Accelerated black holes in de Sitter universe

Non-spinning (charged) C-**metric** — a solution of Einstein–Maxwell equations with positive cosmological constant Λ — represents a pair of accelerated black holes in asymptotically de Sitter universe.

Intuitively, both black holes starts at the opposite poles of de Sitter universe, move toward each other (squeezing a cosmic string between them or stretching the string on other side) until they stop and start to move back to the poles of the universe.

$$\boldsymbol{g} = \frac{\ell^2}{\omega^2}\left(-\mathcal{F}\,\mathbf{d}\tau^2 + \frac{1}{\mathcal{F}}\,\mathbf{d}\upsilon^2 + \frac{1}{\mathcal{G}}\,\mathbf{d}\xi^2 + \mathcal{G}\,\mathbf{d}\varphi^2\right)$$

$$\mathbf{F} = e\,\mathbf{d}\upsilon \wedge \mathbf{d}\tau$$

$$-\mathcal{F} = 1 - \upsilon^2 + 2\,\frac{m}{\ell}\,\mathrm{ch}\,\alpha_o\,\upsilon^3 - \frac{e^2}{\ell^2}\,\mathrm{ch}^2\,\alpha_o\,\upsilon^4$$

$$\mathcal{G} = 1 - \xi^2 + 2\,\frac{m}{\ell}\,\mathrm{sh}\,\alpha_o\,\xi^3 - \frac{e^2}{\ell^2}\,\mathrm{sh}^2\,\alpha_o\,\xi^4$$

$$\omega = -\upsilon\cosh\alpha_o + \xi\sinh\alpha_o$$

Interpretation of coordinates

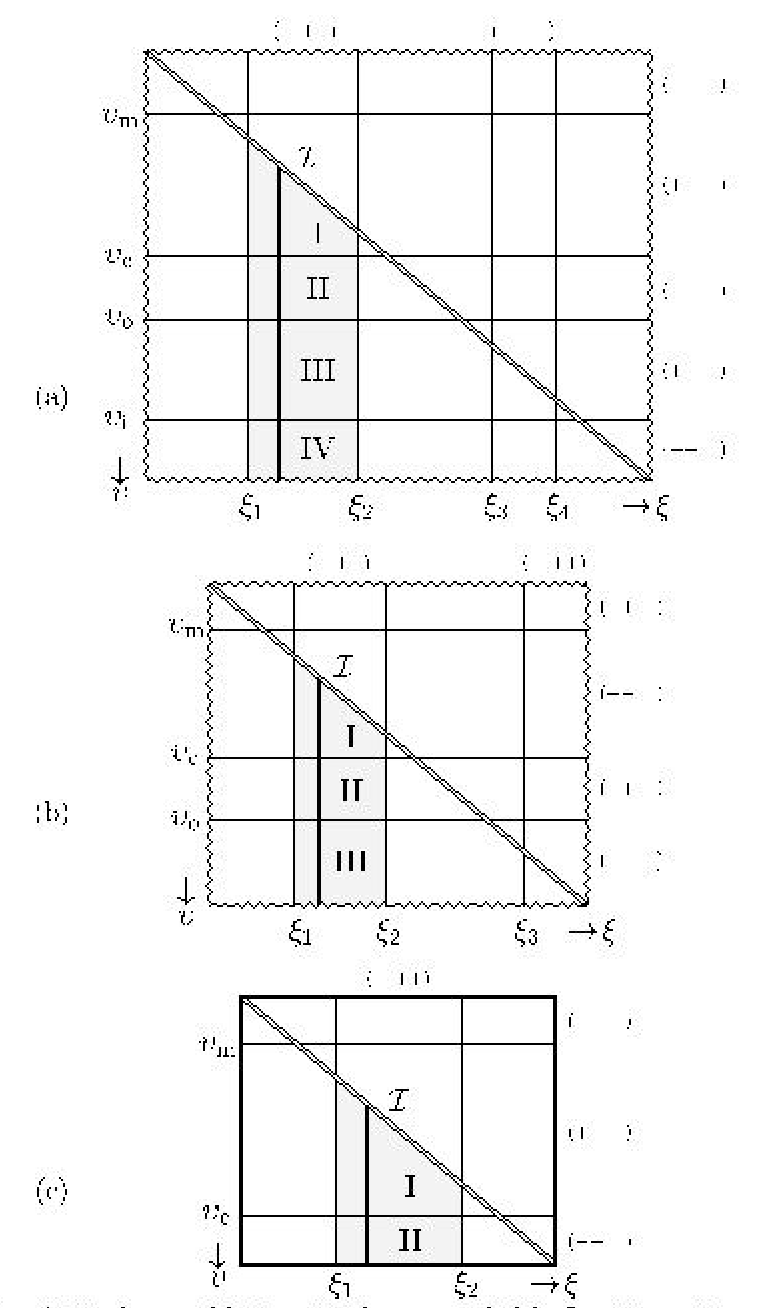
- τ — time coordinate of 'accelerated' observers
- υ — radial coordinate
- ξ — angular coordinate measured from the axis of symmetry
- φ — angular coordinate around the axis of symmetry

