

[Příkaz **restart** zapomene všechno, co jsme provedli od startu programu Maple

[> **restart**;

[>

Seznam

je uspořádaný a jeho prvky se mohou opakovat

[> **[1,2,2,1]**;

[1, 2, 2, 1]

[Počet prvků seznamu nám vrátí funkce **nops**

[> **nops (%)**;

nops ([]);

4

0

[Právě na seznamech spočívá schopnost programů pro symbolické manipulace reprezentovat !

[> **Y:=whattype(a+b*c+d)**;

X:=convert(a+b*c+d,list);

nops(X);

Y(op(X));

$Y := +$

$X := [a, b c, d]$

3

$a + b c + d$

[Protože víme že jde o součet, poslední řádek jsme také mohli zapsat jako

[> **`+`(op(X))**;

$a + b c + d$

[Vnořené seznamy jsou také možné

[> **[1, [2, [3, [4, 5]]]]**;

[1, [2, [3, [4, 5]]]]

[Tak lze nahradit chybějící typ struktura/záznam

[> **["Markéta Lazarová", 162, 1967, ["Vláčil, František", 1924, 1999]]**;

["Marketa Lazarova", 98, 1966, ["Vlacid, Frantisek", 1929, 1999]]

Množina

[> **Y:={1,2,2,1}**;

$Y := \{1, 2\}$

[Opakující se prvky se vyřadí.

[> **{1,2,3} union {2,3,4}**;

{1,2,3} intersect {2,3,4};

{1,2,3} minus {2,3,4};

$\{1, 2, 3, 4\}$

$\{2, 3\}$

$\{1\}$

Indexace []

```

> x[2];
                                p q
> y[2];
                                2
                                op
> op(x);
                                a, b c, d
> op(y);
                                1, 2
> convert(x,set) = {op(x)};
convert(y,list) = [op(y)];
                                {a, d, b c} = {a, d, b c}
                                [1, 2] = [1, 2]

```

Práce se seznamy

```

> x;
                                [a, b c, d]
> x[2]:=p*q;
                                X2 := p q
> x;
                                [a, p q, d]
> x[1..2]; # ubrání posledního
                                [a, p q]
> [op(x),m*n]; # přidání na konec
                                [a, p q, d, m n]
> [-1, op(x)]; # přidání na začátek list
                                [-1, a, p q, d]
> subsop(4=0,8=0,[seq(i,i=0..10)]);
                                [0, 1, 2, 0, 4, 5, 6, 0, 8, 9, 10]
>
>

```

Funkce " -> "

Uvidíme časem, že jde o užitečnou zkratku

(Pokud má funkce provádět sekvenci příkazů, musíme ji deklarovat jinak, viz příště)

```

> F:=x->x^2;
                                F := x → x2
> F(1);
F(t);
                                1
                                t2

```

Můžeme deklarovat funkci více proměnných, pokud jejich seznam dáme do závorek, navíc máme k dispozici **if** s významem podobným ? v C.

[Složitější funkce budeme již explicitně psát jako `G:=proc (....) end;`

> `G:=(x,y) -> x*y;`

$$G := (x, y) \rightarrow xy$$

> `G:=(x,y) -> if (x>0) then y else -y fi;`

`G := proc(x, y) option operator, arrow; if 0 < x then y else -y fi end`

> `G(0,b);`

$$-b$$

>

map

Zobrazení seznamu či množiny

> `map(u->1/(u^2+1), [a,b,c]);`

$$\left[\frac{1}{a^2+1}, \frac{1}{b^2+1}, \frac{1}{c^2+1} \right]$$

[Následuje ukázka použití výše uvedených operací

> `X:=1/2*ln((-2*a+sqrt(rho^2+(z+a)^2)+sqrt(rho^2+(z-a)^2))/(2*a+sqrt(rho^2+(z+a)^2)+sqrt(rho^2+(z-a)^2)));`

$$X := \frac{1}{2} \ln \left(\frac{-2a + \sqrt{\rho^2 + z^2 + 2za + a^2} + \sqrt{\rho^2 + z^2 - 2za + a^2}}{2a + \sqrt{\rho^2 + z^2 + 2za + a^2} + \sqrt{\rho^2 + z^2 - 2za + a^2}} \right)$$

