The Fields of Uniformly Accelerated Charges in de Sitter Spacetime

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The scalar and electromagnetic fields of charges uniformly accelerated in de Sitter spacetime are constructed. They represent the generalization of the Born solutions describing fields of two particles with hyperbolic motion in flat spacetime. In the limit $\Lambda \to 0$, the Born solutions are retrieved. Since in the de Sitter universe the infinities $\mathcal{I}^{\pm}$ are spacelike, the radiative properties of the fields depend on the way in which a given point of $\mathcal{I}^{\pm}$ is approached. The fields must involve both retarded and advanced effects: Purely retarded fields do not satisfy the constraints at the past infinity $\mathcal{I}^{-}$.

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The question of the electromagnetic field and associated radiation from uniformly accelerated charges has been one of the best known "perpetual problems" in classical physics from the beginning of the past century. In the pioneering work in 1909, Born gave the time-symmetric solution for the field of two point particles with opposite charges, uniformly accelerated in opposite directions in Minkowski space. In the 1920s, Sommerfeld, von Laue, Pauli, Schott, and others discussed the properties of the field. The controversial point that the field exhibits radiative features but that the radiation reaction force vanishes for the hyperbolic motion, and related questions, was discussed in many articles from the 1960s onward. Even the December 2000 issue of Annals of Physics contains three papers [1] with numerous references on "electrodynamics of hyperbolically accelerated charges."

In general relativity, solutions of Einstein's equations, representing "uniformly accelerated particles or black holes," are the *only* explicitly known exact *radiative* spacetimes describing *finite* sources. They are asymptotically flat at null infinity [2] (except for some special points) and have been used in gravitational radiation theory, quantum gravity, and numerical relativity (cf. review [3]). One of the best known examples is the C-metric, describing uniformly accelerated black holes. There exists also the C-metric for a nonvanishing cosmological constant Λ. However, no general framework is available to analyze these spacetimes for $\Lambda \neq 0$ as that given in Ref. [2] for $\Lambda = 0$.

In this Letter, we present the generalization of the Born solutions for scalar and electromagnetic fields to the case of two charges uniformly accelerated in a de Sitter universe, and explicitly show how in the limit $\Lambda \to 0$ the Born solutions are retrieved. We also study the asymptotic expansions of the fields in the neighborhood of future infinity $\mathcal{I}^{+}$. In de Sitter spacetime, conformal infinities, $\mathcal{I}^{\pm}$, are *spacelike,* which implies the presence of particle and event horizons. It is known [4] that the radiation field is "less invariantly" defined when $\mathcal{I}^{+}$ is spacelike (it depends on the direction in which $\mathcal{I}^{+}$ is approached), but no explicit model appears to be available thus far. Our solutions can serve as prototypes for studying these issues.

In recent work [5], we analyzed fields of accelerated sources to show the *insufficiency of purely retarded fields in de Sitter spacetime*. Consider a point P near $\mathcal{I}^{-}$ whose past null cone will not cross the particles' world lines (Fig. 1). The field at P should vanish if an incoming field is absent. However, the "Coulomb-type" field of particles cannot vanish there because of Gauss law [6]. The requirement that the field be purely retarded leads, in general, to a bad behavior of the field along the "creation light cone" of the "point" at which a source enters the universe (see Ref. [5] for detailed discussion).

It is natural to use de Sitter space for studying radiating sources in spacetimes which are not asymptotically flat and possess spacelike infinities: It is the space of constant curvature, conformal to Minkowski space, and with the Huygens principle satisfied for conformally invariant fields. The de Sitter universe also plays an important role in cosmology—not only in the context of inflationary theories but also as the "asymptotic state" of standard cosmological models with $\Lambda > 0$, which has been indeed suggested by recent observations. In addition, the Born fields generalized to de Sitter space should be relevant from quantum perspectives: for example, for studying particle production in strong fields, or accelerating detectors in the presence of a cosmological horizon.

