Homework #4

Assigned: 8.12.2021   **Due: 22.12.2021**

Group $GA(1, \mathbb{R})$

The group $GA(1, \mathbb{R})$ is two-dimensional Lie group of linear transformations of the type

$$x' = ax + b, \quad a, b \in \mathbb{R}, \ a \neq 0,$$

parametrized by the real numbers a and b.

1. (*5 points*) Find the left-invariant vector fields on this group and, in a suitable basis, determine the structrure constants of the corresponding Lie algebra.

2. (*5 points*) For a general vector from the Lie algebra, find the corresponding one-parameter subgroup.

Group $SL(2, \mathbb{R})$

Group $SL(2, \mathbb{R})$ is Lie group of real matrices $A_{2 \times 2}$ with unit determinant $\det A = 1$.

1. (*7 points*) Find the one-parametric subgroups and evaluate traces of the matrices belonging to these subgroups. *Hint:* Write the general form of a matrix $C \in \mathfrak{sl}(2, \mathbb{R})$ and evaluate directly the matrix exponential $\exp(tC)$ depending on the sign of $\det C$.

2. (*5 points*) Show that the exponential mapping does not cover the whole group $SL(2, \mathbb{R})$. Is the group compact? Is the group connected? If not, does the exponential map cover the whole connected subgroup?