> `series(X,a):`

`Y:=(convert(%,polynom)):`

`whattype(Y);`

`Y:=convert(Y,list):`

`map(factor,Y);`

$$\left[-\frac{a}{\sqrt{\rho^2 + z^2}}, \frac{1}{6} \frac{(\rho^2 - 2z^2)a^3}{(\rho^2 + z^2)^{(5/2)}, 0, -\frac{1}{40} \frac{(3\rho^4 - 24\rho^2 z^2 + 8z^4)a^5}{(\rho^2 + z^2)^{(9/2)}} \right]$$

>

Tabulky

[Pokud nějaký symbol není ani pole ani množina, vznikne přiřazením do jeho složky **tabulka**, kde se pamatuje, které hodnoty jsou obsazené a jakou hodnotou

> `T[-1]:=Nic;`

$$T_{-1} := Nic$$

> `T;`

$$T$$

> `T[0];`

$$T_0$$

> `T[-1];`

$$Nic$$

[Index nemusí být vůbec číslo

> `T[Praha]:=27.3;`

$T_{Praha} := 27.3$

Co je **T** zač zjistíme výpisem **op(T)**

> **op(T) ;**

```
table([
  Praha = 27.3
  -1 = Nic
])
```

>

>

Rovnice a nerovnice rhs,lhs,evalb

> **a=b;**

$a = b$

> **a>b;**

$b < a$

> **evalb(%);**

$b - a < 0$

> **subs(a=3,b=4,%);**

$1 < 0$

> **evalb(%);**

false

> **rhs(a>b);**

a

> **lhs(a>b);**

b

>

seq

> **seq(i,i=1..10);**

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

> **seq(Int(sin(x)^i,x) = combine(int(sin(x)^i,x),i=1..4);**

$\int \sin(x) dx = -\cos(x), \int \sin(x)^2 dx = -\frac{1}{4} \sin(2x) + \frac{1}{2}x,$

$\int \sin(x)^3 dx = -\frac{3}{4} \cos(x) + \frac{1}{12} \cos(3x), \int \sin(x)^4 dx = \frac{1}{32} \sin(4x) - \frac{1}{4} \sin(2x) + \frac{3}{8}x$

> **rhs(%[3]);**

$-\frac{3}{4} \cos(x) + \frac{1}{12} \cos(3x)$

>

\$

> **[1\$10];**

[1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

>

```
> simplify(exp(x^2)*diff(exp(-x^2),x $ 10));  
-30240 + 302400 x^2 - 403200 x^4 + 161280 x^6 - 23040 x^8 + 1024 x^10
```

```
> i$i=1..3;  
1, 2, 3
```

```
> $4..8,$104..108;  
4, 5, 6, 7, 8, 104, 105, 106, 107, 108
```

Pole

představují to, co jsme si zvykli nazývat polem třeba v Pascalu

```
> array(1..4);  
[?, ?, ?, ?]
```

```
> X:=array(1..4); # ejhle rozdíl!  
X := array(1 .. 4, [ ])
```

```
> X[1]:=1;  
X[2]:=2;  
X[3]:=4;  
X[4]:=8;  
  
X1 := 1  
X2 := 2  
X3 := 4  
X4 := 8
```

Mohli jsme si ale ušetřit trochu psaní a hodnoty pole zadat v rámci deklarace

```
> W:=array(1..4, [1,2,4,8]);  
W := [1, 2, 4, 8]
```

Následující dva řádky ukáží,

```
> X;  
W;  
  
X  
W
```

```
> eval(X);  
eval(W);  
  
[1, 2, 4, 8]  
[1, 2, 4, 8]
```

```
> X[5]:=1;  
Error, 1st index, 5, larger than upper array bound 4
```

```
> Y[5]:=1;  
Y5 := 1
```

```
> Y;  
Y
```

```
> eval(Y);
```

```
table([  
5 = 1
```

