Radiation near C-metric infinity

As noted by Penrose already in sixties, the notion of radiation near *spacelike* infinity is 'less invariantly' defined than near *null* conformal infinity. We have studied this ambiguity on the example of C-metric with $\Lambda > 0$ and we have found explicitly the directional structure of the radiative component.

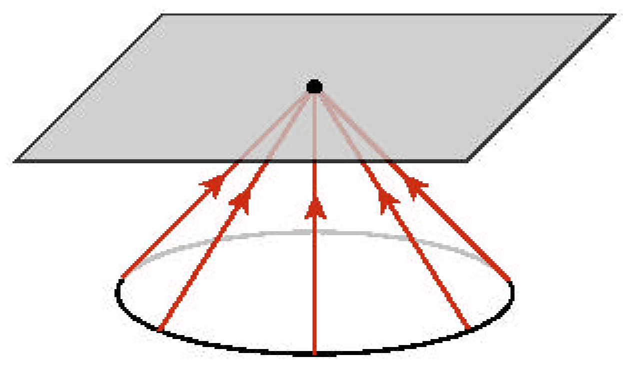
This *directional structure of radiation* naturally supplements the peeling-off property of the fields.
Since radiative component does not vanish along a generic direction, its absence cannot be used to identify radiative sources in a similar way as for spacetimes with null infinity.

Radiative component of the field

component Ψ_4^{i} calculated in the interpretation tetrad parallelly transported along a null geodesic approaching $\mathcal{I}$

For *spacelike* infinity $\mathcal{I}$ the radiative component *depends* on a direction along which $\mathcal{I}$ is approached

[For *null* conformal infinity ($\Lambda = 0$) the radiative component *does not depend* on a direction of a null geodesic approaching a given point on $\mathcal{I}$!]



A point at spacelike infinity can be approached along different null geodesics. Interpretation tetrads parallelly transported along such geodesics differ considerably when transported to the point at infinity. This causes directional dependence of the radiative component Ψ_4^{i}.

Null and orthonormal tetrads

With an orthonormal tetrad $\mathbf{t}$, $\mathbf{q}$, $\mathbf{r}$, $\mathbf{s}$ a complex null tetrad can be associated $\mathbf{k}$, $\mathbf{l}$, $\mathbf{m}$, $\bar{\mathbf{m}}$

$$\mathbf{k} = \tfrac{1}{\sqrt{2}}(\mathbf{t}+\mathbf{q}) \qquad \mathbf{l} = \tfrac{1}{\sqrt{2}}(\mathbf{t}-\mathbf{q})$$
$$\mathbf{m} = \tfrac{1}{\sqrt{2}}(\mathbf{r}-i\mathbf{s}) \qquad \bar{\mathbf{m}} = \tfrac{1}{\sqrt{2}}(\mathbf{r}+i\mathbf{s})$$

Only non-vanishing scalar products are

$$\mathbf{t}\cdot\mathbf{t} = -1 \quad \mathbf{q}\cdot\mathbf{q} = \mathbf{r}\cdot\mathbf{r} = \mathbf{s}\cdot\mathbf{s} = 1 \quad \mathbf{k}\cdot\mathbf{l} = -1 \quad \mathbf{m}\cdot\bar{\mathbf{m}} = 1$$

Interpretation tetrad

Null tetrad $\mathbf{k}_{\mathrm{i}}$, $\mathbf{l}_{\mathrm{i}}$, $\mathbf{m}_{\mathrm{i}}$, $\bar{\mathbf{m}}_{\mathrm{i}}$ which satisfy

- it is parallelly transported along null geodesic
- vector $\mathbf{k}_{\mathrm{i}}$ is tangent to the geodesic
- projection of $\mathbf{k}_{\mathrm{i}}$ on direction normal to $\mathcal{I}$ is fixed independently of the direction of geodesic

Assuming Penrose's asymptotic Einstein condition, vectors $\omega^{-1}\mathbf{k}_{\mathrm{i}}$ and $\omega\mathbf{l}_{\mathrm{i}}$ become asymptotically colinear with the vector $\mathbf{n}$ normal to $\mathcal{I}$.

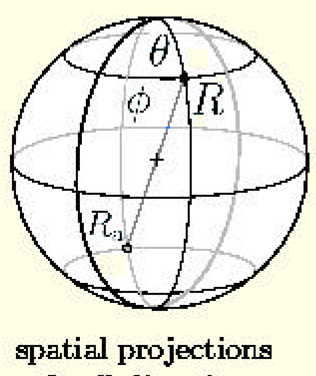
Directional structure of radiation

$$\left|\Psi_4^{\mathrm{i}}\right| \approx 3A^2(m - 2e^2A\xi_\infty)\,\mathcal{G}_\infty\ \frac{1}{\eta}\ \frac{1}{\left(1+|R|^2\right)^2}\ \left|1-\frac{R_1}{R_{\mathrm{a}}}\right|^2\left|1-\frac{R_2}{R_{\mathrm{a}}}\right|^2$$

Radiation component **depends on the direction** along which $\mathcal{I}$ is approached and it is **nonvanishing** in a generic direction. It vanishes only along directions spatially antipodal to the principal null directions, i.e., to the directions from the black holes.

