

Homework 6b

Consider N spinless bosons trapped in the harmonic potential $V(x) = \frac{1}{2}m\omega^2 x^2$ with mutual interaction $V(x_1, x_2) = \lambda \delta(x_1 - x_2)$. What is the ground state of the system for $\lambda = 0$? Try to derive the mean field equation (the Gross-Pitaevskij) using similar approach that we used to derive the Hartree-Fock equations. The situation is simpler now because the bosons collapse all into the same state. The variational wavefunction therefore contains single unknown orbital $\phi_0(x)$:

$$\Psi(x_1, x_2, \dots, x_N) = \phi_0(x_1) \phi_0(x_2) \dots \phi_0(x_N)$$

The resulting equation is also called the non-linear Schrödinger equation since the meanfield potential depends on the solution $\phi_0(x)$.

OPTIONAL TASK:

Try to solve the equation numerically for some value λ .