D)

>

Matice

```
> A:=array(1..3,1..3,[[0,0,1],[0,1,0],[1,0,0]]);  
> X:=array(1..3,[1,2,3]);
```

$$A := \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$X := [1, 2, 3]$$

```
> A&*X;
```

$$A \&* X$$

```
> evalm(A&*X);
```

$$[3, 2, 1]$$

```
> det(A);
```

$$\det(A)$$

Jaktože neumí determinant?

```
> with(linalg);
```

```
> det(A);
```

$$-1$$

```
> A:=matrix(8,8,(a,b)->1/(a+b-1));
```

$$A := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} \\ \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} \end{bmatrix}$$

```
> det(A);
```

$$1$$

$$\frac{1}{365356847125734485878112256000000}$$

Cykly

Když máme pole, potřebujeme cykly. Syntaxe je ale bohužel katastrofální:

```
> for i from 1 to 4 do
  X[i]:=2^(i-1);
od;
```

$X_1 := 1$
 $X_2 := 2$
 $X_3 := 4$
 $X_4 := 8$

```
> for i in [1,2,3,4] do
  print( i );
od;
```

1
2
3
4

```
> for i in [$4..8,$104..108] do
  print( i );
od;
```

4
5
6
7
8
104
105
106
107
108

[Středníky a dvojtečky v cyklech:

```
> for i from 1 to 4 do
  X[i]:=2^(i-1);
od;
```

```
> for i from 1 to 4 do
  X[i]:=2^(i-1):
od;
```

$X_1 := 1$
 $X_2 := 2$
 $X_3 := 4$
 $X_4 := 8$

```
> for i from 1 while i^2<100 do
  print(i);
od;
```

1
2
3
4
5
6
7
8
9

```
> while i<200 do  
  i := i*2;  
od;
```

i:= 20
i:= 40
i:= 80
i:= 160
i:= 320

>

Spojování jmen

```
> X.1;
```

XI

```
> cat(X,1);
```

XI

```
> F:=4;
```

F:= 4

```
> F.1;
```

FI

```
> cat(F,1);
```

4I

```
> sum(a.i*x^i,i=0..4);
```

Error, (in sum) summation variable previously assigned,
argument evaluates to, 5 = 0 .. 4

second

Chyba!! proměnná *i* má přiřazenou hodnotu, takže ...

```
> sum(a.i*x^i,'i'=0..4);
```

$5 a_5 x^5$

To taky není co bychom chtěli

```
> sum('a.i*x^i','i'=0..4);
```

$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$

Tento postup je důležitý když chceme zkonstruovat polynom vyššího řádu

```
> N:=8;
```

N:= 8

```
> F:=sum('a.i*x^i','i'=0..N);
```


$$F := a0 + a1 x + a2 x^2 + a3 x^3 + a4 x^4 + a5 x^5 + a6 x^6 + a7 x^7 + a8 x^8$$

