Accelerated black holes in

(not only)

anti–de Sitter universe

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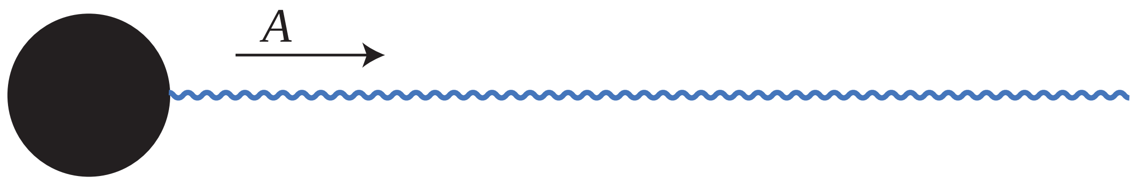
- C-metric – overture
- C-metric with negative Λ
  - Accelerated observers in Minkowski and AdS spacetimes
  - Schwarzschild black hole
- One accelerated black hole in anti-de Sitter universe
  - C-metric with vanishing Λ
  - Anti-de Sitter universe in accelerated coordinates
- Pairs of accelerated black holes in anti-de Sitter universe

C-metric

C-metric represents a black hole uniformly accelerated by a string



- two Killing vectors (boost-rotation symmetric solution)
- Petrov type D (two double degenerated PND)
- belongs to Plebański-Demiański family

Siblings in C-metric family:

- general cosmological constant Λ
- charged solutions
- extremal limits
- spinning black holes

C-metric relatives:

- Born solution
  field of uniformly accelerated charges
  widely discussed in the context of analysis of the radiation and of the radiation reaction force
  M. Born, Ann. Phys. (Leipzig) 30(1909)1; . . . (infinite number of references)
  $\Lambda > 0$: J. Bičák, P. Krtouš: Phys. Rev. Lett. 88(2002)211101; J. Math. Phys. 46(2005)102504
- Black rings
  the metric of the 5-dimensional black ring is composed by
  4-dimensional euclidian C-metric-like piece "warped" with a time direction
  R. Emparan, H. Reall: Phys.Rev.Lett. 88(2002)101101
- Black funnels and droplets
  various degenerated cases and limits of the C-metric
  V. Hubeny, D. Marolf, M. Rangamani: Class.Quant.Grav. 27(2010)025001

C-metric – some applications:

- Numerical relativity
  boost-rotation symmetric solutions has been used as a test bed for numerical simulations
- Cosmological production of black hole pairs
  general Λ, topologically nontrivial identifications for $\Lambda < 0$
  R. Mann, S. Ross: Phys. Rev. D 52(1995)2254; R. Mann: Class. Quantum Grav. 14(1997)L109; etc.
  O. Dias: PhD thesis; O. Dias, J. Lemos: Phys. Rev. D 67(2003)084018; Phys. Rev. D 67(2003)064001
- Randall–Sundrum model in 3+1 dimensions
  3-dimensional brane in a special subcase of AdS C-metric
  R. Emparan, G. Horowitz, R. Myers: JHEP 0001(2000)007
- Ryu-Takayanagi formula for entanglement entropy in AdS/CFT correspondence
  a search for minimal surfaces in C-metric bulk spanned on spherical boundaries in AdS infinity
  P. Krtouš, A. Zelnikov: work in progress

C-metric with general Λ:

$$\boldsymbol{g} = \frac{1}{A^2(x+y)^2}\left(-F\,\mathbf{d}t^2 + \frac{1}{F}\,\mathbf{d}y^2 + \frac{1}{G}\,\mathbf{d}x^2 + G\,\mathbf{d}\varphi^2\right) \qquad \mathbf{F} = e\,\mathbf{d}y \wedge \mathbf{d}t$$

$$\begin{aligned} F &= \ell^{-2}A^{-2} - 1 + y^2 - 2mA\,y^3 + e^2A^2\,y^4 \\ G &= \qquad\qquad\; 1 - x^2 - 2mA\,x^3 - e^2A^2\,x^4 \end{aligned}$$

m	mass parameter
e	charge parameter
A	acceleration parameter
C	conicity parameter: $\varphi \in (-C\pi, C\pi)$
ℓ	cosmological scale: $\ell = \sqrt{-3/\Lambda}$

- Two Killing vectors $\boldsymbol{\partial}_t$, $\boldsymbol{\partial}_\varphi$
- Two double-degenerate principal null directions lying in surfaces $x =$ constant (Petrov type D)
- Conical singularity (cosmic string) on the axis

$\Lambda = 0$

T. Levi-Civita (1917)
H. Weyl (1918)
J. Ehlers, W. Kundt (1962)
W. Kinnersley, M. Walker (1970)
A. Ashtekar, T. Dray (1981)
W. B. Bonnor (1983)
...
J. B. Griffiths, P. Krtouš, J. Podolský (2006)

$\Lambda > 0$

J. Plebański, M. Demiański (1976)
...
J. Podolský, J. B. Griffiths (2001)
O. J. C. Dias, J. P. S. Lemos (2003)
P. Krtouš, J. Podolský (2003)

$\Lambda < 0$

J. Plebański, M. Demiański (1976)
...
O. J. C. Dias, J. P. S. Lemos (2003)
J. Podolský, M. Ortaggio, P. Krtouš (2003)
P. Krtouš (2005)

CAdSI: $A < 1/\ell$

$$\ell A = \sin\chi_o$$

$$\tau = \cot\chi_o \; t \qquad \upsilon = \tan\chi_o \; y \qquad \xi = -x$$

$$-\mathcal{F} = -1 - \upsilon^2 + 2\,\frac{m}{\ell}\,\cos\chi_o\,\upsilon^3 - \frac{e^2}{\ell^2}\,\cos^2\chi_o\,\upsilon^4$$

$$\mathcal{G} = \quad 1 - \xi^2 + 2\,\frac{m}{\ell}\,\sin\chi_o\,\xi^3 - \frac{e^2}{\ell^2}\,\sin^2\chi_o\,\xi^4$$

$$\omega = \upsilon\,\cos\chi_o - \xi\,\sin\chi_o$$

CAdSII: $A > 1/\ell$

$$\ell A = \cosh\alpha_o$$

$$\tau = \tanh\alpha_o \; t \qquad \upsilon = \coth\alpha_o \; y \qquad \xi = -x$$

$$-\mathcal{F} = 1 - \upsilon^2 + 2\,\frac{m}{\ell}\,\mathrm{sh}\,\alpha_o\,\upsilon^3 - \frac{e^2}{\ell^2}\,\mathrm{sh}^2\,\alpha_o\,\upsilon^4$$

$$\mathcal{G} = 1 - \xi^2 + 2\,\frac{m}{\ell}\,\mathrm{ch}\,\alpha_o\,\xi^3 - \frac{e^2}{\ell^2}\,\mathrm{ch}^2\,\alpha_o\,\xi^4$$

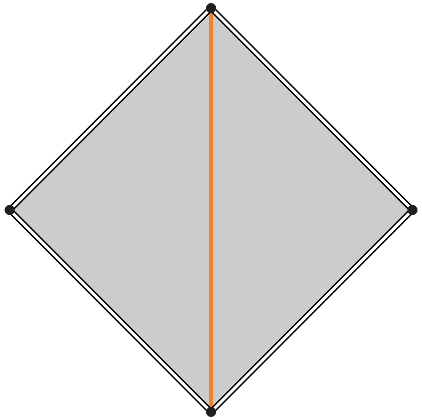
$$\omega = \upsilon\,\mathrm{sh}\,\alpha_o - \xi\,\mathrm{ch}\,\alpha_o$$

$$\boldsymbol{g} = \frac{\ell^2}{\omega^2}\left(-\mathcal{F}\,\mathbf{d}\tau^2 + \frac{1}{\mathcal{F}}\,\mathbf{d}\upsilon^2 + \frac{1}{\mathcal{G}}\,\mathbf{d}\xi^2 + \mathcal{G}\,\mathbf{d}\varphi^2\right)$$