The de Sitter universe has topology $S^3 \times \mathbb{R}$. The metric in standard "spherical" coordinates (note [7]) is

$$g_{\rm dS} = -d\tau^2 + \alpha^2 \cosh^2(\tau/\alpha)\,(d\chi^2 + \sin^2\chi\, d\omega^2)\,, \tag{1}$$

where $d\omega^2 = d\vartheta^2 + \sin^2\vartheta\, d\varphi^2$, $\tau \in \mathbb{R}$, and $\alpha^2 = 3/\Lambda$. Putting $\chi = \tilde{r}$, $\tau = \alpha \log\tan(\tilde{t}/2)$, $\tilde{t} \in \langle 0, \pi \rangle$, in Eq. (1), the de Sitter metric can be written in the form

$$g_{\rm dS} = \alpha^2 \sin^{-2}\tilde{t}(-d\tilde{t}^2 + d\tilde{r}^2 + \sin^2\tilde{r}\, d\omega^2)\,. \tag{2}$$

The lines $\tilde{r} = \pi$ and $\tilde{r} = -\pi$ are identified, the spacelike hypersurfaces $\tilde{t} = 0, \pi$ represent $\mathcal{I}^{-}$ and $\mathcal{I}^{+}$ (Fig. 1).

By employing conformal techniques, we recently studied [5] two particles moving with uniform acceleration

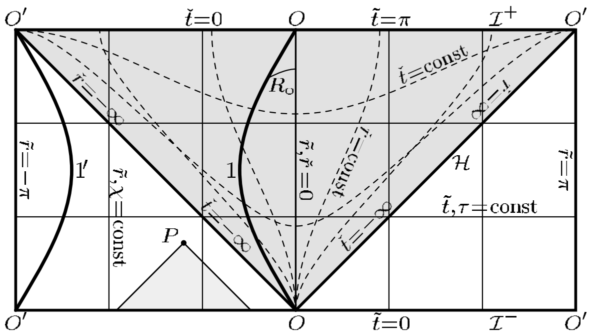

FIG. 1. The conformal diagram of de Sitter spacetime. Uniformly accelerated particles move along world lines 1 and $1'$. The shaded region is the domain of influence of 1, its boundary $\mathcal{H}$ is the "creation light cone" of this particle "born" at $\tilde{t} = 0$ at "point" O. Retarded fields of 1 and $1'$ cannot affect point P; a Coulomb-type field, however, cannot vanish there.

(note [8]) in de Sitter space. Their world lines are plotted in Fig. 1 as 1, $1'$ [for explicit formulas see Ref. [5], Eq. (4.4); see also Eqs. (6) and (10) below]. Both particles start at antipodes of the spatial section of de Sitter space at $\mathcal{I}^-$ and move one towards the other until $\tilde{t} = \pi/2$, the moment of the maximal contraction of de Sitter space. Then they move, in a time-symmetric manner, apart from each other until they reach future infinity at the antipodes from which they started. Their physical velocities, as measured in the "comoving" coordinates $\{\tau, \chi, \vartheta, \varphi\}$, have simple forms $v_\chi = \sqrt{g_{\chi\chi}}\, d\chi/d\tau = \mp a_o \alpha \tanh(\tau/\alpha) \times [1 + a_o^2\alpha^2 \tanh^2(\tau/\alpha)]^{-1/2}$, where $|a_o|$ is the magnitude of their acceleration. In contrast to the flat space case, the particles do not approach the velocity of light in the "natural" global coordinate system. They are causally disconnected (Fig. 1) as in the flat space case: No signal from one particle can reach the other particle.