> **ERR:=int((F-sin(x))^2,x=0..2*Pi);**

$$\begin{aligned} ERR := & -20160 a7 \pi + \frac{8192}{13} a6^2 \pi^{13} + 13440 a7 \pi^3 - 2688 a7 \pi^5 + \frac{1024}{5} a1 a8 \pi^{10} \\ & + 64 a1 a6 \pi^8 + \frac{64}{3} a0 a5 \pi^6 + \frac{16384}{13} a5 a7 \pi^{13} + \frac{4096}{11} a2 a8 \pi^{11} + 64 a2 a5 \pi^8 \\ & + \frac{32768}{15} a7^2 \pi^{15} + \frac{2048}{11} a5^2 \pi^{11} + \frac{131072}{17} a8^2 \pi^{17} + 4 a1 \pi + 8 a1 a2 \pi^4 + \frac{64}{5} a0 a4 \pi^5 \\ & + 8 a0 a3 \pi^4 + \frac{128}{7} a3^2 \pi^7 + \frac{512}{9} a4^2 \pi^9 + \frac{8}{3} a1^2 \pi^3 + \frac{32}{5} a2^2 \pi^5 + 2 a0^2 \pi + 16 a3 \pi^3 - 24 a3 \pi \\ & + 8 a2 \pi^2 + 32 a4 \pi^4 - 96 a4 \pi^2 + \frac{16}{3} a0 a2 \pi^3 + 4 a0 a1 \pi^2 + 64 a3 a4 \pi^8 + \frac{256}{7} a2 a4 \pi^7 \\ & + \frac{64}{3} a2 a3 \pi^6 + \frac{64}{3} a1 a4 \pi^6 + \frac{64}{5} a1 a3 \pi^5 + \frac{16384}{7} a6 a7 \pi^{14} + \frac{1024}{5} a2 a7 \pi^{10} + 64 a0 a7 \pi^8 \\ & + \frac{1024}{9} a3 a5 \pi^9 + \frac{1024}{5} a3 a6 \pi^{10} + \frac{65536}{15} a6 a8 \pi^{15} + \frac{1024}{9} a2 a6 \pi^9 + \frac{1024}{9} a1 a7 \pi^9 \\ & + \frac{256}{7} a1 a5 \pi^7 + \frac{2048}{3} a4 a7 \pi^{12} + \frac{4096}{11} a4 a6 \pi^{11} + \frac{16384}{13} a4 a8 \pi^{13} + \frac{1024}{9} a0 a8 \pi^9 \\ & + \frac{256}{7} a0 a6 \pi^7 + \frac{2048}{3} a5 a6 \pi^{12} + \frac{4096}{11} a3 a7 \pi^{11} + \frac{1024}{5} a4 a5 \pi^{10} + \frac{2048}{3} a3 a8 \pi^{12} \\ & + \frac{16384}{7} a5 a8 \pi^{14} + 8192 a7 a8 \pi^{16} - 960 a6 \pi^4 + 512 a8 \pi^8 - 161280 a8 \pi^2 + 53760 a8 \pi^4 \\ & - 7168 a8 \pi^6 + 64 a5 \pi^5 - 320 a5 \pi^3 + 480 a5 \pi + 256 a7 \pi^7 + \pi + 128 a6 \pi^6 + 2880 a6 \pi^2 \end{aligned}$$

funkce seq mi neschvali ani 'i', take musim

> **i:='i';**

i := i

> **Nezn:={seq(a.i,i=0..N)};**

Nezn := {a6, a8, a5, a7, a0, a4, a1, a2, a3}

> **Rov:={seq(diff(ERR,a.i),i=0..N)};**

>

$$\begin{aligned} Rov := & \left\{ \frac{16384}{13} a6 \pi^{13} + 64 a1 \pi^8 + \frac{16384}{7} a7 \pi^{14} + \frac{1024}{5} a3 \pi^{10} + \frac{65536}{15} a8 \pi^{15} + \frac{1024}{9} a2 \pi^9 \right. \\ & + \frac{4096}{11} a4 \pi^{11} + \frac{256}{7} a0 \pi^7 + \frac{2048}{3} a5 \pi^{12} - 960 \pi^4 + 128 \pi^6 + 2880 \pi^2, -20160 \pi + 13440 \pi^3 \\ & - 2688 \pi^5 + \frac{16384}{13} a5 \pi^{13} + \frac{65536}{15} a7 \pi^{15} + \frac{16384}{7} a6 \pi^{14} + \frac{1024}{5} a2 \pi^{10} + 64 a0 \pi^8 \\ & + \frac{1024}{9} a1 \pi^9 + \frac{2048}{3} a4 \pi^{12} + \frac{4096}{11} a3 \pi^{11} + 8192 a8 \pi^{16} + 256 \pi^7, \frac{1024}{5} a1 \pi^{10} + \frac{4096}{11} a2 \pi^{11} \\ & \left. + \frac{262144}{17} a8 \pi^{17} + \frac{65536}{15} a6 \pi^{15} + \frac{16384}{13} a4 \pi^{13} + \frac{1024}{9} a0 \pi^9 + \frac{2048}{3} a3 \pi^{12} + \frac{16384}{7} a5 \pi^{14} \right\} \end{aligned}$$