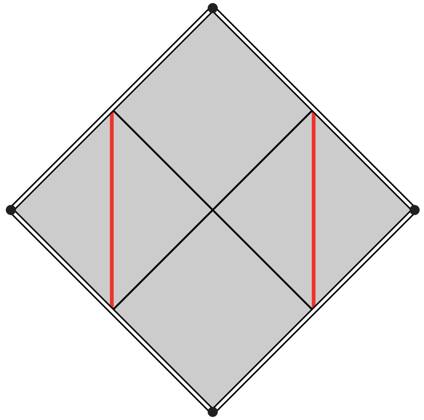
Accelerated observers in Minkowski and AdS spacetimes

Minkowski spacetime

$A = 0$

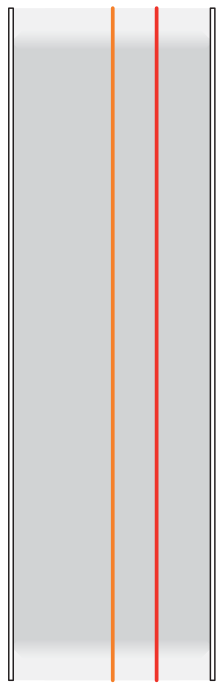


$A > 0$

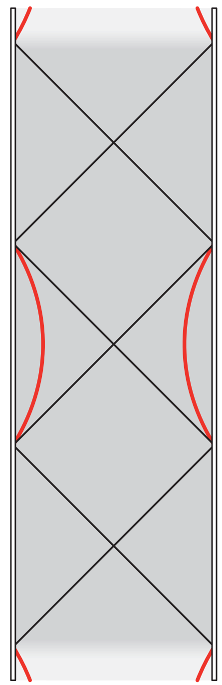


Anti-de Sitter spacetime

$A < \dfrac{1}{\ell}$



$A > \dfrac{1}{\ell}$



Interpretation of coordinates $\tau,\ \upsilon,\ \xi,\ \varphi$:

$$\boldsymbol{g} = \frac{\ell^2}{\omega^2}\left(-\mathcal{F}\,\mathbf{d}\tau^2 + \frac{1}{\mathcal{F}}\mathbf{d}\upsilon^2 + \frac{1}{\mathcal{G}}\mathbf{d}\xi^2 + \mathcal{G}\,\mathbf{d}\varphi^2\right)$$

τ	time coordinate of ‘accelerated’ observers outside black hole	
υ	radial coordinate	$R = \ell/\upsilon$
ξ	angular coordinate measured from the axis of symmetry	$\Theta = \int \frac{1}{\sqrt{\mathcal{G}}}\, d\xi$
φ	angular coordinate around the axis of symmetry	

zeros of $\mathcal{G}$ — axes of φ symmetry

$$\boldsymbol{g} = \frac{\ell^2}{\omega^2}\left(-\mathcal{F}\,\mathbf{d}\tau^2 + \frac{1}{\mathcal{F}}\,\mathbf{d}\upsilon^2 + \frac{1}{\mathcal{G}}\,\mathbf{d}\xi^2 + \mathcal{G}\,\mathbf{d}\varphi^2\right)$$

4 zeros, $\xi_\mathrm{b} < \xi_\mathrm{f}$ the smallest ones:

ξ_f axis in 'forward' direction
ξ_b axis in 'backward' direction

zeros of $\mathcal{F}$ — horizons

$$\boldsymbol{g} = \frac{\ell^2}{\omega^2}\left(-\mathcal{F}\,\mathbf{d}\tau^2 + \frac{1}{\mathcal{F}}\,\mathbf{d}\upsilon^2 + \frac{1}{\mathcal{G}}\,\mathbf{d}\xi^2 + \mathcal{G}\,\mathbf{d}\varphi^2\right)$$

CAdSI: 2 zeros $\upsilon_\mathrm{o} < \upsilon_\mathrm{i}$

CAdSII: 4 zeros $\upsilon_\mathrm{c} < \upsilon_\mathrm{a} < \upsilon_\mathrm{o} < \upsilon_\mathrm{i}$

υ_c cosmological horizon
υ_a acceleration horizon
υ_o outer black hole horizon
υ_i inner black hole horizon

assuming $m \neq 0$, $e \neq 0$

zeros of ω — conformal infinity $\mathcal{I}$

$$\boldsymbol{g} = \frac{\ell^2}{\omega^2}\left(-\mathcal{F}\,\mathbf{d}\tau^2 + \frac{1}{\mathcal{F}}\,\mathbf{d}\upsilon^2 + \frac{1}{\mathcal{G}}\,\mathbf{d}\xi^2 + \mathcal{G}\,\mathbf{d}\varphi^2\right)$$

CAdSI: $\upsilon = \tan\chi_\mathrm{o}\ \xi$

CAdSII: $\upsilon = \coth\alpha_\mathrm{o}\ \xi$

Black hole in accelerated coordinates T, R, Θ, Φ:

$$T = \ell\tau \qquad R = \frac{\ell}{v} \qquad \mathbf{d}\Theta = \frac{1}{\sqrt{\mathcal{G}}}\,\mathbf{d}\xi \qquad \Phi = \varphi$$

$$\boldsymbol{g} = \frac{\ell^2}{\omega^2 R^2}\left(-\mathcal{H}\,\mathbf{d}T^2 + \frac{1}{\mathcal{H}}\,\mathbf{d}R^2 + R^2\left(\mathbf{d}\Theta^2 + \mathcal{G}\,\mathbf{d}\Phi^2\right)\right)$$

CAdSI: $A < 1/\ell$

$$\mathcal{H} = 1 + \frac{R^2}{\ell^2} - \cos\chi_o\,\frac{2m}{R} + \cos^2\chi_o\,\frac{e^2}{R^2}$$

$$\frac{\omega R}{\ell} = \cos\chi_o - \frac{R}{\ell}\,\xi\,\sin\chi_o$$

CAdSII: $A > 1/\ell$

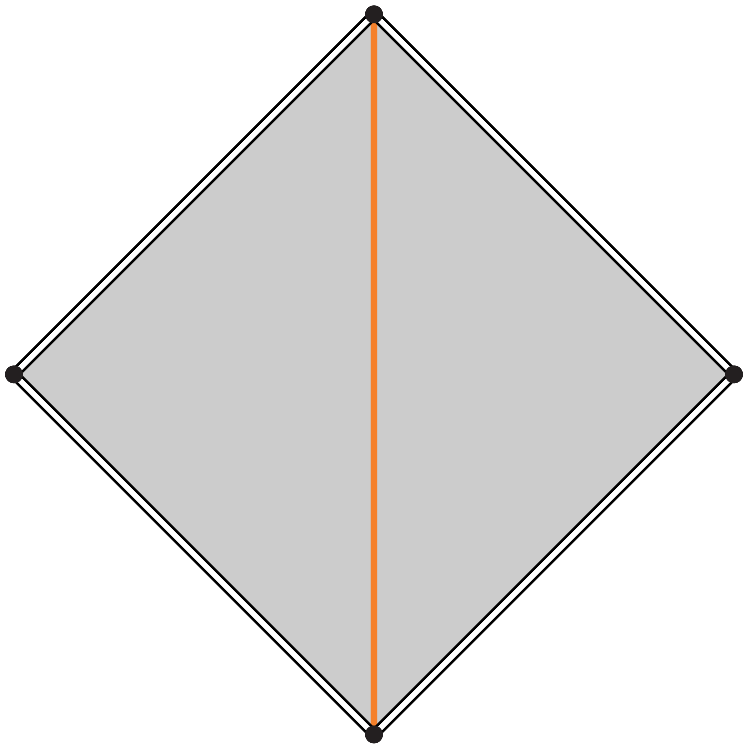
$$\mathcal{H} = 1 - \frac{R^2}{\ell^2} - \operatorname{sh}\alpha_o\,\frac{2m}{R} + \operatorname{sh}^2\alpha_o\,\frac{e^2}{R^2}$$

$$\frac{\omega R}{\ell} = \operatorname{sh}\alpha_o - \frac{R}{\ell}\,\xi\,\operatorname{ch}\alpha_o$$

