

Various limits of Kerr-NUT-(A)dS spacetimes

[deformed black holes and nutty spacetimes]

Pavel Krtouš

Charles University, Prague, Czech Republic

`Pavel.Krtous@utf.mff.cuni.cz`
<http://utf.mff.cuni.cz/~krtous/>

in collaboration with

I. Kolář, D. Kubizňák, V. P. Frolov

21st International Conference on General Relativity and Gravitation
July 12, 2016; New York, U.S.A.

Outline

- **Off-shell Kerr–NUT–(A)dS metric**
geometry possessing the principal CCKY tensor
- **On-shell Kerr–NUT–(A)dS metric**
Euclidian instanton, generally rotating black hole
- **Limits of vanishing rotations**
spherical, deformed and twisted black holes
- **Nutty spacetimes**
Taub–NUT-like limit

higher dimensions ($D = 2N$)

Off-shell Kerr–NUT–(A)dS metric

Kerr–NUT–(A)dS metric

(for simplicity only in even dimensions $D = 2N$)

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

x_{μ} radial and latitudinal coordinates ($\mu = 1, \dots, N$)
 ψ_k temporal and longitudinal coordinates ($k = 0, \dots, N-1$)
 $X_{\mu} = X_{\mu}(x_{\mu})$ metric functions to be determined by the Einstein equations

$$A^{(k)} = \sum_{\substack{\nu_1, \dots, \nu_k \\ \nu_1 < \dots < \nu_k}} x_{\nu_1}^2 \dots x_{\nu_k}^2$$

$$A_{\mu}^{(k)} = \sum_{\substack{\nu_1, \dots, \nu_k \\ \nu_1 < \dots < \nu_k, \nu_i \neq \mu}} x_{\nu_1}^2 \dots x_{\nu_k}^2$$

$$U_{\mu} = \prod_{\substack{\nu \\ \nu \neq \mu}} (x_{\nu}^2 - x_{\mu}^2)$$

- Myers R. C., Perry M. J.: *Black Holes in Higher Dimensional Space-Times*, Ann.Phys. 172 (1986) 304
- Gibbons G. W., L H., Page D. N., Pope C. N.: *Rotating Black Holes in Higher Dimensions with a Cosmological Constant*, Phys.Rev.Lett. 93 (2004) 171102
- Chen W., L H., Pope C. N.: *General Kerr-NUT-AdS Metrics in All Dimensions*, Class.Quant.Grav. 23 (2006) 5323

Kerr–NUT–(A)dS metric

- “Kerr” — rotating black holes
- “NUT” — nontrivial NUT parameters
- “(A)dS” — arbitrary cosmological constant

Properties

- Uniquely determined by the existence of the principal CCKY tensor
- Integrability of geodesic motion
- Separability of the Hamilton–Jacobi equations
- Commuting scalar and Dirac symmetry operators
- Separability of the Klein–Gordon and Dirac equations

Explicit and hidden symmetries

Tower of various Killing objects build form the principal CCKYtensor

Explicit symmetries — N independent **Killing vectors**:

$$\mathbf{l}_{(j)} = \partial_{\psi_j} \quad j = 0, \dots, N - 1$$

\Rightarrow observables linear in momenta

Hidden symmetries — N independent **Killing tensors of rank 2**:

$$\mathbf{k}_{(j)} = \sum_{\mu} A_{\mu}^{(j)} \left[\frac{U_{\mu}}{X_{\mu}} \mathbf{d}x_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} \mathbf{d}\psi_k \right)^2 \right] \quad j = 0, \dots, N - 1$$

$$\mathbf{k}_{(0)} = \mathbf{g} \quad \text{metric for } j = 0$$

\Rightarrow observables quadratic in momenta
(generalization of the Carter constant)

Kerr–NUT–(A)dS metric

— coordinate ranges

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

$$X_{\mu} = X_{\mu}(x_{\mu})$$

$$U_{\mu} = \prod_{\substack{\nu \\ \nu \neq \mu}} (x_{\nu}^2 - x_{\mu}^2)$$

Zeros of X_{μ} correspond to:

- axes of rotational symmetry
- Killing horizon of temporal symmetry

Zeros of X_{μ} determine ranges of x_{μ} :

- x_{μ} from spatial sector (angles) run between two roots of X_{μ} (two semi-axes)
- x_{μ} from Lorentzian sector (radius) is not restricted by roots of X_{μ} (horizons)

Kerr–NUT–(A)dS metric

— rewinding Killing angles

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

Any constant linear combination of Killing coordinates gives Killing coordinates:

$$\phi_{\mu} = \sum_k \alpha_{\mu}^k \psi_k$$

Which coordinates should be periodic?