$$\begin{aligned}
& + 8192 a7 \pi^{16} + 512 \pi^8 - 161280 \pi^2 + 53760 \pi^4 - 7168 \pi^6, \frac{64}{3} a5 \pi^6 + \frac{64}{5} a4 \pi^5 + 8 a3 \pi^4 \\
& + 4 a0 \pi + \frac{16}{3} a2 \pi^3 + 4 a1 \pi^2 + 64 a7 \pi^8 + \frac{1024}{9} a8 \pi^9 + \frac{256}{7} a6 \pi^7, \frac{1024}{5} a8 \pi^{10} + 64 a6 \pi^8 \\
& + 4 \pi + 8 a2 \pi^4 + \frac{16}{3} a1 \pi^3 + 4 a0 \pi^2 + \frac{64}{3} a4 \pi^6 + \frac{64}{5} a3 \pi^5 + \frac{1024}{9} a7 \pi^9 + \frac{256}{7} a5 \pi^7, \\
& \frac{4096}{11} a8 \pi^{11} + 64 a5 \pi^8 + 8 a1 \pi^4 + \frac{64}{5} a2 \pi^5 + 8 \pi^2 + \frac{16}{3} a0 \pi^3 + \frac{256}{7} a4 \pi^7 + \frac{64}{3} a3 \pi^6 \\
& + \frac{1024}{5} a7 \pi^{10} + \frac{1024}{9} a6 \pi^9, 8 a0 \pi^4 + \frac{256}{7} a3 \pi^7 + 16 \pi^3 - 24 \pi + 64 a4 \pi^8 + \frac{64}{3} a2 \pi^6 \\
& + \frac{64}{5} a1 \pi^5 + \frac{1024}{9} a5 \pi^9 + \frac{1024}{5} a6 \pi^{10} + \frac{4096}{11} a7 \pi^{11} + \frac{2048}{3} a8 \pi^{12}, \frac{64}{5} a0 \pi^5 + \frac{1024}{9} a4 \pi^9 \\
& + 32 \pi^4 - 96 \pi^2 + 64 a3 \pi^8 + \frac{256}{7} a2 \pi^7 + \frac{64}{3} a1 \pi^6 + \frac{2048}{3} a7 \pi^{12} + \frac{4096}{11} a6 \pi^{11} + \frac{16384}{13} a8 \pi^{13} \\
& + \frac{1024}{5} a5 \pi^{10}, \frac{64}{3} a0 \pi^6 + \frac{16384}{13} a7 \pi^{13} + 64 a2 \pi^8 + \frac{4096}{11} a5 \pi^{11} + \frac{1024}{9} a3 \pi^9 + \frac{256}{7} a1 \pi^7 \\
& + \frac{2048}{3} a6 \pi^{12} + \frac{1024}{5} a4 \pi^{10} + \frac{16384}{7} a8 \pi^{14} + 64 \pi^5 - 320 \pi^3 + 480 \pi \}
\end{aligned}$$

>

> **Res := solve (Rov, Nezn) ;**

$$Res := \{ a5 = -\frac{63063}{8} \frac{17145 \pi^2 + \pi^6 - 375 \pi^4 - 133650}{\pi^{12}},$$

$$a6 = \frac{45045}{16} \frac{\pi^6 - 378 \pi^4 + 17325 \pi^2 - 135135}{\pi^{13}}, a7 = -\frac{6435}{16} \frac{\pi^6 - 378 \pi^4 + 17325 \pi^2 - 135135}{\pi^{14}},$$