$\chi_o = 0 \quad \Rightarrow \quad \mathcal{H} = 1 + \frac{R^2}{\ell^2} - \frac{2m}{R} + \frac{e^2}{R^2} \qquad \mathcal{G} = \sin^2\Theta$

$\Rightarrow$ a black hole in anti-de Sitter universe

(Un)accelerated observer in Minkowski spacetimes



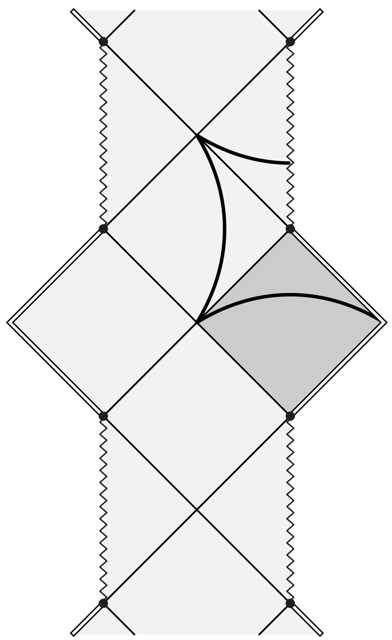
$A = 0$

Black hole in asymptotically flat spacetime, $\Lambda = 0$

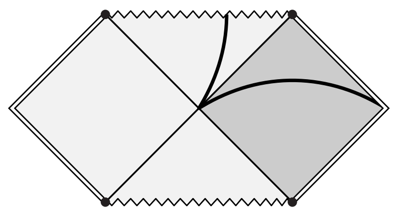
$$\boldsymbol{g} = -\mathcal{H}\,\mathbf{d}T^2 + \frac{1}{\mathcal{H}}\,\mathbf{d}R^2 + R^2\big(\mathbf{d}\Theta^2 + \sin^2\Theta\,\mathbf{d}\Phi^2\big)$$

$$\mathcal{H} = 1 - \frac{2m}{R} + \frac{e^2}{R^2}$$

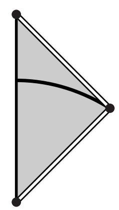
T-R diagrams



$m \neq 0 \quad e \neq 0$



$m \neq 0 \quad e = 0$



$m = 0 \quad e = 0$

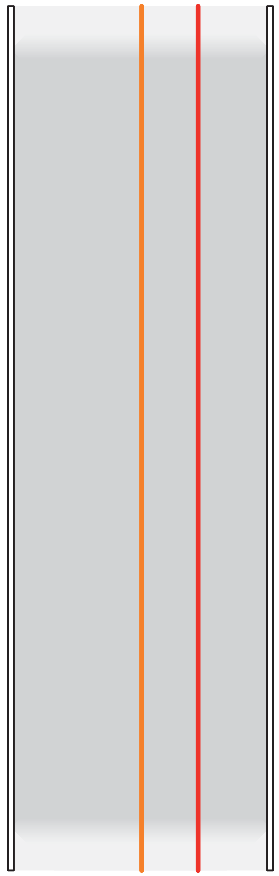
$\Lambda = 0$

Black hole in asymptotically flat spacetime

just reminding

animation . . .

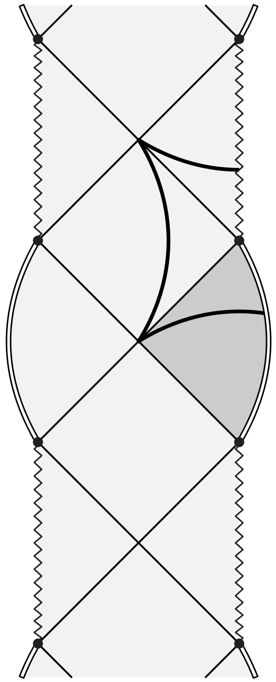
Subcritically accelerated observer in AdS spacetimes



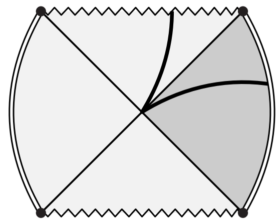
$$A < \frac{1}{\ell}$$

CAdSI: $A < 1/\ell$

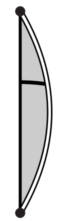
τ-υ diagrams
$\xi, \varphi = \text{const.}$