Kerr–NUT–(A)dS metric

($D = 2N$)

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

$$X_{\mu} = X_{\mu}(x_{\mu})$$

off-shell metric

On-shell Kerr–NUT–(A)dS metric

Kerr–NUT–(A)dS metric

($D = 2N$)

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

Einstein equations

with $\Lambda = (2N - 1)(N - 1)\lambda$

\Downarrow

$$X_{\mu} = \lambda \mathcal{J}(x_{\mu}^2) - 2b_{\mu} x_{\mu}$$

$\mathcal{J}(x^2)$ polynomial of degree N in x^2

$$\mathcal{J}(x^2) = \prod_{\mu} (a_{\mu}^2 - x^2) = \sum_{k=0}^N \mathcal{A}^{(k)} (-x^2)^{N-k}$$

Kerr–NUT–(A)dS metric

— explicit parameters of the solutions

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

$$X_{\mu} = \lambda \prod_{\mu} (a_{\mu}^2 - x_{\mu}^2) - 2 b_{\mu} x_{\mu}$$

λ	\propto cosmological constant	1
b_{μ}	mass and NUT charges	N
a_{μ}	rotational parameters	N

Kerr–NUT–(A)dS metric

— explicit parameters of the solutions

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

$$X_{\mu} = \lambda \prod_{\mu} (a_{\mu}^2 - x_{\mu}^2) - 2 b_{\mu} x_{\mu}$$

λ	\propto cosmological constant	1
b_{μ}	mass and NUT charges	N
a_{μ}	rotational parameters	N

scaling freedom

$$x_{\mu} \rightarrow s x_{\mu}, \quad \psi_k \rightarrow s^{-(k+1)} \psi_k,$$

$$a_{\mu} \rightarrow s a_{\mu}, \quad b_{\mu} \rightarrow s^{2N-1} b_{\mu}, \quad \lambda \rightarrow \lambda$$

\Rightarrow elimination of one parameter by a gauge condition – 1

$$\lambda a_N^2 = -1$$

$2N - 1$ parameters of the solution + 1 cosmological constant

Euclidian instanton

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

$$b_{\mu} \neq 0 \quad \lambda > 0 \quad \Rightarrow \quad X_{\mu} = \lambda \mathcal{J}(x_{\mu}^2) - 2b_{\mu} x_{\mu}$$

let ${}^{(-)}x_{\mu}, {}^{(+)}x_{\mu}$ be roots of X_{μ} close to $a_{\mu-1}$ and a_{μ}

\Rightarrow natural ranges of coordinates:

$${}^{(-)}x_{\mu} < x_{\mu} < {}^{(+)}x_{\mu}$$

for suitable signs of b_{μ} one get $x_1 < x_2 < \dots < x_N$

- Euclidian signature
- geometry depend both on b_{μ} and a_{μ}

Maximally symmetric spaces

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

$$b_{\mu} = 0 \quad \Rightarrow \quad X_{\mu} = \lambda \mathcal{J}(x_{\mu}^2) = \lambda \prod_{\mu} (a_{\mu}^2 - x_{\mu}^2)$$

⇓

- maximally symmetric spaces
 - sphere, hyperbolic space, (A)dS
- geometry does not depend on a_{μ}
 - just a choice of coordinates

Black hole

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

$$X_{\mu} = \lambda \mathcal{J}(x_{\mu}^2) - 2 b_{\mu} x_{\mu}$$

Wick rotation of radial coordinate:

$$x_N = ir \quad b_N = im$$

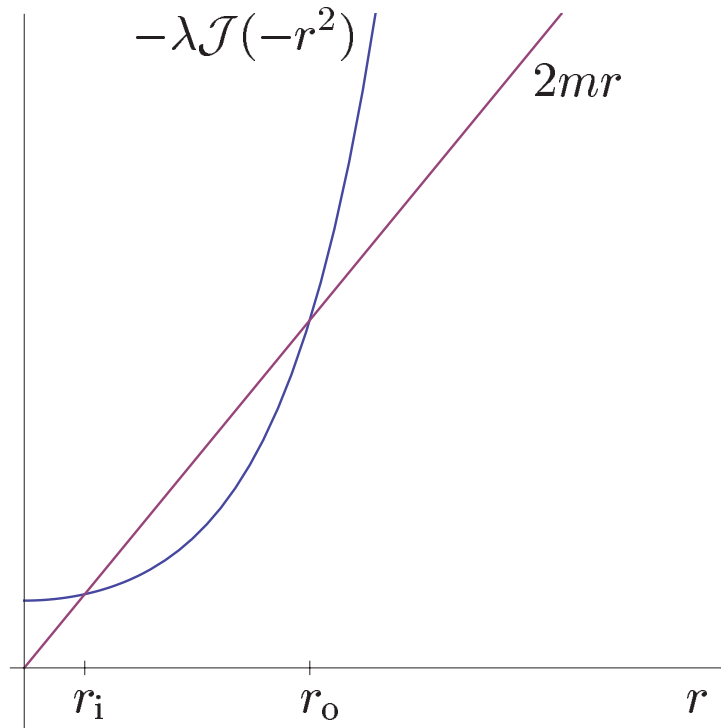
Gauge condition:

$$a_N^2 = -\frac{1}{\lambda}$$

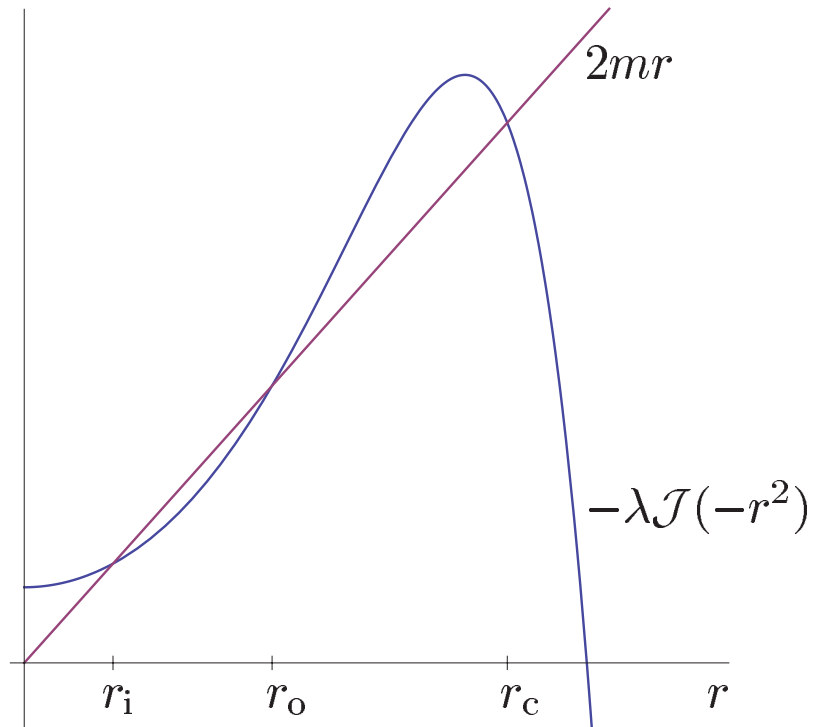
Black hole

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

$$X_N = \lambda \mathcal{J}(-r^2) + 2mr \quad x_N = ir \quad b_N = im \quad a_N^2 = -\lambda^{-1}$$



$\lambda \leq 0$



$\lambda > 0$

Limit of vanishing rotations

- Krtouš P., Kubizňák D., Frolov V. P., Kolář I.: *Deformed and twisted black holes with NUTs*, Class. Quant. Grav. 33 (2016) 115016

Kerr–NUT–(A)dS metric

— rewinding Killing angles

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_k A_{\mu}^{(k)} d\psi_k \right)^2 \right]$$

special choice of Killing angles:

$$\phi_{\mu} = \lambda a_{\mu} \sum_k \mathcal{A}_{\mu}^{(k)} \psi_k$$

⇓

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_{\nu} \frac{J_{\mu}(a_{\nu}^2)}{\lambda a_{\mu} \mathcal{U}_{\nu}} d\phi_{\nu} \right)^2 \right]$$

where

$$\mathcal{A}_{\mu}^{(k)} = \sum_{\substack{\nu_1, \dots, \nu_k \\ \nu_1 < \dots < \nu_k, \nu_i \neq \mu}} a_{\nu_1}^2 \dots a_{\nu_k}^2$$

$$\mathcal{U}_{\mu} = \prod_{\substack{\nu \\ \nu \neq \mu}} (a_{\nu}^2 - a_{\mu}^2)$$