Parameters

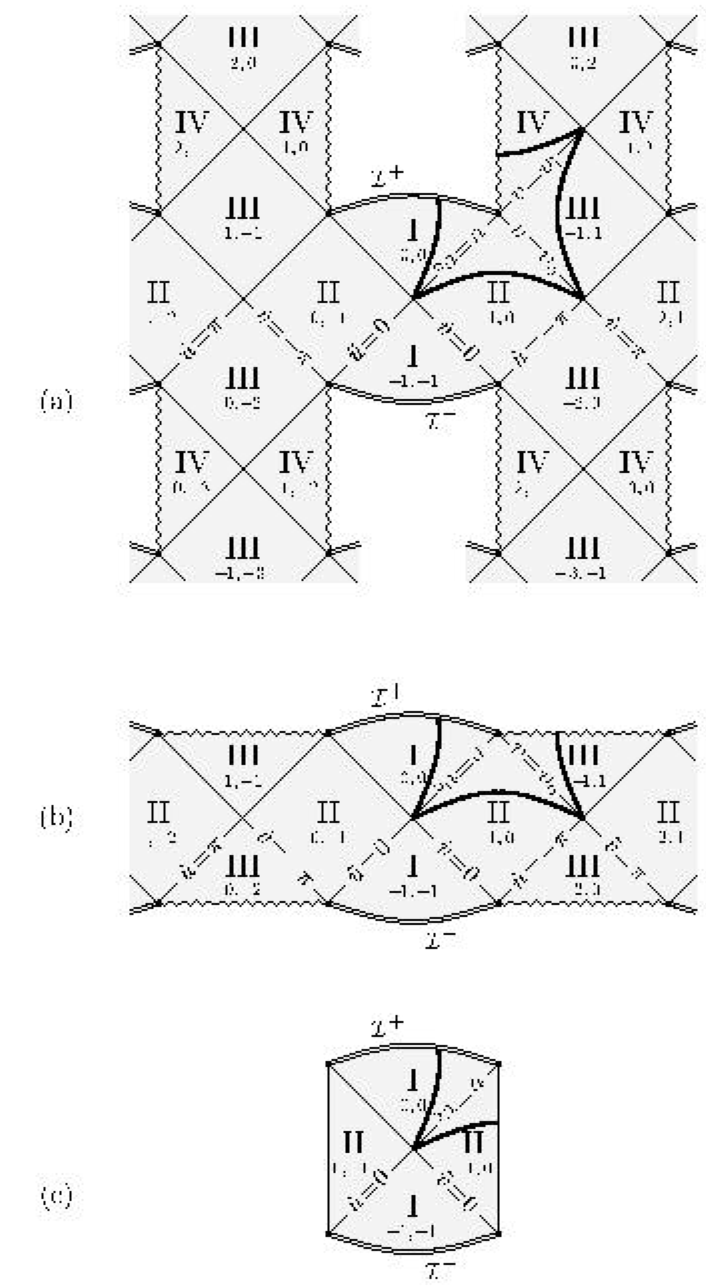
- m — mass parameter
- e — charge parameter
- A — acceleration parameter: $\sinh\alpha_o = \ell A$
- C — conicity parameter: $\varphi \in (-C\pi, C\pi)$
- ℓ — cosmological scale: $\ell = \sqrt{3/\Lambda}$

Basic properties of the spacetime

- Two Killing vectors $\boldsymbol{\partial}_\tau$, $\boldsymbol{\partial}_\varphi$ and one conformal Killing tensor
- Two double-degenerate principal null directions lying in surfaces $\xi = \text{const.}$ (Petrov type D) pointing 'radially' from the black holes
- Spacelike past and future conformal infinities $\mathcal{I}^-$ and $\mathcal{I}^+$
- Two disconnected event horizons (a pair of black holes) and one cosmological/acceleration horizon in one asymptotically de Sitter domain
- Conical singularity along the axis of symmetry – cosmic string 'accelerating' black holes



