Test of Point Symmetries of Ordinary or Partial Differential Equations

Run TMF064.Package.m first!

```
Clear["Global`*"]
```

- **Variables and differential equations in the form R(x,u,∂*u*,...) = 0**
  (as an example it is used classical linear harmonic oscillator but try some different equation)

```
(* Independent variables *)
IndepVar = {t};
(* Dependent variables *)
DepVar = {x};
(* PDEs, only the functions R(...) without "== 0" *)
PDEs = {m D[x[t], t, t] + k x[t]}
```

```
{k x[t] + m x''[t]}
```

Expression to substitute for in the infinitesimal criterion of invariance

```
subs = {D[x[t], t, t]};
sol = Solve[PDEs == 0, subs]
```

$$\left\{\left\{x''[t] \to -\frac{k\,x[t]}{m}\right\}\right\}$$

- **Using the infinitesimal criterion to check that specified infinitesimals give a point symmetry**
  (try different infinitesimals *ξ*[t] and *η*[x] and see if you get zero)

```
(* Infinitesimals for all variables *)
ξ[t] = c[1];
η[x] = c[2] x[t];
(* Next expression should return zero(es)
 if infinitesimals give a point symmetry of PDEs *)
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

```
x[0] = x[t]
x[1] = x'[t]
x[2] = x''[t]
```

```
{0}
```

- **Infinitesimal generators, point transformations and commutator table from the last ansatz**

```
ShowPointSymmetriesAndCommutationRelations[X, f, ϵ, IndepVar, DepVar, ξ, η, c, 2, {}]
```

Infinitesimal operators:

$X[1]f[t, x] = f^{(1,0)}[t, x]$

$X[2]f[t, x] = x f^{(0,1)}[t, x]$

Corresponding global transformations:

X[1] gives $\{t[\epsilon] \to t + \epsilon, x[\epsilon] \to x\}$

X[2] gives $\{t[\epsilon] \to t, x[\epsilon] \to x\, e^{\epsilon}\}$

Commutator table:

	1	2
1	0	0
2	0	0