$$a8 = 0, a4 = \frac{45045}{8} \frac{2 \pi^6 - 735 \pi^4 + 33390 \pi^2 - 259875}{\pi^{11}},$$

$$a2 = \frac{3465}{2} \frac{2 \pi^6 - 654 \pi^4 + 28665 \pi^2 - 221130}{\pi^9},$$

$$a3 = -\frac{17325}{2} \frac{-353 \pi^4 + \pi^6 + 15834 \pi^2 - 122850}{\pi^{10}},$$

$$a1 = -315 \frac{2 \pi^6 - 561 \pi^4 + 23595 \pi^2 - 180180}{\pi^8}, a0 = 9 \frac{30030 \pi^2 + 4 \pi^6 - 770 \pi^4 - 225225}{\pi^7} \}$$

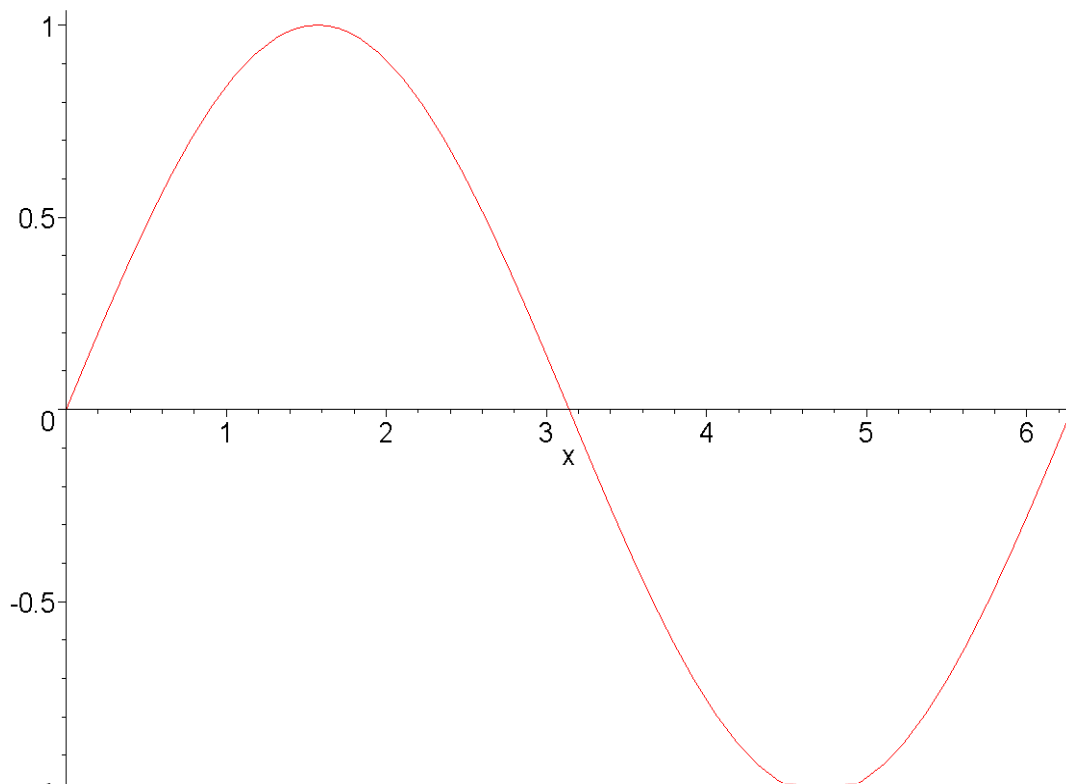
> **F.N := subs (Res, F) ;**

$$F8 := 9 \frac{30030 \pi^2 + 4 \pi^6 - 770 \pi^4 - 225225}{\pi^7} - 315 \frac{(2 \pi^6 - 561 \pi^4 + 23595 \pi^2 - 180180) x}{\pi^8}$$