Two charges moving along the orbits of the boost Killing vector in flat space are *at rest* in the Rindler coordinate system and have a constant distance from the spacetime origin, as measured along the slices orthogonal to the Killing vector. Similarly, the world lines 1 and $1'$ are the orbits of the "static" Killing vector $\partial/\partial T$ of de Sitter space. In static coordinates $\{T, R, \vartheta, \varphi\}$, $T = \frac{\alpha}{2} \log[(\cos\tilde{r} - \cos\tilde{t})/(\cos\tilde{r} + \cos\tilde{t})]$, $R = \alpha \sin\tilde{r}/\sin\tilde{t}$, the particles 1, $1'$ are at rest at $R = \pm R_o = \mp a_o \alpha^2/\sqrt{1 + a_o^2\alpha^2}$, with four accelerations $-(R_o/\alpha^2)\partial/\partial R$. The particle 1 ($1'$) has, as measured at fixed T, a constant proper distance from the origin $\tilde{t} = \pi/2$, $\tilde{r} = 0$ ($\tilde{r} = \pi$). As with Rindler coordinates in Minkowski space, the static coordinates cover only a "half" of de Sitter space; in the other half the Killing vector $\partial/\partial T$ becomes spacelike.

By the conformal transformation of the boosted Coulomb fields in Minkowski space, we constructed [5] test scalar and electromagnetic fields produced by charges moving along the world lines 1, $1'$ in de Sitter space. The scalar field from two *identical* scalar charges s is given by

$$\Phi_{\rm sym} = (s/4\pi)\mathcal{Q}^{-1}, \tag{3}$$

$$\mathcal{Q} = [\alpha^2(\sqrt{1 + a_o^2\alpha^2} + a_o R \cos\vartheta)^2 - \alpha^2 + R^2]^{1/2} \tag{4}$$

[Ref. [5], Eq. (5.4)], whereas the electromagnetic field due to *opposite* charges $+e$ and $-e$ is [Ref. [5], Eq. (5.7)]

$$F_{\rm sym} = -\frac{e}{4\pi}\frac{1}{\mathcal{Q}^3}\frac{a_o\alpha^4}{\sin^3\tilde{t}}[\cos\tilde{t}\sin^2\tilde{r}\sin\vartheta\, d\tilde{r}\wedge d\vartheta + (a_o^{-1}\sqrt{a_o^2 + \alpha^{-2}}\sin\tilde{r} + \sin\tilde{t}\cos\vartheta)d\tilde{t}\wedge d\tilde{r} - \sin\tilde{t}\cos\tilde{r}\sin\tilde{r}\sin\vartheta\, d\tilde{t}\wedge d\vartheta]. \tag{5}$$

We call these smooth (outside the sources) fields symmetric because they can be written as a symmetric combination of retarded and advanced effects from both charges.

Although Eqs. (3) and (5) represent fields due to uniformly accelerated charges in de Sitter space, their relation to the Born solutions is not transparent because the sources are not located symmetrically with respect to $\tilde{r} = 0$. Hence, we consider the world lines 2 and $2'$ (Fig. 2) which, due to homogeneity and isotropy of de Sitter space, also represent uniformly accelerated particles. These world lines and the resulting fields can be obtained from Eqs. (3)–(5) by a spatial rotation by $\pi/2$. We find the world lines 2, $2'$ to be given by

$$\cot\tilde{t} = -\sinh(\lambda_{\rm dS}\alpha^{-1}\sqrt{1 + a_o^2\alpha^2})/\sqrt{1 + a_o^2\alpha^2}\,,$$
$$\tan\tilde{r} = \pm\cosh(\lambda_{\rm dS}\alpha^{-1}\sqrt{1 + a_o^2\alpha^2})/(a_o\alpha)\,, \tag{6}$$

$\vartheta = 0$, $\varphi = 0$. The scalar and electromagnetic fields are

$$\Phi_{\rm BdS} = (s/4\pi)\sin\tilde{t}(\sin\tilde{t} + \cos\tilde{r})^{-1}\mathcal{R}^{-1}, \tag{7}$$