- Normalization factor specific for C-metric geometry
- Radiative dependence on affine parameter η
- Dependence on a direction along which $\mathcal{I}$ is approached

R is stereographic parametrization of the direction of the null geodesic with respect of the reference tetrad
$R_{\mathrm{a}} = -\bar{R}^{-1}$ is a direction spatially antipodal to the direction given by R
R_1, R_2 are parameters of principal null directions $\mathbf{k}_1$, $\mathbf{k}_2$



spatial projections of null directions

The radiation does *not* vanish even for non-accelerating black holes ($A = 0$). In this case $\left|\Psi_4^{\mathrm{i}}\right|$ simplifies to

$$\left|\Psi_4^{\mathrm{i}}\right| \approx \frac{3}{4}\frac{m}{\ell^2}\frac{1}{\eta}\sin^2\theta$$

It vanishes only along *radial* directions coming from black holes.

Reference tetrad

Null tetrad $\mathbf{k}_{\mathrm{o}}$, $\mathbf{l}_{\mathrm{o}}$, $\mathbf{m}_{\mathrm{o}}$, $\bar{\mathbf{m}}_{\mathrm{o}}$ or associated orthonormal tetrad $\mathbf{t}_{\mathrm{o}}$, $\mathbf{q}_{\mathrm{o}}$, $\mathbf{r}_{\mathrm{o}}$, $\mathbf{s}_{\mathrm{o}}$ chosen at infinity $\mathcal{I}$. It is adjusted to infinity in the sense that

- unit time vector $\mathbf{t}_{\mathrm{o}}$ is normal to $\mathcal{I}$
- vectors $\mathbf{q}_{\mathrm{o}}, \mathbf{r}_{\mathrm{o}}, \mathbf{s}_{\mathrm{o}}$ are tangent to $\mathcal{I}$
- vectors $\mathbf{k}_{\mathrm{i}}, \mathbf{l}_{\mathrm{i}}$ are coplanar with normal to $\mathcal{I}$

With respect to the reference tetrad we parametrize null directions at infinity. Namely, null direction $\mathbf{k}$ can be parametrized by $R \in \mathbb{C}$:

$$\mathbf{k} \propto \mathbf{k}_{\mathrm{o}} + \bar{R}\,\mathbf{m}_{\mathrm{o}} + R\,\bar{\mathbf{m}}_{\mathrm{o}} + R\bar{R}\,\mathbf{l}_{\mathrm{o}}$$

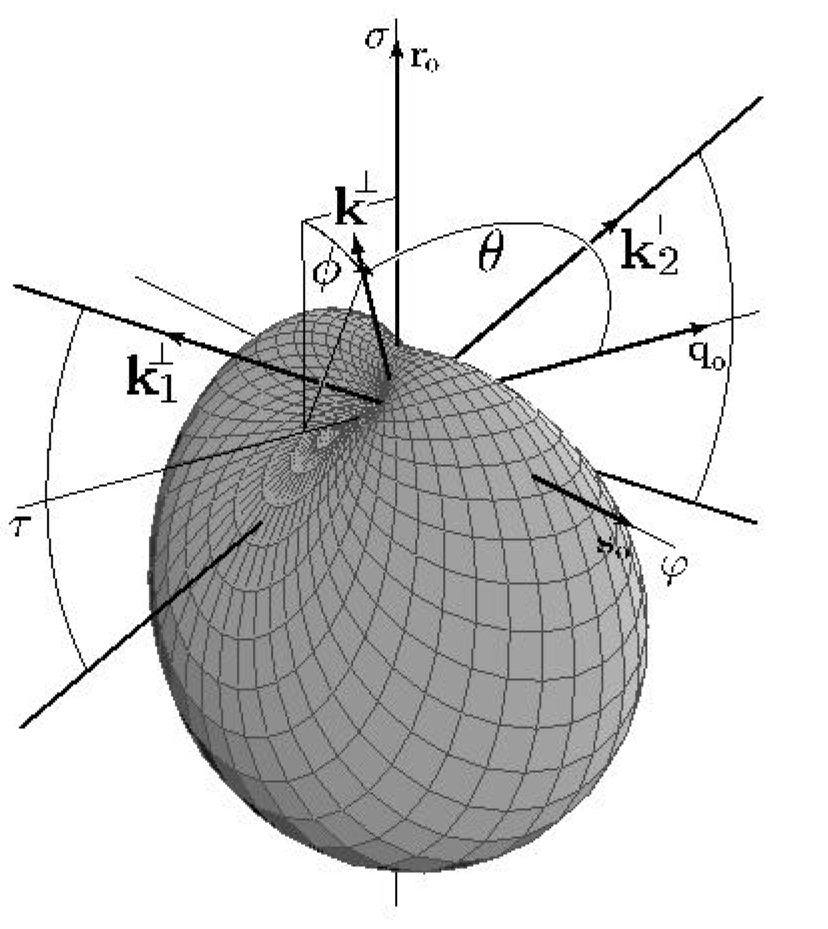
R is stereographic representation of spherical angles θ, ϕ measured with respect spatial frame $\mathbf{q}_{\mathrm{o}}), \mathbf{r}_{\mathrm{o}}), \mathbf{s}_{\mathrm{o}})$