$m \neq 0 \quad e \neq 0$

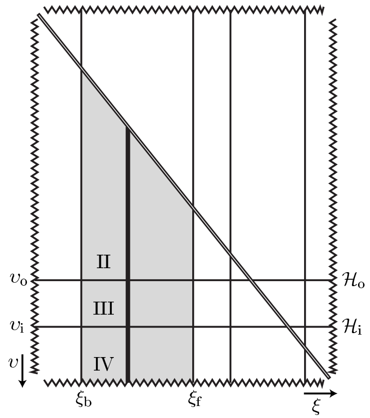


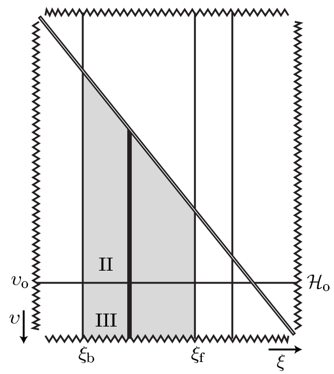
$m \neq 0 \quad e = 0$

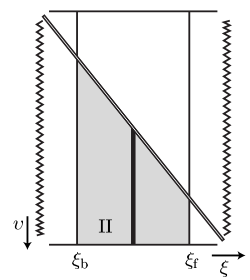


$m = 0 \quad e = 0$

ξ-υ diagrams
$\tau, \varphi = \text{const.}$



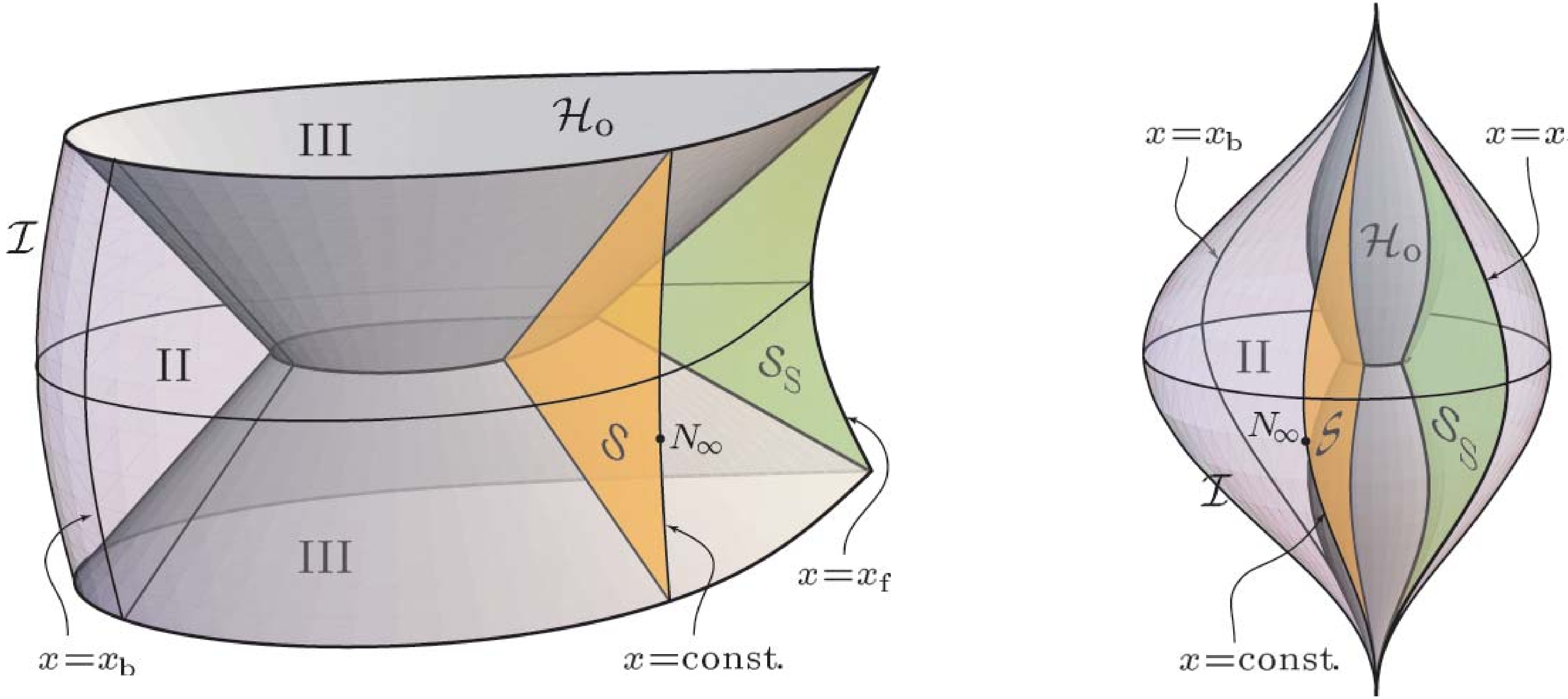




CAdSI: $A < 1/\ell$

One accelerated black hole in AdS

animation . . .

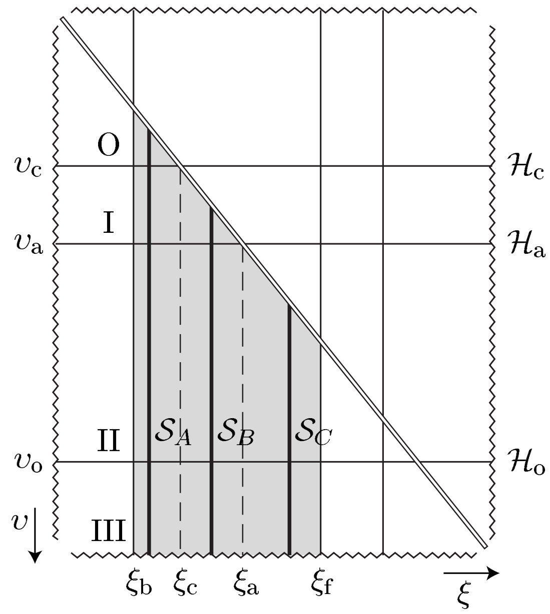


III
$\mathcal{H}_o$
$\mathcal{I}$
II
$\mathcal{S}_S$
$\mathcal{S}$
N_∞
III
$x=x_f$
$x=x_b$
$x=$const.
$x=x_b$
$x=x_f$
$\mathcal{H}_o$
II
N_∞
$\mathcal{S}$
$\mathcal{S}_S$
$\mathcal{I}$
$x=$const.

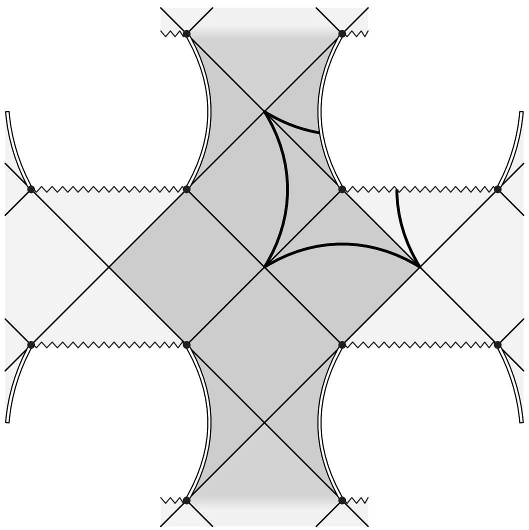
CAdSII: $A > 1/\ell$

$m \neq 0 \quad e = 0$

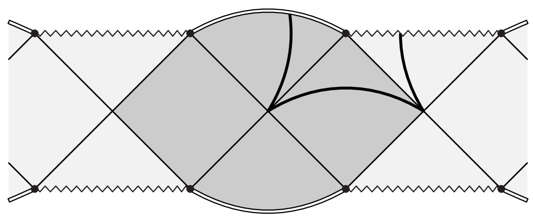
ξ-υ diagram
$\tau, \varphi = \text{const.}$



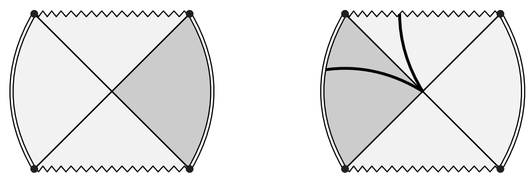
τ-υ diagram $\quad \xi, \varphi = \text{const.}$



$\mathcal{S}_A$: $\quad \xi \in (\xi_{\rm b}, \xi_{\rm c})$



$\mathcal{S}_B$: $\quad \xi \in (\xi_{\rm c}, \xi_{\rm a})$

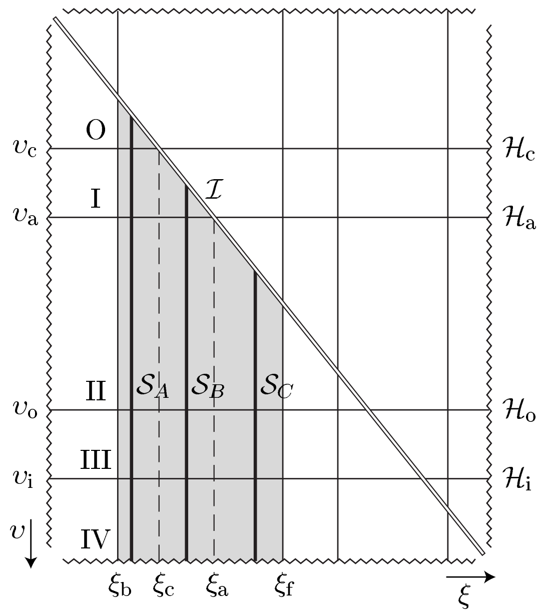


$\mathcal{S}_C$: $\quad \xi \in (\xi_{\rm a}, \xi_{\rm f})$

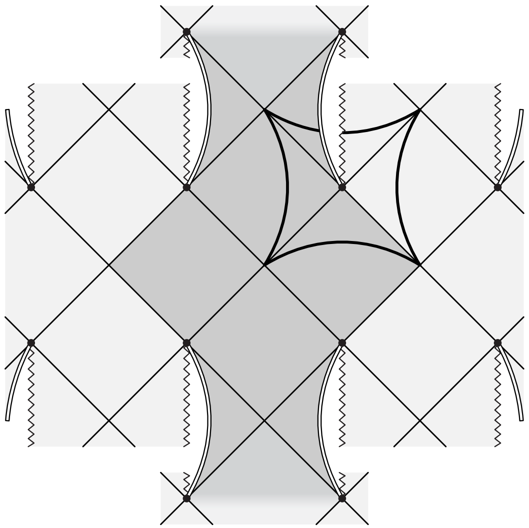
CAdSII: $A > 1/\ell$

$m \neq 0 \quad e \neq 0$

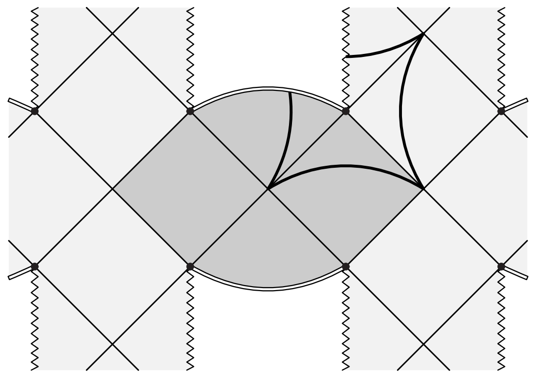
ξ-υ diagram
$\tau, \varphi = \mathrm{const.}$