$$F_{\rm BdS} = -\frac{e}{4\pi}\frac{\alpha^3}{\mathcal{R}^3}\frac{a_o\alpha\sin\vartheta}{(\sin\tilde{t} + \cos\tilde{r})^3}[\sin^2\tilde{r}\cos\tilde{t}\, d\tilde{r}\wedge d\vartheta - (a_o^{-1}\sqrt{a_o^2 + \alpha^{-2}}\cos\tilde{r} - \sin\tilde{t})\cot\vartheta\, d\tilde{t}\wedge d\tilde{r} + (a_o^{-1}\sqrt{a_o^2 + \alpha^{-2}} - \cos\tilde{r}\sin\tilde{t})\sin\tilde{r}\, d\tilde{t}\wedge d\vartheta], \tag{8}$$

$$\frac{\mathcal{R}}{\alpha} = \frac{[(a_o\alpha\sin\tilde{t} - \sqrt{1 + a_o^2\alpha^2}\cos\tilde{r})^2 + \sin^2\tilde{r}\sin^2\vartheta]^{1/2}}{\sin\tilde{t} + \cos\tilde{r}}.$$

In order to understand explicitly the relation of these fields to the classical Born solutions, consider Minkowski spacetime with spherical coordinates $\{t, r, \vartheta, \varphi\}$ with metric $g_{\rm M} = -dt^2 + dr^2 + r^2 d\omega^2$. If we set

$$t = -\alpha\cos\tilde{t}/(\cos\tilde{r} + \sin\tilde{t})\,,$$
$$r = \alpha\sin\tilde{r}/(\cos\tilde{r} + \sin\tilde{t})\,,$$

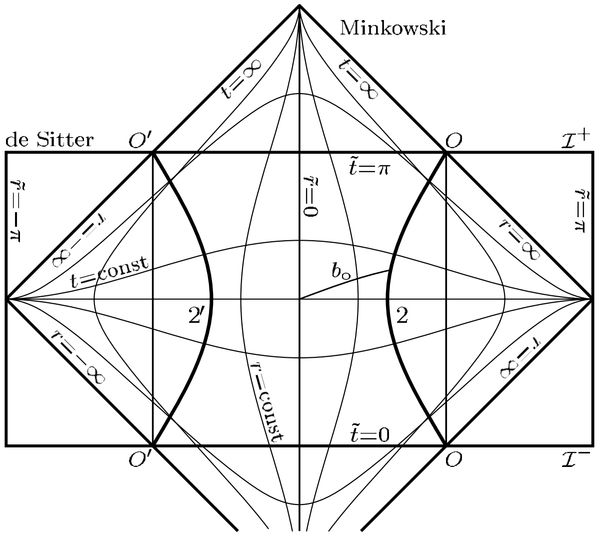

FIG. 2. The world lines 2, 2′ of uniformly accelerated charges symmetrically located with respect to the origins of both de Sitter and conformally related Minkowski spacetimes.

with ϑ, φ unchanged, we find that this Minkowski space is conformally related to de Sitter space as follows (Fig. 2):

$$g_{\mathrm{dS}} = \Omega^2 g_{\mathrm{M}}, \qquad \Omega = \frac{\cos\tilde{r} + \sin\tilde{t}}{\sin\tilde{t}} = \frac{2\alpha^2}{\alpha^2 - t^2 + r^2}. \tag{9}$$

In coordinates $\{t, r, \vartheta, \varphi\}$, which can also be used in de Sitter space (note [9]), the world lines 2, 2′, Eqs. (6), acquire the simple form: $\vartheta = 0$, $\varphi = 0$, and

$$t = b_o \sinh(\lambda_{\mathrm{M}}/b_o), \qquad r = \pm b_o \cosh(\lambda_{\mathrm{M}}/b_o), \tag{10}$$

where λ_{M} is the proper time as measured by g_{M}, and $b_o/\alpha = \sqrt{1 + a_o^2\alpha^2} - a_o\alpha$. The world lines (10) are just two hyperbolas (Fig. 2), representing particles with uniform acceleration $1/b_o$ as measured in Minkowski space.