$$J_{\mu}(a^2) = \prod_{\substack{\nu \\ \nu \neq \mu}} (x_{\nu}^2 - a^2)$$

Limit of vanishing rotations

— parametrization of rotations

- for $m \neq 0$, $b_\mu = 0$, $\lambda = 0$
 - rotations parametrized by a_μ
(comparing generators of horizon and static observers at infinity)
- for nontrivial b_μ
 - unclear identification of rotations
(how to distinguish local nutty rotations from global rotations?)
- qualitatively estimated by a ‘length’ of the x -coordinate range
 $^{(+)}x_\mu - ^{(-)}x_\mu$

vanishing rotations in \bar{N} directions



$a_{\bar{\mu}} \rightarrow 0$ for these directions

Limit of vanishing rotations

— scaling of coordinates and parameters

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_{\nu} \frac{J_{\mu}(a_{\nu}^2)}{u_{\nu}} d\phi_{\nu} \right)^2 \right]$$

$$X_{\mu} = \lambda \mathcal{J}(x_{\mu}^2) - 2 b_{\mu} x_{\mu}$$

- setting $a_{\bar{\mu}} = 0$ yields degenerate ranges of coordinates
- setting $a_{\bar{\mu}} = 0$ yields degenerate metric
- a suitable scaling of coordinates necessary!

Limit of vanishing rotations

— scaling of coordinates and parameters

$$g = \sum_{\mu} \left[\frac{U_{\mu}}{X_{\mu}} dx_{\mu}^2 + \frac{X_{\mu}}{U_{\mu}} \left(\sum_{\nu} \frac{J_{\mu}(a_{\nu}^2)}{\lambda a_{\mu} \mathcal{U}_{\nu}} d\phi_{\nu} \right)^2 \right]$$

$$X_{\mu} = \lambda \mathcal{J}(x_{\mu}^2) - 2b_{\mu} x_{\mu}$$

$$\phi_{\mu} = \lambda a_{\mu} \sum_k \mathcal{A}_{\mu}^{(k)} \psi_k$$

regular sector

$$a_{\bar{N}+\tilde{\mu}} = \tilde{a}_{\tilde{\mu}}$$

$$x_{\bar{N}+\tilde{\mu}} = \tilde{x}_{\tilde{\mu}}$$

$$\phi_{\bar{N}+\tilde{\mu}} = \tilde{\phi}_{\tilde{\mu}}$$

$$\tilde{\mu} = 1, \dots, \tilde{N}$$

unspined sector

$$a_{\bar{\mu}} = \varepsilon \bar{a}_{\bar{\mu}}$$

$$x_{\bar{\mu}} = \varepsilon \bar{x}_{\bar{\mu}}$$

$$\phi_{\bar{\mu}} = \bar{\phi}_{\bar{\mu}}$$

$$\bar{\mu} = 1, \dots, \bar{N}$$

$$\varepsilon \rightarrow 0$$

Limit of vanishing rotations

— limit of the metric

regular sector

$$\begin{aligned} a_{\bar{N}+\tilde{\mu}} &= \tilde{a}_{\tilde{\mu}} \\ x_{\bar{N}+\tilde{\mu}} &= \tilde{x}_{\tilde{\mu}} \\ \phi_{\bar{N}+\tilde{\mu}} &= \tilde{\phi}_{\tilde{\mu}} \\ \tilde{\mu} &= 1, \dots, \bar{N} \end{aligned}$$

unspined sector

$$\begin{aligned} a_{\bar{\mu}} &= \varepsilon \bar{a}_{\bar{\mu}} \\ x_{\bar{\mu}} &= \varepsilon \bar{x}_{\bar{\mu}} \\ \phi_{\bar{\mu}} &= \bar{\phi}_{\bar{\mu}} \\ \bar{\mu} &= 1, \dots, \bar{N} \end{aligned}$$

$$g = \tilde{g} + \tilde{w}^2 \bar{g}$$

$$\tilde{w}^2 = \frac{\tilde{A}^{(\bar{N})}}{\bar{A}^{(\bar{N})}}$$

$$\tilde{g} = \sum_{\tilde{\mu}} \left[\frac{\tilde{U}_{\tilde{\mu}}}{\tilde{X}_{\tilde{\mu}}} d\tilde{x}_{\tilde{\mu}}^2 + \frac{\tilde{X}_{\tilde{\mu}}}{\tilde{U}_{\tilde{\mu}}} \left(\sum_{\tilde{\nu}} \frac{\tilde{J}_{\tilde{\mu}}(\tilde{a}_{\tilde{\nu}}^2)}{\lambda \tilde{a}_{\tilde{\nu}} \tilde{U}_{\tilde{\nu}}} d\phi_{\tilde{\nu}} \right)^2 \right]$$