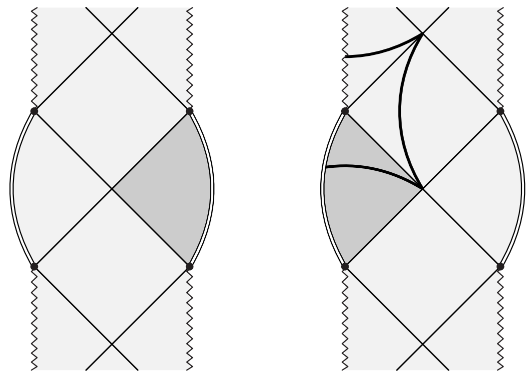
τ-υ diagram $\quad \xi, \varphi = \mathrm{const.}$



$\mathcal{S}_A$: $\quad \xi \in (\xi_{\mathrm{b}}, \xi_{\mathrm{c}})$

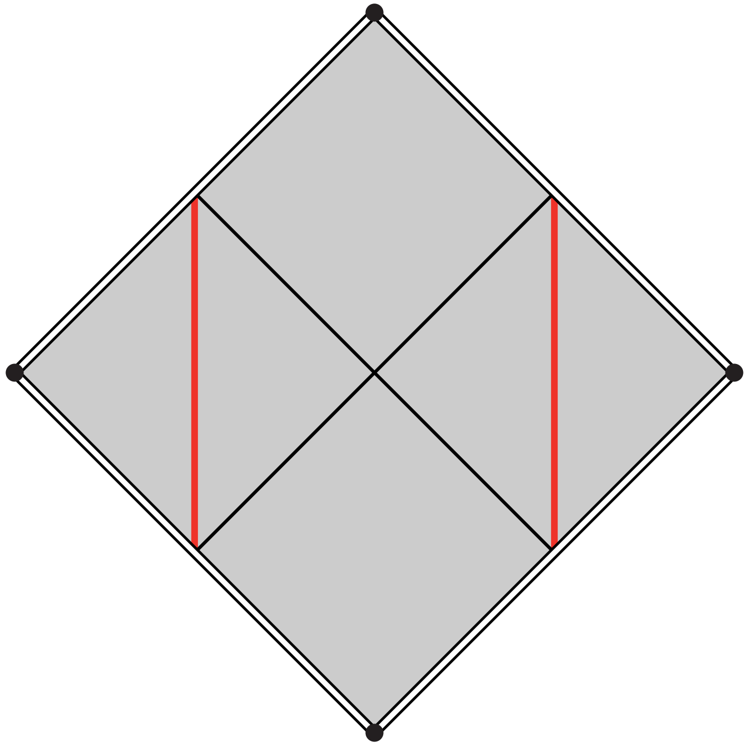


$\mathcal{S}_B$: $\quad \xi \in (\xi_{\mathrm{c}}, \xi_{\mathrm{a}})$



$\mathcal{S}_C$: $\quad \xi \in (\xi_{\mathrm{a}}, \xi_{\mathrm{f}})$

Accelerated observers in Minkowski spacetime



$A > 0$

$\Lambda = 0$

Two accelerated observers in flat spacetime

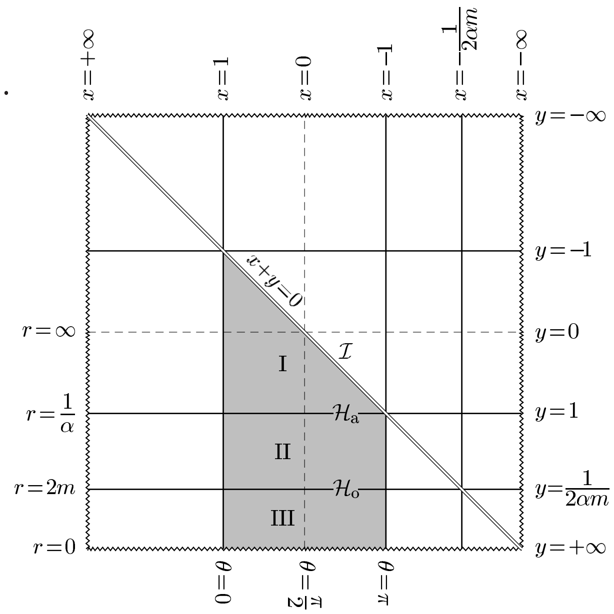
Two accelerated black holes in asymptotically flat spacetime

animation . . .

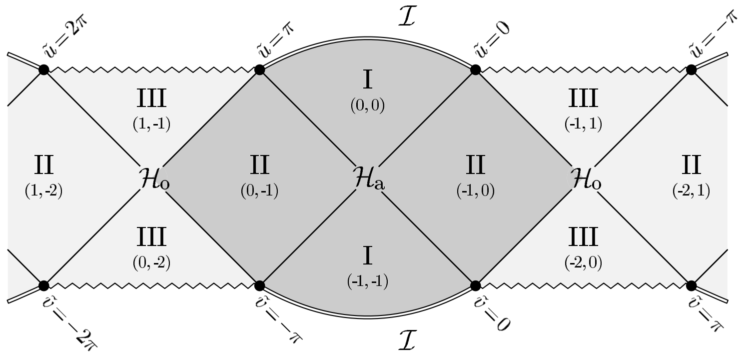
C-metric $\Lambda = 0$

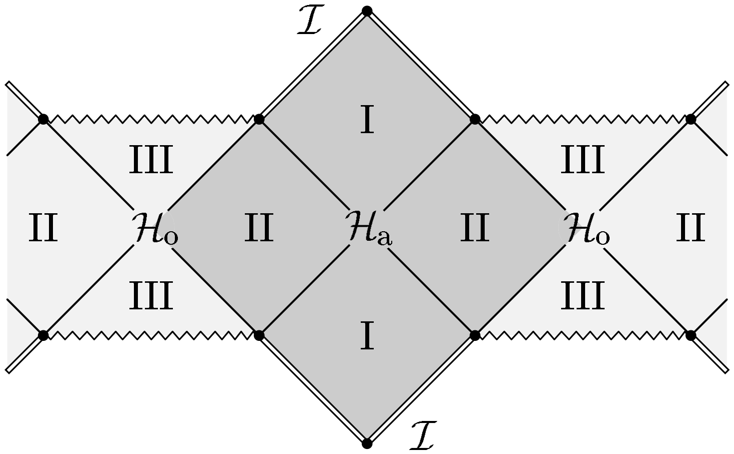
$m \neq 0 \quad e = 0$

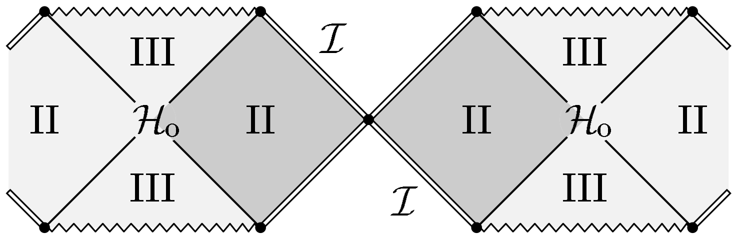
ξ-υ diagram
$\tau, \varphi = \text{const.}$