Transforming the fields (7) and (8) into conformally flat coordinates $\{t, r, \vartheta, \varphi\}$, we obtain

$$\Phi_{\mathrm{BdS}} = (s/4\pi)\Omega^{-1}\mathcal{R}^{-1}, \tag{11}$$

$$\begin{aligned} F_{\mathrm{BdS}} = &-\frac{e}{4\pi}\frac{\alpha^3}{2b_o}\frac{\sin\vartheta}{\mathcal{R}^3} \\ &\times [r(b_o^2 + t^2 + r^2)dt \wedge d\vartheta - (b_o^2 + t^2 - r^2) \\ &\quad \times \cot\vartheta\, dt \wedge dr - 2tr^2 dr \wedge d\vartheta], \end{aligned} \tag{12}$$

the factor $\mathcal{R}$ now being given by

$$\mathcal{R} = [(b_o^2 + t^2 - r^2)^2 + 4b_o^2 r^2 \sin^2\vartheta]^{1/2}/(2b_o). \tag{13}$$

Expressions (7), (8), (11), and (12) represent *the generalized Born scalar and electromagnetic fields from the sources moving with constant acceleration a_o along the world lines (6), respectively (10), in de Sitter universe.*

To connect these fields with their counterparts in flat space, note that they are conformally related by transformation (9). Under the conformal transformation, the field Φ_{BdS} in (11) has to be multiplied by factor Ω, which gives $\Phi_{\mathrm{BdS}} = (s/4\pi)\mathcal{R}^{-1}$, and F_{BdS} in (12) remains unchanged. The transformed fields then precisely coincide with the classical Born fields; see, e.g., Refs. [1,2,10].

In order to see the limit for $\Lambda \to 0$, we parametrize the sequence of de Sitter spaces by Λ, identifying them in terms of coordinates $\{t, r, \vartheta, \varphi\}$. As $\Lambda = 3/\alpha^2 \to 0$, Eq. (9) implies $\Omega_\Lambda \to 2$, $g_{\mathrm{dS}\,\Lambda} \to 4g_{\mathrm{M}}$. After the trivial rescaling of t, r by factor 2, the standard Minkowski metric is obtained. The limit of the fields (11) and (12), in which b_o is kept constant [cf. $a_o = (1 - b_o^2\alpha^{-2})/(2b_o)$], leads to the scalar and electromagnetic Born fields in flat space. Because of the rescaling of coordinates by factor 2, we get the physical acceleration $1/b_o = 2a_o$, and the scalar field rescaled by $1/2$.

What is the character of the generalized Born fields? Focusing on the electromagnetic case, we first decompose the field (8) into the orthonormal tetrad $\{e_\mu\}$ tied to coordinates $\{\tilde{t}, \tilde{r}, \vartheta, \varphi\}$; for example, $e_{\tilde{t}} = (\alpha^{-1}\sin\tilde{t})\partial/\partial\tilde{t}$, etc., the dual tetrad $e^{\tilde{t}} = (\alpha/\sin\tilde{t})d\tilde{t}$, etc. Splitting the field into the electric and the magnetic parts, $F_{\mathrm{BdS}} = E \wedge e^{\tilde{t}} + B \cdot e^{\tilde{r}} \wedge e^{\vartheta} \wedge e^{\varphi}$, we get

$$\begin{aligned} E = &\frac{e}{4\pi}\frac{\alpha\sin^2\tilde{t}}{\mathcal{R}^3(\sin\tilde{t} + \cos\tilde{r})^3} \\ &\times [-(\sqrt{1 + a_o^2\alpha^2}\cos\tilde{r} - a_o\alpha\sin\tilde{t})\cos\vartheta\, e_{\tilde{r}} \\ &\quad + (\sqrt{1 + a_o^2\alpha^2} - a_o\alpha\sin\tilde{t}\cos\tilde{r})\sin\vartheta\, e_{\vartheta}], \\ B = &-\frac{e}{4\pi}\frac{a_o\alpha^2\sin^2\tilde{t}}{\mathcal{R}^3(\sin\tilde{t} + \cos\tilde{r})^3}\cos\tilde{t}\sin\tilde{r}\sin\vartheta\, e_{\varphi}. \end{aligned} \tag{14}$$

The fields exhibit some features typical for the classical Born solution. The toroidal electric field, E_φ, vanishes; only B_φ is nonvanishing. At $\tilde{t} = \pi/2$, the moment of time symmetry, $B_\varphi = 0$. It vanishes also for $\vartheta = 0$—there is no Poynting flux along the axis of symmetry.