$$\tilde{X}_{\tilde{\mu}} = \lambda \tilde{\mathcal{J}}(\tilde{x}_{\tilde{\mu}}^2) - 2 \tilde{b}_{\tilde{\mu}} \tilde{x}_{\tilde{\mu}}^{1-2\bar{N}}$$

$$\bar{g} = \sum_{\bar{\mu}} \left[\frac{\bar{U}_{\bar{\mu}}}{\bar{X}_{\bar{\mu}}} d\bar{x}_{\bar{\mu}}^2 + \frac{\bar{X}_{\bar{\mu}}}{\bar{U}_{\bar{\mu}}} \left(\sum_{\bar{\nu}} \frac{\bar{J}_{\bar{\mu}}(\bar{a}_{\bar{\nu}}^2)}{\lambda \bar{a}_{\bar{\nu}} \bar{U}_{\bar{\nu}}} d\phi_{\bar{\nu}} \right)^2 \right]$$

$$\bar{X}_{\bar{\mu}} = \lambda \bar{\mathcal{J}}(\bar{x}_{\bar{\mu}}^2) - 2 \bar{b}_{\bar{\mu}} \bar{x}_{\bar{\mu}}$$

off-shell Kerr–NUT–(A)dS
Lorentzian part

on-shell Kerr–NUT–(A)dS
Euclidian instanton

Limit of vanishing rotations

— counting parameters

$$\mathbf{g} = \tilde{\mathbf{g}} + \tilde{w}^2 \bar{\mathbf{g}}$$

parameters $\bar{a}_{\bar{\mu}}$ survive the limit $a_{\bar{\mu}} \rightarrow 0$

$$\frac{1}{\varepsilon} a_{\bar{\mu}} \rightarrow \bar{a}_{\bar{\mu}}$$

Limit of vanishing rotations

— counting parameters

original metric

$$2N - 1$$

$$N = \tilde{N} + \bar{N}$$

regular sector

\tilde{N} parameters $\tilde{a}_{\tilde{\mu}}$

\tilde{N} parameters $\tilde{b}_{\tilde{\mu}}$

unspined sector

\bar{N} parameters $\bar{a}_{\bar{\mu}}$

\bar{N} parameters $\bar{b}_{\bar{\mu}}$

Limit of vanishing rotations

— counting parameters

original metric

$$2N - 1$$

$$N = \tilde{N} + \bar{N}$$

regular sector

\tilde{N} parameters $\tilde{a}_{\tilde{\mu}}$

\tilde{N} parameters $\tilde{b}_{\tilde{\mu}}$

−1 scaling freedom

$$2\tilde{N} - 1$$

unspined sector

\bar{N} parameters $\bar{a}_{\bar{\mu}}$

\bar{N} parameters $\bar{b}_{\bar{\mu}}$

−1 scaling freedom

$$2\bar{N} - 1$$

$$2N - 2$$

1 parameter less after the limit

Limit of vanishing rotations

— interpretation

$$g = \tilde{g} + \tilde{w}^2 \bar{g} \qquad \tilde{w}^2 = \frac{\tilde{A}^{(\tilde{N})}}{\tilde{\mathcal{A}}^{(\tilde{N})}}$$

Lorentzian part

$$\tilde{g} = \sum_{\tilde{\mu}} \left[\frac{\tilde{U}_{\tilde{\mu}}}{\tilde{X}_{\tilde{\mu}}} d\tilde{x}_{\tilde{\mu}}^2 + \frac{\tilde{X}_{\tilde{\mu}}}{\tilde{U}_{\tilde{\mu}}} \left(\sum_{\tilde{\nu}} \frac{\tilde{J}_{\tilde{\mu}}(\tilde{a}_{\tilde{\nu}}^2)}{\lambda \tilde{a}_{\tilde{\nu}} \tilde{\mathcal{U}}_{\tilde{\nu}}} d\phi_{\tilde{\nu}} \right)^2 \right]$$

$$\tilde{X}_{\tilde{\mu}} = \lambda \tilde{\mathcal{J}}(\tilde{x}_{\tilde{\mu}}^2) - 2\tilde{b}_{\tilde{\mu}} \tilde{x}_{\tilde{\mu}}^{1-2\tilde{N}}$$