A qualitative diagram of the ξ-υ section (τ, $\varphi = \text{const.}$) of the C-metric spacetime. The three cases correspond to (a) charged accelerated black holes in asymptotically de Sitter universe ($e \neq 0$, $m \neq 0$), (b) uncharged black holes ($e = 0$, $m \neq 0$), and (c) de Sitter universe ($e = 0$, $m = 0$). Horizontal lines indicate the horizons, vertical lines are axes of φ symmetry. The diagonal double line $\upsilon = \xi\tanh\alpha_o$ corresponds to infinity $\mathcal{I}$. Singularities are depicted by 'zig-zag' lines. The boundary of each diagram corresponds to ξ, $\upsilon = \pm\infty$. Different columns and rows correspond to different signs of the functions $\mathcal{G}$ and $\mathcal{F}$, respectively, and thus to different signatures of the metric, which are indicated on the sides of the diagrams. C-metric describes, in general, four distinct spacetimes — the domains in columns $\xi_b < \xi < \xi_f$ and $\xi_3 < \xi < \xi_4$, separated in addition by infinity (the diagonal line). The physically most reasonable spacetime has the coordinates ξ, υ in the ranges $\xi_b < \xi < \xi_f$ and $\upsilon > \xi\tanh\alpha_o$ (the shaded areas). Sections $\xi = \text{const.}$ which correspond to the conformal diagrams on the right are indicated by thick lines.



The conformal diagrams of the τ-υ section (ξ, $\varphi = \text{const.}$). Three diagrams correspond again to the cases of (a) charged accelerated black holes, (b) uncharged black holes, and (c) de Sitter universe (in accelerated coordinates). The conformal infinities $\mathcal{I}$ are indicated by double lines, the singularities are drawn by 'zig-zag' lines, and horizons by thin lines. The horizons $\upsilon = \upsilon_c$, υ_o, υ_i divide the conformal diagrams into four types of blocks, labeled by I–IV, which correspond to the regions I–IV on the left. The sections $\tau = \text{const.}$ (drawn on the left) are indicated by thick lines. Only a part of the complete conformal diagram is shown in the cases (a) and (b), however, the rest of the diagram would have a similar structure as the part shown. The complete diagram depends on a freedom in the choice of a global topology of the spacetime given by identifications of different blocks of the conformal diagram. In the case (c), the diagram does not contain any black hole — it is 'closed on its sides' by poles of a spacelike section S^3 of de Sitter universe.

Relation to Kinnersley–Walker form

$$g = \frac{1}{A^2(x+y)^2}\left(-F\,dt^2 + \frac{1}{F}\,dy^2 + \frac{1}{G}\,dx^2 + G\,d\varphi^2\right)$$

$$F = -\ell^{-2}A^{-2} - 1 + y^2 - 2mA\,y^3 + e^2A^2\,y^4$$

$$G = 1 - x^2 - 2mA\,x^3 - e^2A^2\,x^4$$

$$\tau = \coth\alpha_o\;t \qquad \upsilon = \tanh\alpha_o\;y \qquad \xi = -x$$

Black hole coordinates

$$g = \frac{\ell^2}{\omega^2R^2}\left(-\mathcal{H}\,dT^2 + \frac{1}{\mathcal{H}}\,dR^2 + R^2(d\Theta^2 + \mathcal{G}\,d\Phi^2)\right)$$

$$\mathcal{H} = 1 - \frac{R^2}{\ell^2} - \cosh\alpha_o\,\frac{2m}{R} + \cosh^2\alpha_o\,\frac{e^2}{R^2}$$

$$T = \ell\tau \quad R = \frac{\ell}{\upsilon} \quad d\Theta = \frac{1}{\sqrt{\mathcal{G}}}\,d\xi \quad \Phi = \varphi$$

For vanishing acceleration $\alpha_o = 0$ we get

$$R\omega = \ell \qquad \xi = \cos\Theta \qquad \mathcal{H} = 1 - \frac{R^2}{\ell^2} - \frac{2m}{R} + \frac{e^2}{R^2}$$

C-metric becomes the Reissner–Nordström–de Sitter or Schwarzschild–de Sitter solution, respectively.

zeros of $\mathcal{G}$ — axes of φ symmetry

4 zeros, $\xi_b < \xi_f$ the smallest ones:

- ξ_f — axis in 'forward' direction
- ξ_b — axis in 'backward' direction

zeros of $\mathcal{F}$ — horizons

4 zeros $\upsilon_i > \upsilon_o > \upsilon_c > \upsilon_m$ (assuming $m \neq 0$, $e \neq 0$):

- υ_i — inner black hole horizon
- υ_o — outer black hole horizon
- υ_c — cosmological/acceleration horizon
- υ_m — non-physical value

zeros of ω — conformal infinity $\mathcal{I}$

$\upsilon = \tanh\alpha_o\,\xi$

Few references on C-metric

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