The classical Born field decays rapidly ($E \sim r^{-4}$, $B \sim r^{-5}$) at spatial infinity, but it is "radiative" ($E, B \sim r^{-1}$) if we expand it along null geodesics $t - r =$ const, approaching thus null infinity. In de Sitter spacetime with standard slicing, the space is finite (S^3). However, we can approach infinity along spacelike hypersurfaces if, for example, we consider the "steady-state" half of the de Sitter universe (cf. Fig. 1) with flat-space slices, i.e., if we take the "conformally flat" time $\check{t} =$ const (note [9]). Introducing the orthogonal tetrad tied to conformally flat coordinates $\{\check{t}, \check{r}, \vartheta, \varphi\}$, the tetrad components of the fields decay as $\check{r}^{-2}$ at $\check{t} =$ const, $\check{r} \to \infty$, so that the Poynting flux falls off as $\check{r}^{-4}$.

The fields decay very rapidly along *timelike* world lines as $\mathcal{I}^+$ is approached. This is caused by the exponential expansion of slices $\tau =$ const [cf. Eq. (1)]. As $\tau \to \infty$, the electric field (14) becomes radial, $E_{\tilde{r}} \sim \exp(-2\tau/\alpha)$, and $B_\varphi \sim \exp(-2\tau/\alpha)$. The energy density, $u = \frac{1}{2}(E^2 + B^2)$, decays as (expansion factor)$^{-4}$—as energy density in the radiation dominated standard cosmologies. The

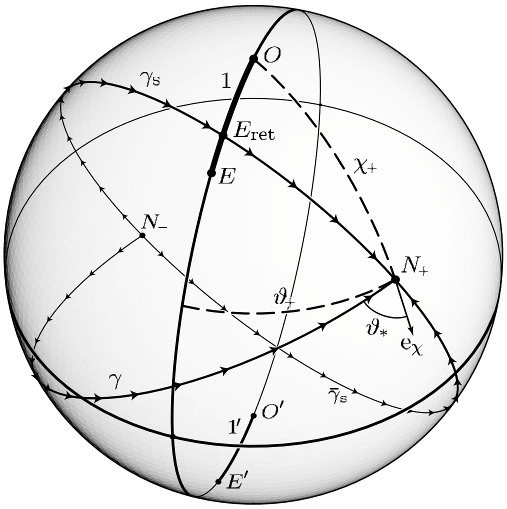

FIG. 3. Space trajectories of null geodesics γ, γ_s, and $\bar{\gamma}_s$ indicated on the slice $\tilde{t} = \text{const}$ ($\varphi = 0$). Charges 1, 1′ move along $\vartheta = 0$ from poles O, O' to points E, E' and back. γ, γ_s, and $\bar{\gamma}_s$ start at N_- at $\tilde{t} = 0$ and arrive at N_+ (with coordinates χ_+, ϑ_+) at $\tilde{t} = \pi$. The direction of γ at N_+ is specified by angles ϑ_*, φ_* (φ_* describes rotation around e_χ in the dimension not seen). γ_s crosses the world line of particle 1 at E_{ret}; $\bar{\gamma}_s$ reaches N_+ from the opposite direction.

density of the conserved energy $u_{conf} = (\alpha/\sin\tilde{t})u \sim \exp(-3\tau/\alpha)$ (determined by a timelike conformal Killing vector $\partial/\partial\tilde{t}$) gets rarified at the same rate that the volume increases.