$\tilde{b}_{\tilde{\mu}}$ mass and NUT charges

$\tilde{a}_{\tilde{\mu}}$ rotations

Euclidian instanton

$$\bar{g} = \sum_{\bar{\mu}} \left[\frac{\bar{U}_{\bar{\mu}}}{\bar{X}_{\bar{\mu}}} d\bar{x}_{\bar{\mu}}^2 + \frac{\bar{X}_{\bar{\mu}}}{\bar{U}_{\bar{\mu}}} \left(\sum_{\bar{\nu}} \frac{\bar{J}_{\bar{\mu}}(\bar{a}_{\bar{\nu}}^2)}{\lambda \bar{a}_{\bar{\nu}} \bar{\mathcal{U}}_{\bar{\nu}}} d\phi_{\bar{\nu}} \right)^2 \right]$$

$$\bar{X}_{\bar{\mu}} = \lambda \bar{\mathcal{J}}(\bar{x}_{\bar{\mu}}^2) - 2\bar{b}_{\bar{\mu}} \bar{x}_{\bar{\mu}}$$

$\bar{b}_{\bar{\mu}}$ deformation of the instanton

$\bar{a}_{\bar{\mu}}$ twisting/intertwining of angles

Partially rotating deformed twisted black hole

Switching-off all rotations

regular sector

Lorentzian part

$$\tilde{N} = 1$$

unspined sector

Euclidian instanton

$$\bar{N} = N - 1$$

$$g = -\left(1 - \lambda r^2 - 2mr^{3-2N}\right) dt^2 + \frac{1}{1 - \lambda r^2 - 2mr^{3-2N}} dr^2 + r^2 \bar{g}$$

$\bar{b}_{\bar{\mu}} = 0$ **Schwarzschild–Tangherlini–(A)dS black hole**

- \bar{g} sphere
- $\bar{a}_{\bar{\mu}}$ just a choice of coordinates

$\bar{b}_{\bar{\mu}} \neq 0$ **Non-rotating deformed twisted black hole**

- \bar{g} Euclidian instanton
- $\bar{b}_{\bar{\mu}}$ deformation parameters
- $\bar{a}_{\bar{\mu}}$ intertwining parameters

Deformed black hole

switching-off all rotations and twists by repeating the limiting procedure

$$\mathbf{g} = -f dt^2 + \frac{1}{f} dr^2 + r^2 \left[\mathbf{q}_{N-1} + \xi_{N-1}^2 \left(\mathbf{q}_{N-2} + \cdots + \xi_3^2 \left(\mathbf{q}_2 + \xi_2^2 \mathbf{q}_1 \right) \right) \right]$$

$$f = 1 - \lambda r^2 - \frac{2m}{r^{2N-3}}$$

$$\mathbf{q}_{\bar{\mu}} = \frac{1}{\Delta_{\bar{\mu}}} d\xi_{\bar{\mu}}^2 + \Delta_{\bar{\mu}} d\phi_{\bar{\mu}}^2 \quad \Delta_{\bar{\mu}} = 1 - \xi_{\bar{\mu}}^2 - 2\beta_{\bar{\mu}} \xi_{\bar{\mu}}^{-2\bar{\mu}+3}$$

- \mathbf{q}_1 is just a spherical geometry with a conical singularity
- $\mathbf{q}_{\bar{\mu}}$ with $\bar{\mu} > 1$ are nontrivial 2-dimensional geometries

Static deformed non-twisted black hole

Nutty spacetimes

- Kolář I., Krtouš P.: *Limits of Kerr-NUT-(A)dS spacetimes*, in preparation

Nutty spacetimes

— generalization of Taub-NUT-(A)dS

Limit near double roots of X_μ

$$X_\mu = \lambda \mathcal{J}(x_\mu^2) - 2b_\mu x_\mu$$

Polynomial $\mathcal{J}(x^2)$ can be parametrized by:

— by roots a_μ

$$\mathcal{J}(x^2) = \prod_\mu (a_\mu^2 - x^2)$$

— by ‘tangent points’ \hat{x}_μ

$$\mathcal{J}(x^2) = \sum_{k=0}^N \frac{2N-1}{2N-2k-1} \hat{A}^{(k)} (-x^2)^{N-1-k}$$

$$\text{where } \hat{A}^{(k)} = \sum_{\substack{\nu_1, \dots, \nu_k \\ \nu_1 < \dots < \nu_k}} \hat{x}_{\nu_1}^2 \dots \hat{x}_{\nu_k}^2$$