τ-υ diagram $\quad \xi, \varphi = \text{const.}$

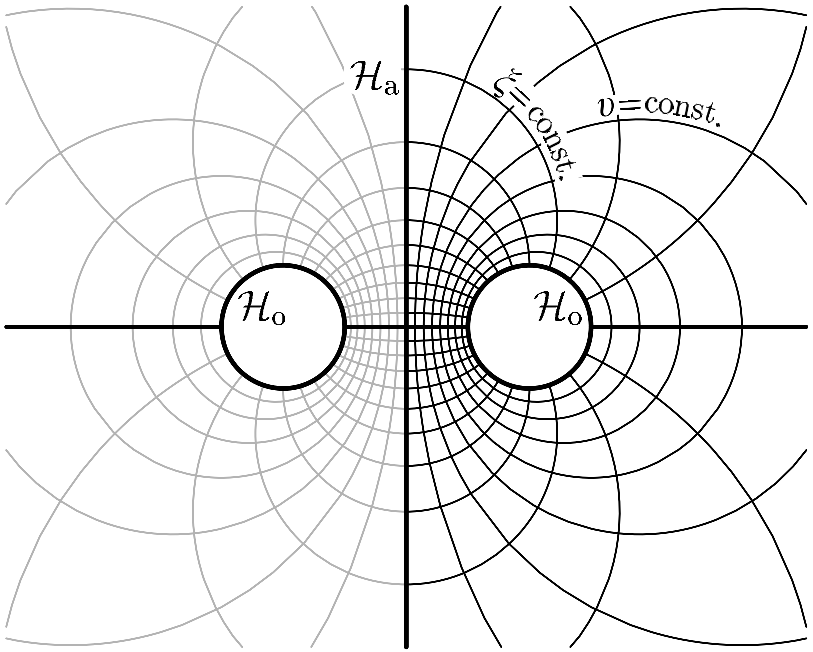




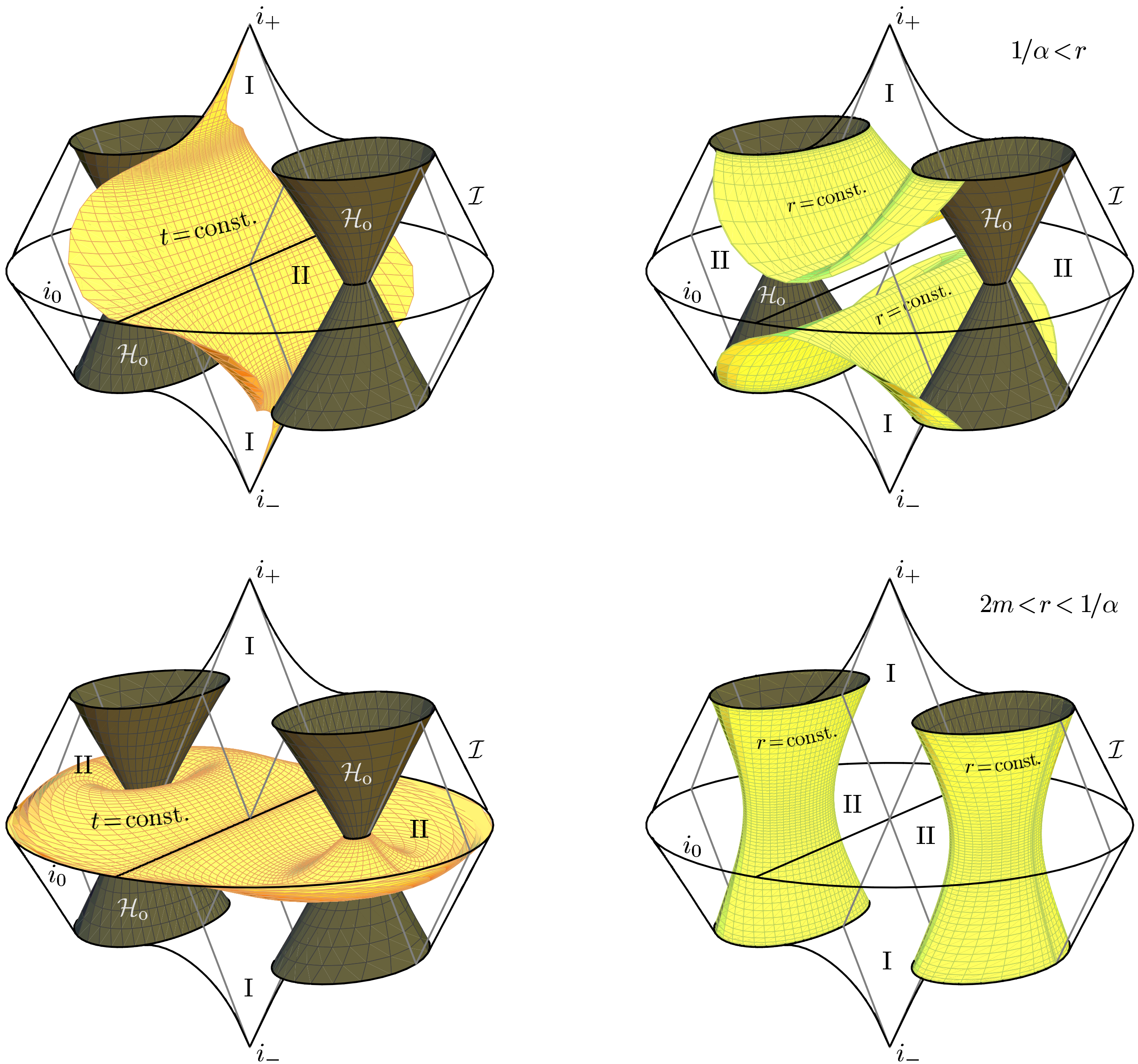


Bispherical coordinates

coordinates adjusted to two centers — two black holes

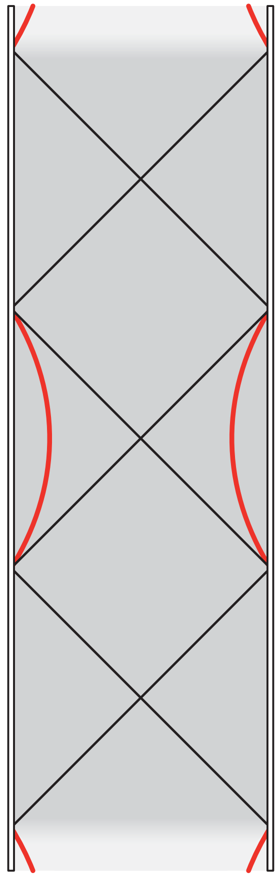


- υ is a 'radial' coordinate running between two holes
- ξ labels lines joining two holes



i_+
I
$t = \text{const.}$
$\mathcal{H}_o$
II
i_0
$\mathcal{I}$
i_-
$1/\alpha < r$
$r = \text{const.}$
$2m < r < 1/\alpha$