Will a slower decay occur if $\mathcal{I}^+$ is approached along null geodesics? To study the asymptotic behavior of a field along a null geodesic (see, e.g., Ref. [4]), we have to (i) find a geodesic and parametrize it by an affine parameter ζ, (ii) construct a tetrad parallelly propagated along the geodesic, and (iii) study the asymptotic expansion of the tetrad components of the field. We find that along null geodesics lying in the axis $\vartheta = 0$ (thus crossing the particles' world lines) the "radiation field," i.e., the coefficient of the leading term in $1/\zeta$, vanishes, as could have been anticipated—particles do not radiate in the direction of their acceleration. The radiation field also vanishes along null geodesics reaching infinity along directions *opposite* to those of geodesics emanating from the particles (see Fig. 3). Along all other geodesics, the field *has* radiative character. Along a null geodesic coming from a general direction to a general point on $\mathcal{I}^+$, we find the electric and magnetic fields (in a parallelly transported tetrad $\{f_\mu\}$) to be perpendicular one to the other, equal in magnitude, and proportional to ζ^{-1}. The magnitude of Poynting flux, $|S_{(f)}| = |E_{(f)}|^2 = |B_{(f)}|^2$, is

$$|S_{(f)}| = \frac{e^2}{(4\pi)^2}\frac{a_o^2\sin^2\vartheta_+\csc^4\chi_+}{4(1+a_o^2\alpha^2\cos^2\vartheta_+)^3}[\cos^2\vartheta_*\sin^2\varphi_* + (\cos\varphi_* + a_o^{-1}\sqrt{a_o^2+\alpha^{-2}}\sin\vartheta_*\csc\vartheta_+)^2]\zeta^{-2} \tag{15}$$

(see Fig. 3 for the definition of angles χ_+, ϑ_+, ϑ_*, φ_*). These results are typical for a *radiative* field. Most interestingly, this radiative aspect depends on the specific geodesic along which a given point on *spacelike* $\mathcal{I}^+$ is approached (cf. [4]). Moreover, the radiative character does not disappear even for static sources but it does along null geodesics emanating from such sources.

Since the field can be interpreted as the combination of retarded and advanced effects, similarly to the flat space case [2], the radiation reaction force also vanishes.

In summary, we have constructed the fields of uniformly accelerated charges in a de Sitter universe which go over to classical Born fields in the limit $\Lambda \to 0$. Aside from some similarities found, the generalized fields provide the models showing how a positive cosmological constant implies essential differences from physics in flat spacetime: Advanced effects occur inevitably, and the character of the far fields depends substantially on the way in which future (spacelike) infinity is approached. Since vacuum energy seems to be dominant in the universe, it is of interest to understand fundamental physics in the vacuum dominated de Sitter spacetime.

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[7] It is convenient to allow the angular coordinates χ, ϑ, φ on S^3 to attain values in $\mathbb{R}$ and use the identifications "mod2π," and $\{\chi, \vartheta, \varphi\} \cong \{-\chi, \pi - \vartheta, \varphi + \pi\}$, $\{\chi, \vartheta, \varphi\} \cong \{\chi, -\vartheta, \varphi + \pi\}$. Thus, the "radial" coordinate χ can be negative, the points with $\chi < 0$ being identical to those with $|\chi| > 0$, located "symmetrically" with respect to the origin $\chi = 0$. The same convention is used for $\tilde{r}$, r, $\check{r}$, and R (see Appendix in [5] for details).

[8] "A uniform acceleration" means that the (proper) time derivative $\dot{a}^\alpha$ of the acceleration, projected into the hypersurface orthogonal to the four-velocity, vanishes. The magnitude of the acceleration is constant.

[9] $\{t, r, \vartheta, \varphi\}$ differ from the standard conformally flat coordinates $\{\check{t}, \check{r}, \vartheta, \varphi\}$ by the shift in $\tilde{t}$ direction by $\pi/2$ (Figs. 1 and 2). $\{\check{t}, \check{r}, \vartheta, \varphi\}$ are related to the usual "steady-state" coordinates $\{\check{\eta}, \check{r}, \vartheta, \varphi\}$ of exponentially expanding $k = 0$ cosmologies by $\check{t} = -\alpha\exp(-\check{\eta}/\alpha)$.

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