Homework 3b

Try to find the stationary solutions of rotating body with hamiltonian $\hat{H} = \frac{1}{2}\left(\frac{\hat{P}_x^2}{I_x} + \frac{\hat{P}_y^2}{I_y} + \frac{\hat{P}_z^2}{I_z} \right)$ using perturbation theory in parameter $\lambda = \frac{1}{I_x} - \frac{1}{I_y}$ starting from symetric-top solution or from variational principle by expanding the wave function

$$|\psi^j_m\rangle = \sum_{m'} p_{m'} D^j_{m m'}(\alpha,\beta,\gamma)^*$$

where $p_{m'}$ are unknown coefficients. Find all states for $j=0,1$ and try also $j=2$. Explain meaning of the quantum numbers j, m.

You will need relations ($\hbar = 1$)

$$\hat{J}_z D^{j\,*}_{m'm} = m' D^{j\,*}_{m'm}$$

$$(\hat{J}_x \pm i \hat{J}_y) D^{j\,*}_{m'm} = \sqrt{(j \mp m')(j \pm m'+1)}\, D^{j\,*}_{m'\pm 1, m}$$

$$\hat{P}_z D^{j\,*}_{m'm} = m D^{j\,*}_{m'm}$$

$$(\hat{P}_x \mp \hat{P}_y) D^{j\,*}_{m'm} = \sqrt{(j \mp m)(j \pm m+1)}\, D^{j}_{m', m\pm 1}$$

and orthogonality:

$$\int_0^{2\pi} d\alpha \int_0^{\pi} d\beta \sin\beta \int_0^{2\pi} d\gamma\, D^{j'}_{m'k'}(\alpha,\beta,\gamma)^* D^{j}_{mk}(\alpha,\beta,\gamma) = \frac{8\pi^2}{2j+1} \delta_{m'm} \delta_{k'k} \delta_{j'j}$$