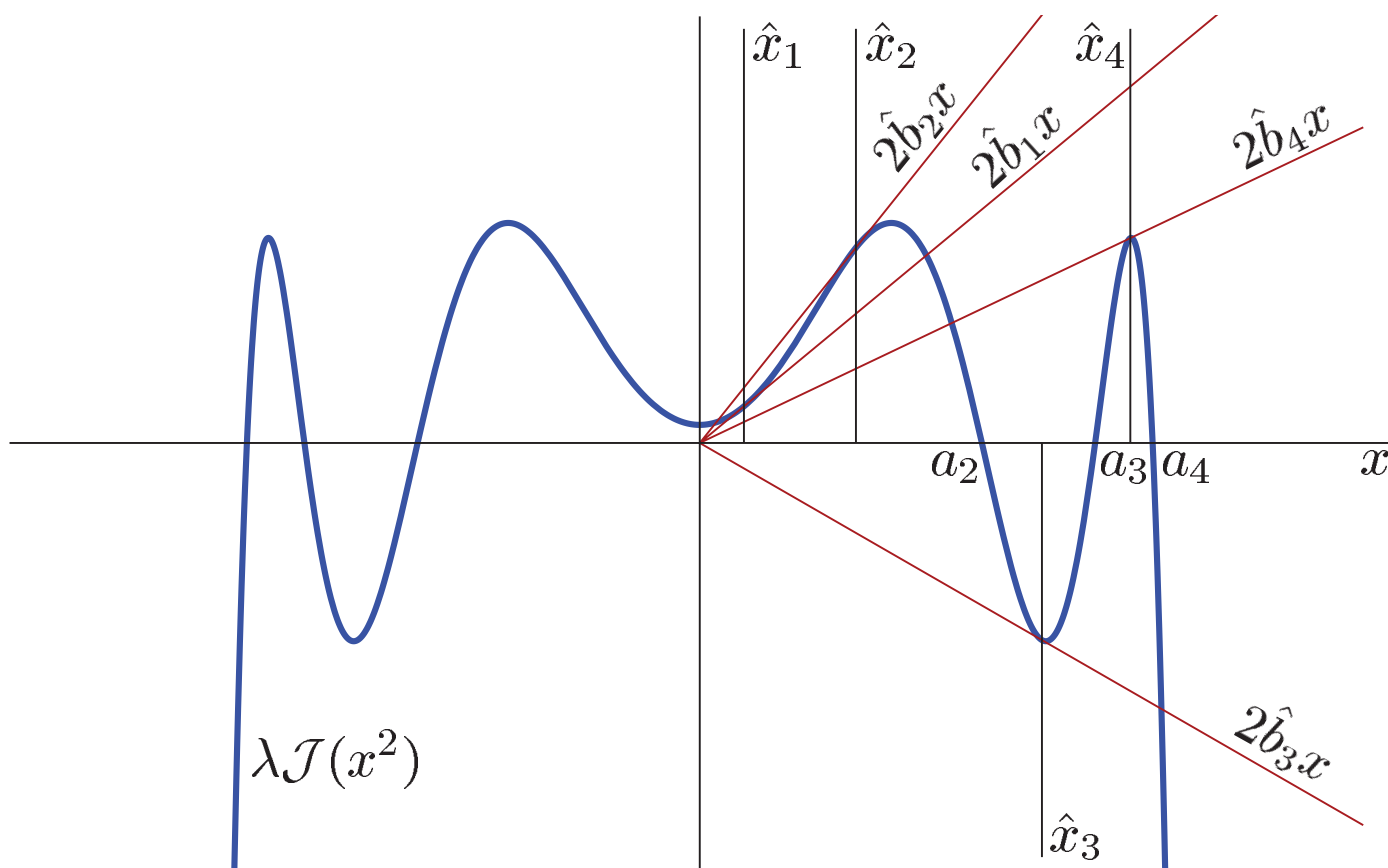
tangent points \hat{x}_μ are double roots of X_μ for suitable values $b_\mu = \hat{b}_\mu$

Nutty spacetimes

— generalization of Taub-NUT-(A)dS

tangent points \hat{x}_μ are double roots of X_μ for suitable values $b_\mu = \hat{b}_\mu$

$$X_\mu = \lambda \mathcal{J}(x^2) - 2\hat{b}_\mu x$$



Nutty spacetimes

— generalization of Taub-NUT-(A)dS

N roots a_μ

N tangent points \hat{x}_μ

gauge condition:

$$a_N = -\frac{1}{\lambda}$$

tangent point in Lorentzian sector:

$$\hat{r}^2 = \frac{1 \sum_k \frac{1}{2N-2k-1} \lambda^k \hat{A}^{(k)}}{\lambda \sum_k \frac{1}{2N-2k-3} \lambda^k \hat{A}^{(k)}}$$