$$R = \tan\tfrac{\theta}{2}\,\exp(-i\phi)$$

In particular, for C-metric we choose the reference frame with $\mathbf{t}_{\mathrm{o}}$ normal to $\mathcal{I}$ and $\mathbf{q}_{\mathrm{o}}$ along Killing vector $\boldsymbol{\partial}_\tau$. A choice of the vectors $\mathbf{r}_{\mathrm{o}}$ and $\mathbf{s}_{\mathrm{o}}$ is irrelevant for calculation of the *magnitude* $\left|\Psi_4^{\mathrm{i}}\right|$ of the radiative component.

The leading terms of gravitational field as a function of a direction along which the point at infinity is approached — the *directional structure of radiation.*

Directions from the origin of the diagram correspond to spatial projections of null directions at infinity. The magnitude of the fields measured along a null geodesic with a tangent vector $\mathbf{k}$ is drawn in the spatial direction $-\mathbf{k}^\perp$ *from* which the geodesic arrives. The angles θ, ϕ parametrize the null direction with respect to the reference tetrad $\mathbf{q}_{\mathrm{o}}, \mathbf{r}_{\mathrm{o}}, \mathbf{s}_{\mathrm{o}}$. Principal null directions $\mathbf{k}_1$ and $\mathbf{k}_2$, are tangent to geodesics coming from the 'left' and 'right' black holes — they are pointing 'from the sources'. The radiative term of the field vanishes completely along directions spatially 'antipodal' to the principal null directions.



Components of gravitational field

Gravitational field is characterized by the Weyl tensor $\mathbf{C}_{abcd}$ and with respect to a null tetrad $\mathbf{k}$, $\mathbf{l}$, $\mathbf{m}$, $\bar{\mathbf{m}}$ can be parametrized by five complex coefficients

$$\Psi_0 = \mathbf{C}_{abcd}\,\mathbf{k}^a\,\mathbf{m}^b\,\mathbf{k}^c\,\mathbf{m}^d \qquad \Psi_3 = \mathbf{C}_{abcd}\,\mathbf{l}^a\,\mathbf{k}^b\,\mathbf{l}^c\,\bar{\mathbf{m}}^d$$
$$\Psi_1 = \mathbf{C}_{abcd}\,\mathbf{k}^a\,\mathbf{l}^b\,\mathbf{k}^c\,\mathbf{m}^d \qquad \Psi_2 = \mathbf{C}_{abcd}\,\mathbf{k}^a\,\mathbf{m}^b\,\bar{\mathbf{m}}^c\,\mathbf{l}^d$$
$$\Psi_4 = \mathbf{C}_{abcd}\,\mathbf{l}^a\,\bar{\mathbf{m}}^b\,\mathbf{l}^c\,\bar{\mathbf{m}}^d$$

Gravitational field of C-metric

Weyl tensor of C-metric has two double degenerate principal null directions $\mathbf{k}_1$, $\mathbf{k}_2$. With respect of the null tetrad aligned along $\mathbf{k}_1$ and $\mathbf{k}_2$ only nonvanishing component of the Weyl tensor is thus Ψ_2^{s}:

$$\Psi_2^{\mathrm{s}} = -\left(m - 2e^2A\xi - \ell^{-1}e^2\omega\right)\ell^{-3}\omega^3$$
$$\Psi_0^{\mathrm{s}} = \Psi_1^{\mathrm{s}} = \Psi_3^{\mathrm{s}} = \Psi_4^{\mathrm{s}} = 0$$

At infinity, the principal null directions are parameterized with respect of the reference tetrad by angles $\phi = 0$ and θ_{s} or $\pi - \theta_{\mathrm{s}}$, where $\tan\theta_{\mathrm{s}} = \ell A\sqrt{\mathcal{G}_\infty}$. The field components with respect to the reference tetrad are

$$\Psi_2^{\mathrm{o}} = \tfrac{1}{2}\,\Psi_2^{\mathrm{s}}\left(3\cos^{-2}\theta_{\mathrm{s}} - 1\right)$$
$$\Psi_1^{\mathrm{o}} = \Psi_3^{\mathrm{o}} = -\tfrac{3}{2}\,\Psi_2^{\mathrm{s}}\sin\theta_{\mathrm{s}}\cos^{-2}\theta_{\mathrm{s}}$$
$$\Psi_0^{\mathrm{o}} = \Psi_4^{\mathrm{o}} = \tfrac{3}{2}\,\Psi_2^{\mathrm{s}}\tan^2\theta_{\mathrm{s}}$$