Supercritically accelerated observers in AdS spacetime

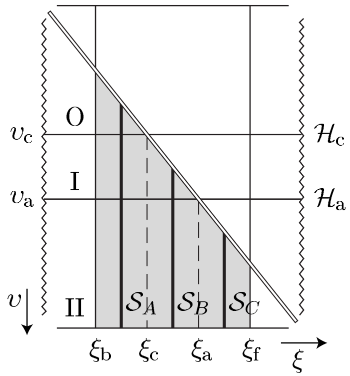


$$A > \frac{1}{\ell}$$

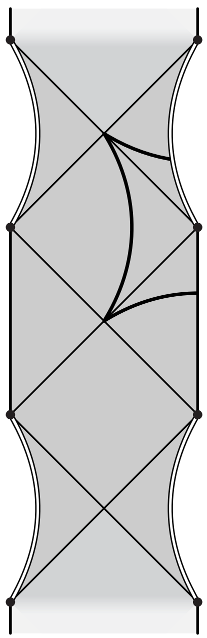
Anti-de Sitter in C-metric form $\quad m = 0 \quad e = 0 \quad A > 1/\ell$

$$\boldsymbol{g}_{\mathrm{AdS}} = \frac{\ell^2}{\omega^2}\Big(-(v^2-1)\,\mathbf{d}\tau^2 + \frac{1}{v^2-1}\,\mathbf{d}v^2 + \frac{1}{\xi^2-1}\,\mathbf{d}\xi^2 + (1-\xi^2)\,\mathbf{d}\varphi^2\Big)$$

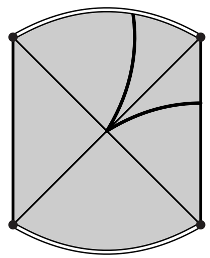
$$= \frac{\ell^2}{\omega^2 R^2}\bigg(-\Big(1-\frac{R^2}{\ell^2}\Big)\,\mathbf{d}T^2 + \Big(1-\frac{R^2}{\ell^2}\Big)^{-1}\mathbf{d}R^2 + R^2\,\big(\mathbf{d}\Theta^2 + \sin^2\Theta\,\mathbf{d}\Phi^2\big)\bigg)$$



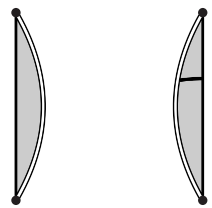
$$\omega = v\,\mathrm{sh}\,\alpha_{\mathrm{o}} - \xi\,\mathrm{ch}\,\alpha_{\mathrm{o}} \qquad\qquad \frac{\omega R}{\ell} = \mathrm{sh}\,\alpha_{\mathrm{o}} + \frac{R}{\ell}\,\mathrm{ch}\,\alpha_{\mathrm{o}}\cos\Theta$$



$\mathcal{S}_A$: $\quad \xi \in (\xi_{\mathrm{b}}, \xi_{\mathrm{c}})$



$\mathcal{S}_B$: $\quad \xi \in (\xi_{\mathrm{c}}, \xi_{\mathrm{a}})$



$\mathcal{S}_C$: $\quad \xi \in (\xi_{\mathrm{a}}, \xi_{\mathrm{f}})$

Anti-de Sitter in cosmological and accelerated frame

$$\boldsymbol{g}_{\mathrm{AdS}} = \frac{\ell^2}{\cos^2\tilde{r}}\Big(-\mathbf{d}\tilde{t}^2 + \mathbf{d}\tilde{r}^2 + \sin^2\tilde{r}\,\big(\mathbf{d}\vartheta^2 + \sin^2\vartheta\,\mathbf{d}\varphi^2\big)\Big)$$

cosmological coordinates

$$= \frac{\ell^2}{\xi_{\mathrm{II}}^2}\Big(-(\upsilon_{\mathrm{II}}^2-1)\,\mathbf{d}\tau_{\mathrm{II}}^2 + \frac{1}{\upsilon_{\mathrm{II}}^2-1}\mathbf{d}\upsilon_{\mathrm{II}}^2 + \frac{1}{\xi_{\mathrm{II}}^2-1}\mathbf{d}\xi_{\mathrm{II}}^2 + (1-\xi_{\mathrm{II}}^2)\,\mathbf{d}\varphi^2\Big)$$

static type II coordinates

$$= \frac{\xi^2}{\omega^2}\,\frac{\ell^2}{\xi^2}\Big(-(\upsilon^2-1)\,\mathbf{d}\tau^2 + \frac{1}{\upsilon^2-1}\mathbf{d}\upsilon^2 + \frac{1}{\xi^2-1}\mathbf{d}\xi^2 + (1-\xi^2)\,\mathbf{d}\varphi^2\Big)$$

C-metric coordinates

$$= \Omega^2\,\frac{\ell^2}{\cos^2\tilde{r}'}\Big(-\mathbf{d}\tilde{t}'^2 + \mathbf{d}\tilde{r}'^2 + \sin^2\tilde{r}'\,\big(\mathbf{d}\vartheta'^2 + \sin^2\vartheta'\,\mathbf{d}\varphi^2\big)\Big)$$

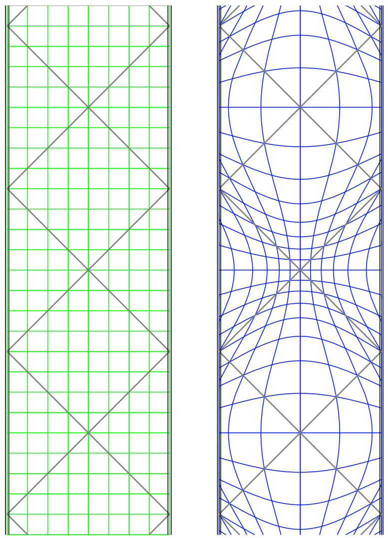
accelerated coordinates

$$\Omega = \Big(\operatorname{ch}\alpha_{\mathrm{o}} + \operatorname{sh}\alpha_{\mathrm{o}}\frac{\sin\tilde{t}'}{\cos\tilde{r}'}\Big) = \Big(\operatorname{ch}\alpha_{\mathrm{o}} - \operatorname{sh}\alpha_{\mathrm{o}}\frac{\sin\tilde{t}}{\cos\tilde{r}}\Big)^{-1}$$

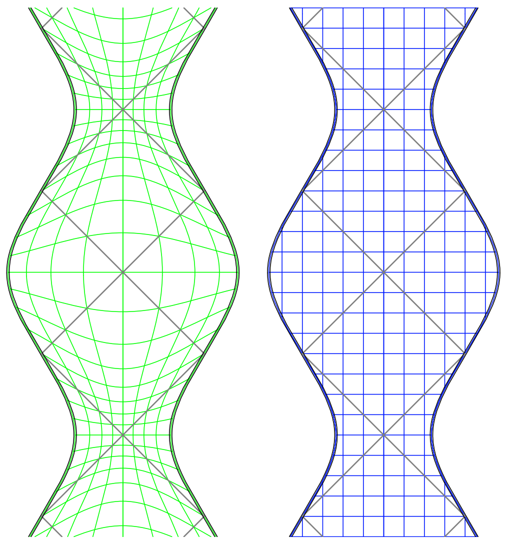
$$\tan\tilde{t}' = \frac{\operatorname{ch}\alpha_{\mathrm{o}}\sin\tilde{t} - \operatorname{sh}\alpha_{\mathrm{o}}\cos\tilde{r}}{\cos\tilde{t}}$$

$$\cot\tilde{r}' = \frac{-\operatorname{sh}\alpha_{\mathrm{o}}\sin\tilde{t} + \operatorname{ch}\alpha_{\mathrm{o}}\cos\tilde{r}}{\sin\tilde{r}}$$

$$\vartheta' = \vartheta$$



cosmological frame



accelerated frame

AdS: $A > 1/\ell$

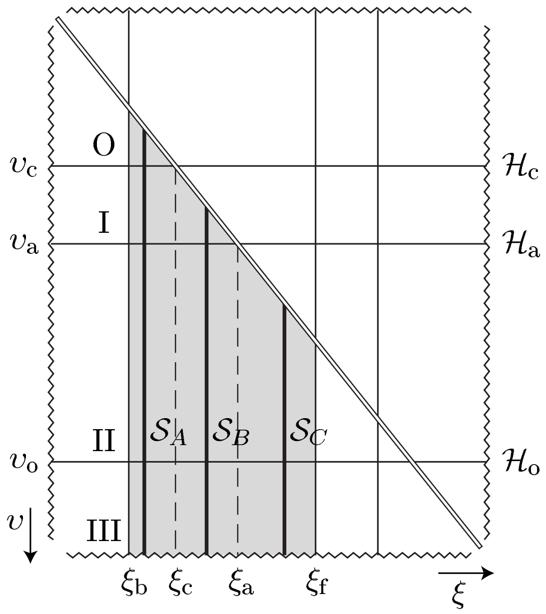
Accelerated coordinates in AdS

animation . . .