$N - 1$ ‘nutty’ parameters $\hat{x}_{\bar{\mu}}$

(encode critical combination of rotations and NUT parameters)

Nutty spacetimes

— generalization of Taub-NUT-(A)dS

spacetime with near-critical NUTs
 x -coordinate ranges near double roots $\hat{x}_{\bar{\mu}}$

$$\begin{aligned}\mu &= N && \text{– Lorentzian sector} \\ \bar{\mu} &= 1, \dots, N-1 && \text{– spatial directions}\end{aligned}$$

non-critical value of mass:

$$m \neq \hat{m}$$

NUT charges near the critical values:

$$b_{\bar{\mu}} = \hat{b}_{\bar{\mu}} + \mathcal{O}(\varepsilon^2)$$

scaling coordinates near double roots:

$$x_{\bar{\mu}} = \hat{x}_{\bar{\mu}} + \varepsilon \delta x_{\bar{\mu}} \xi_{\bar{\mu}}$$

scaling angles:

$$\psi_{\bar{k}+1} = \frac{1}{\varepsilon} \Psi_{\bar{k}}$$

shifting time:

$$t = \psi_0 + \frac{1}{\varepsilon} \sum_{\bar{k}} \overset{\circ}{A}^{(\bar{k}+1)} \Psi_{\bar{k}}$$

rewinding angles:

$$\varphi_{\bar{\mu}} = \sum_{\bar{k}} \overset{\circ}{A}_{\bar{\mu}}^{(\bar{k})} \Psi_{\bar{k}}$$

Nutty spacetimes

— generalization of Taub-NUT-(A)dS

spacetime with near-critical NUTs
 x -coordinate ranges near double roots $\hat{x}_{\bar{\mu}}$

non-critical value of mass:

$$m \neq \hat{m}$$

NUT charges near the critical values:

$$b_{\bar{\mu}} = \hat{b}_{\bar{\mu}} + \mathcal{O}(\varepsilon^2)$$

scaling coordinates near double roots:

$$x_{\bar{\mu}} = \hat{x}_{\bar{\mu}} + \frac{\varepsilon}{\delta_{\bar{\mu}}} \xi_{\bar{\mu}}$$

scaling angles:

$$\psi_{\bar{k}+1} = \frac{1}{\varepsilon} \Psi_{\bar{k}}$$

shifting time:

$$t = \psi_0 + \frac{1}{\varepsilon} \sum_{\bar{k}} \overset{\circ}{A}^{(\bar{k}+1)} \Psi_{\bar{k}}$$

rewinding angles:

$$\varphi_{\bar{\mu}} = \sum_{\bar{k}} \overset{\circ}{A}_{\bar{\mu}}^{(\bar{k})} \Psi_{\bar{k}}$$

$$g = -\frac{\Delta}{\Sigma} \left(dt + \sum_{\bar{\mu}} \frac{2\hat{x}_{\bar{\mu}}}{\delta_{\bar{\mu}}} (\xi_{\bar{\mu}} - \xi_{\bar{\mu}}^{\circ}) d\varphi_{\bar{\mu}} \right)^2 + \frac{\Sigma}{\Delta} dr^2 + \sum_{\bar{\mu}} \frac{r^2 + \hat{x}_{\bar{\mu}}^2}{\delta_{\bar{\mu}}} \left(\frac{1}{1 - \xi_{\bar{\mu}}^2} d\xi_{\bar{\mu}}^2 + (1 - \xi_{\bar{\mu}}^2) d\varphi_{\bar{\mu}}^2 \right)$$

$$\Delta = -\lambda \mathcal{J}(-r^2) - 2mr$$

$$\Sigma = \prod_{\bar{\nu}} (r^2 + \hat{x}_{\bar{\nu}}^2)$$

$$\delta_{\mu} = \lambda(2N - 1)(\hat{r}^2 + \hat{x}_{\bar{\mu}}^2)$$

- Mann R. B., Stelea C.: *New multiply nutty spacetimes*, Phys.Lett. B634 (2006) 448

Summary

- Kerr–NUT–(A)dS describes wide family of geometries
- a full interpretation still open
- **non-rotating limits leads to deformed and twisted black holes**
- interpretation of spatial NUT parameters — deformation (?)
- interpretation of rewinding Killing angles — twisting (?)
- **nutty spacetimes as double-root limits of Kerr–NUT–(A)dS**
- an analogous double-root limit in the Lorentzian sector
— near horizon limits of Kerr–NUT–(A)dS