_k, \hat{J}_l] = i\,\varepsilon_{klm} J_m, \quad [\hat{P}_k, \hat{P}_l] = -i\,\varepsilon_{klm} P_m, \quad [\hat{J}_k, \hat{P}_l] = 0$$

and the relation

$$\hat{J}^2 \equiv \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2 = \hat{P}^2 = P_x^2 + P_y^2 + P_z^2$$

I remind that J_k are components of angular momentum operators in lab-frame:

$$\hat{J}_x = i\left[\cos\alpha \frac{\cos\beta}{\sin\beta}\partial_\alpha + \sin\alpha\,\partial_\beta - \frac{\cos\alpha}{\sin\beta}\partial_\gamma\right],$$

$$\hat{J}_y = i\left[\sin\alpha \frac{\cos\beta}{\sin\beta}\partial_\alpha - \cos\alpha\,\partial_\beta - \frac{\sin\alpha}{\sin\beta}\partial_\gamma\right],$$

$$\hat{J}_z = -i\,\partial_\alpha,$$

and P_k are the angular momentum components in body-frame

$$\hat{P}_x = i\left[\frac{\cos\gamma}{\sin\beta}\partial_\alpha - \sin\gamma\,\partial_\beta - \frac{\cos\beta}{\sin\beta}\cos\gamma\,\partial_\gamma\right],$$

$$\hat{P}_y = i\left[-\frac{\sin\gamma}{\sin\beta}\partial_\alpha - \cos\gamma\,\partial_\beta + \frac{\cos\beta}{\sin\beta}\sin\gamma\,\partial_\gamma\right],$$

$$\hat{P}_z = -i\,\partial_\gamma$$

Find the explicit expression for J^2.