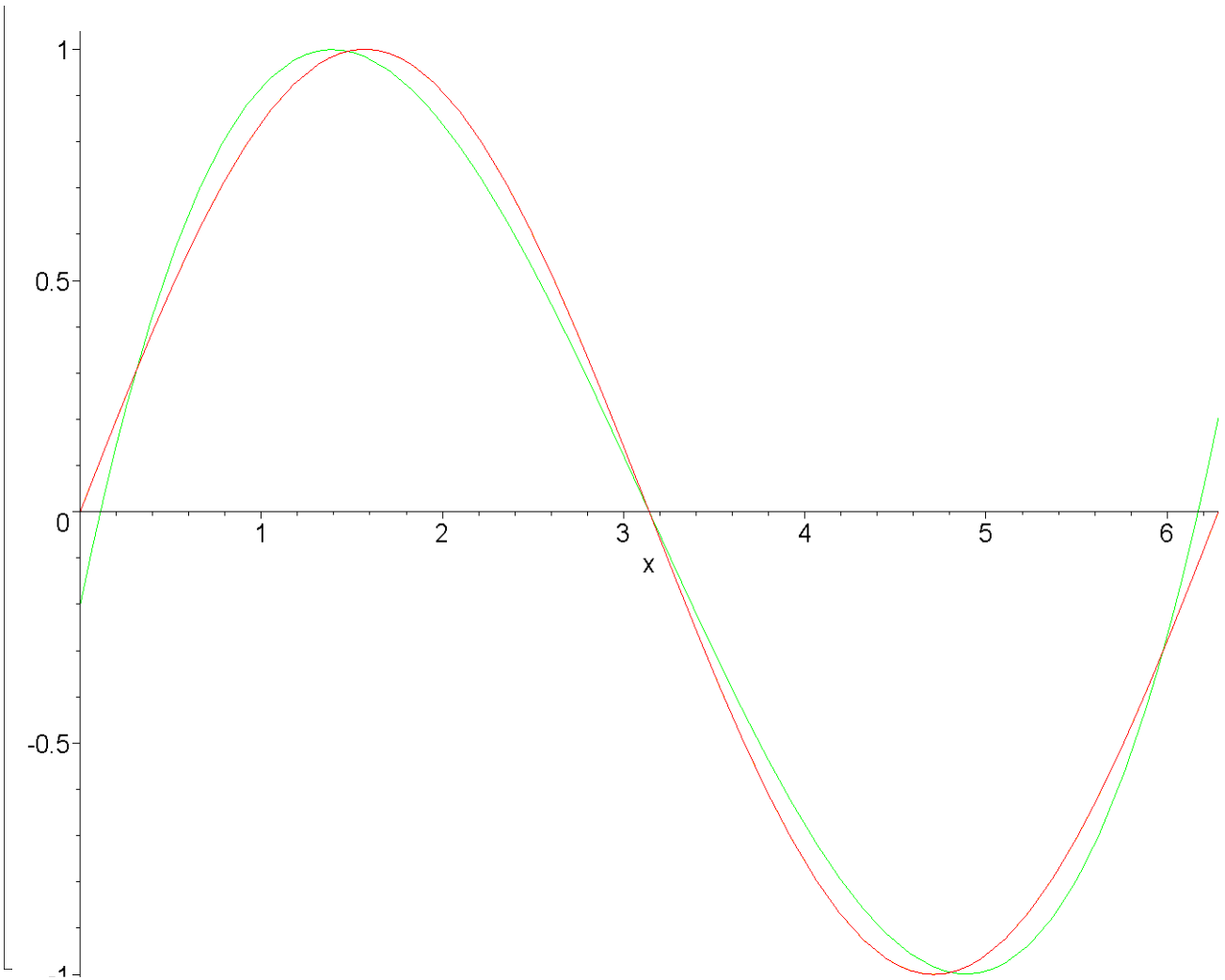
$$+ \frac{3465}{2} \frac{(2 \pi^6 - 654 \pi^4 + 28665 \pi^2 - 221130) x^2}{\pi^9}$$