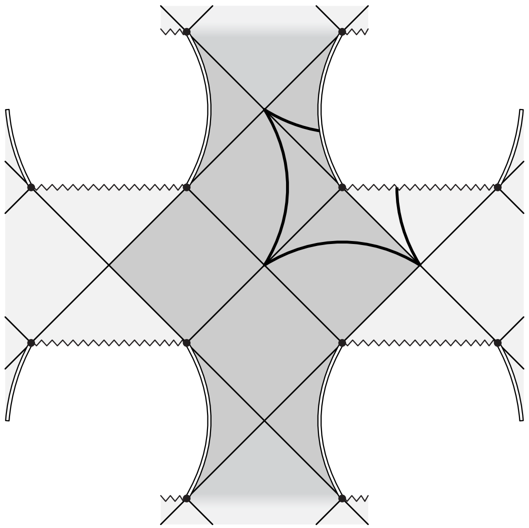
CAdSII: $A > 1/\ell$

$m \neq 0 \quad e = 0$

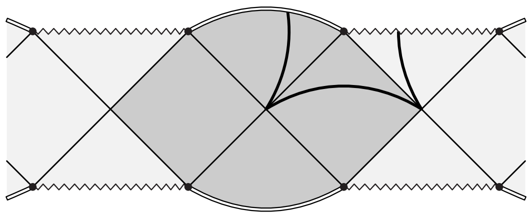
ξ-υ diagram
$\tau, \varphi = \text{const.}$



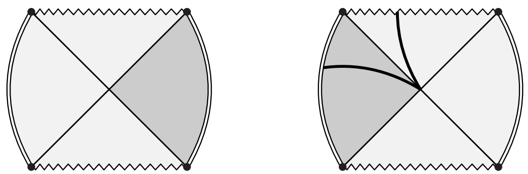
τ-υ diagram $\quad \xi, \varphi = \text{const.}$



$\mathcal{S}_A$: $\quad \xi \in (\xi_{\mathrm{b}}, \xi_{\mathrm{c}})$



$\mathcal{S}_B$: $\quad \xi \in (\xi_{\mathrm{c}}, \xi_{\mathrm{a}})$

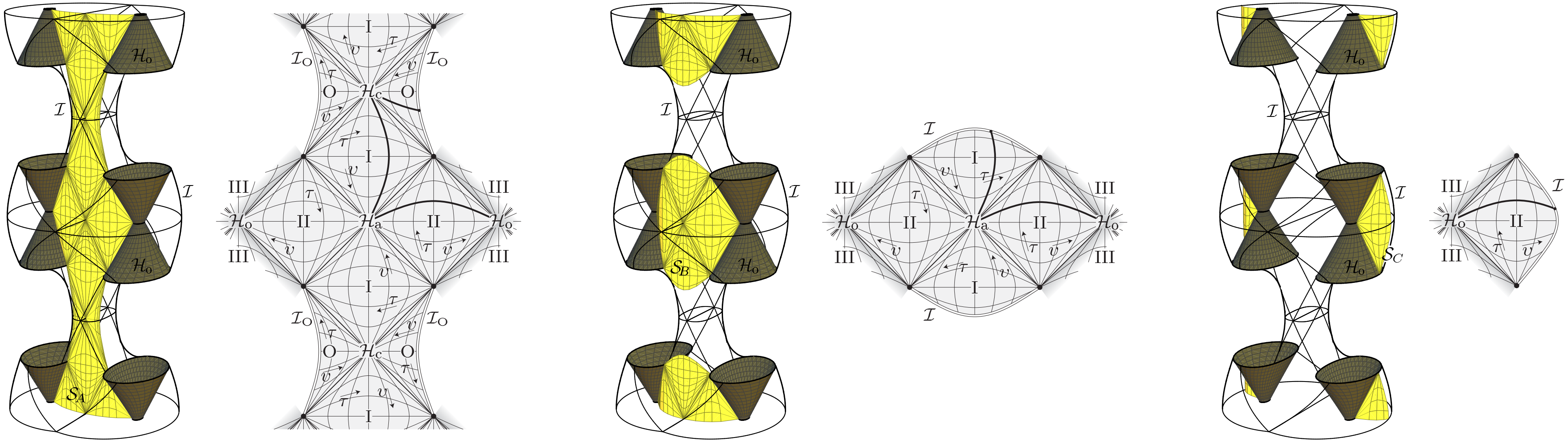


$\mathcal{S}_C$: $\quad \xi \in (\xi_{\mathrm{a}}, \xi_{\mathrm{f}})$

CAdSII: $A > 1/\ell$

Pairs of accelerated black holes in AdS

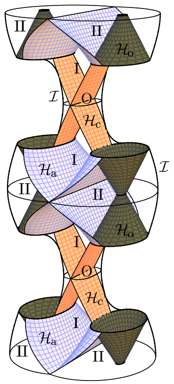
animation ...



$\mathcal{H}_o$
$\mathcal{I}$
$\mathcal{S}_A$
I
$\mathcal{I}_O$
O
$\mathcal{H}_c$
τ
v
II
III
$\mathcal{H}_a$
$\mathcal{S}_B$
$\mathcal{S}_C$

This presentation and the animations can be found at:
http://utf.mff.cuni.cz/~krtous/

Related work



C-metric with non-vanishing Λ

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Accelerated black holes in anti-de Sitter universe

Podolský J., Ortaggio M., Krtouš P.: Phys. Rev. D **68**, 124004 (2003)
Radiation from accelerated black holes in an anti-de Sitter universe

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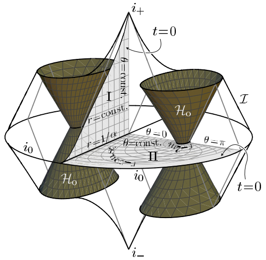
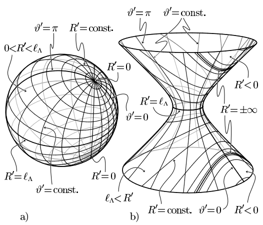
Radiation from accelerated black holes in a de Sitter universe



C-metric with vanishing Λ

Griffiths J. B., Krtouš P., Podolský J.: Class. Quantum Grav. 23, 6745 (2006), gr-qc/0609056
Interpreting the C-metric



Accelerated observers in (anti-)de Sitter universe

Bičák J., Krtous P.: Phys. Rev. D **63** (2001) 124020
Accelerated sources in de Sitter spacetime and the insufficiency of retarded fields

Bičák J., Krtouš P.: Phys. Rev. Lett. **88**, 211101 (2002)
The fields of uniformly accelerated charges in de Sitter spacetime

Bičák J., Krtouš P.: J. Math. Phys. **46**, J. Math. Phys. 46, 102504 (2005)
Fields of accelerated sources: Born in de Sitter