$$\begin{aligned}
& - \frac{17325 (-353 \pi^4 + \pi^6 + 15834 \pi^2 - 122850) x^3}{2 \pi^{10}} \\
& + \frac{45045 (2 \pi^6 - 735 \pi^4 + 33390 \pi^2 - 259875) x^4}{8 \pi^{11}} \\
& - \frac{63063 (17145 \pi^2 + \pi^6 - 375 \pi^4 - 133650) x^5}{8 \pi^{12}} \\
& + \frac{45045 (\pi^6 - 378 \pi^4 + 17325 \pi^2 - 135135) x^6}{16 \pi^{13}} \\
& - \frac{6435 (\pi^6 - 378 \pi^4 + 17325 \pi^2 - 135135) x^7}{16 \pi^{14}}
\end{aligned}$$

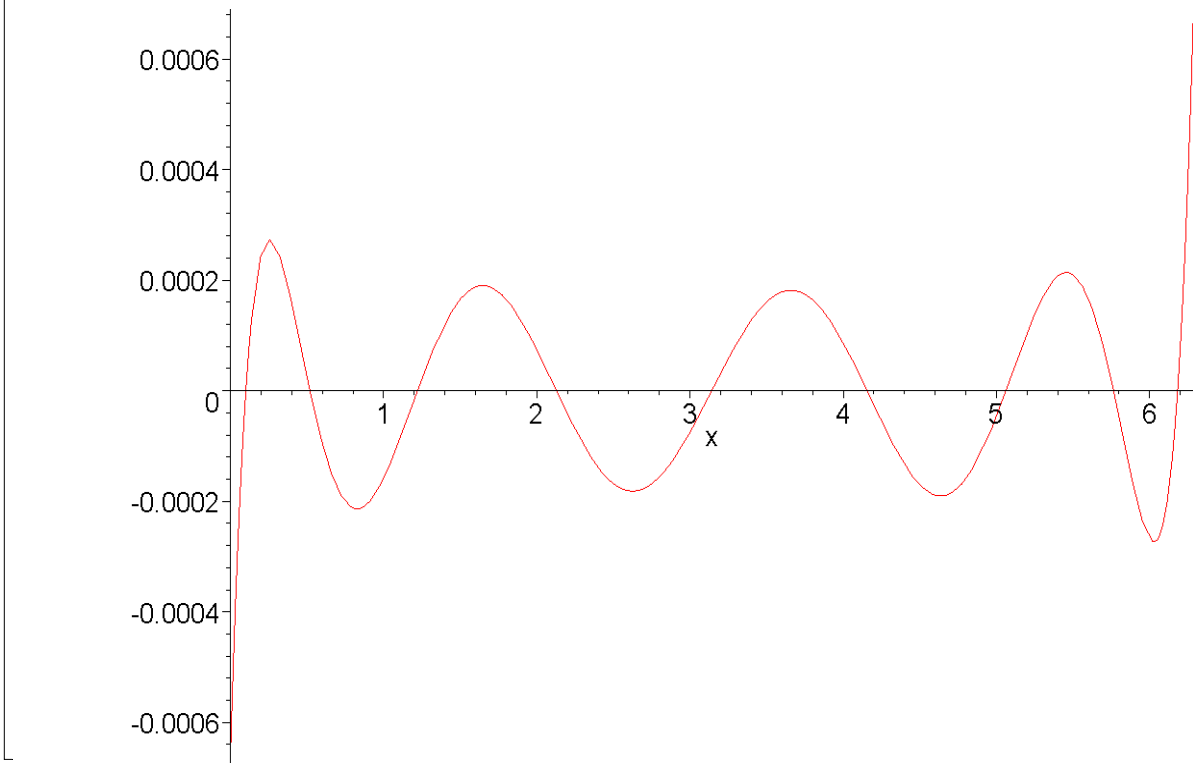
```
> plot(F.N,x=0..2*Pi);
```



```
> plot([F8,F4],x=0..2*Pi);
```



```
> plot(F8-sin(x), x=0..2*Pi);
```



[>