

# Maple @ UTF (2/2004)

## simplify( ... )

> F := (x^7 - y^7) / (x - y);

$$F := \frac{x^7 - y^7}{x - y}$$

> simplify(F);

$$x^6 + yx^5 + y^2x^4 + y^3x^3 + y^4x^2 + y^5x + y^6$$

## solve(f(x)=0,x); RootOf(...); allvalues(...);

> solve(exp(x)/x=y, x);

$$-\text{LambertW}\left(-\frac{1}{y}\right)$$

> evalf(eval(%), y=4);

.3574029562

> solve({z=s\*x\*y, rho=s\*sqrt(1-y^2)\*sqrt(x^2-1)}, {x, y});  
allvalues(%)[3];

$$\left. \begin{aligned} & \{y = \text{RootOf}(s^2 Z^4 + (-s^2 - z^2 - \rho^2) Z^2 + z^2), x = \frac{z}{s \text{RootOf}(s^2 Z^4 + (-s^2 - z^2 - \rho^2) Z^2 + z^2)}\} \\ & \left\{ y = \frac{1}{2} \frac{\sqrt{2} \sqrt{s^2 + z^2 + \rho^2} - \sqrt{s^4 - 2s^2 z^2 + 2s^2 \rho^2 + z^4 + 2\rho^2 z^2 + \rho^4}}{s}, \right. \\ & \left. x = \frac{z\sqrt{2}}{\sqrt{s^2 + z^2 + \rho^2} - \sqrt{s^4 - 2s^2 z^2 + 2s^2 \rho^2 + z^4 + 2\rho^2 z^2 + \rho^4}} \right\} \end{aligned}$$

## diff(f(x),x)

> F := 1/(x+1/(y+1/(x+y)));  
diff(F, y, x) - diff(F, x, y);  
simplify(%);

$$F := \frac{1}{x + \frac{1}{y + \frac{1}{x+y}}}$$

$$\begin{aligned} & 2 \frac{1 - \frac{1}{(x+y)^2}}{\left(x + \frac{1}{y + \frac{1}{x+y}}\right)^2 \left(y + \frac{1}{x+y}\right)^3 (x+y)^2} + 2 \frac{1}{\left(x + \frac{1}{y + \frac{1}{x+y}}\right)^2 \left(y + \frac{1}{x+y}\right)^2 (x+y)^3} \\ & - 2 \frac{1 - \frac{1}{(x+y)^2}}{\left(y + \frac{1}{x+y}\right)^3 (x+y)^2} - 2 \frac{1}{\left(y + \frac{1}{x+y}\right)^2 (x+y)^3} \\ & + \frac{\left(x + \frac{1}{y + \frac{1}{x+y}}\right)^2}{\left(y + \frac{1}{x+y}\right)^2} \end{aligned}$$

0

## int(f(x),x)

> Int((rho^2-z^2)/sqrt(z^2+rho^2), z) = int((rho^2-z^2)/sqrt(z^2+rho^2), z);

$$\int \frac{\rho^2 - z^2}{\sqrt{z^2 + \rho^2}} dz = \frac{3}{2} \rho^2 \ln(z + \sqrt{z^2 + \rho^2}) - \frac{1}{2} z \sqrt{z^2 + \rho^2}$$

> Int(1/sqrt(a+b\*cos(x)+c\*sin(x)),x);

$$\int \frac{1}{\sqrt{a + b \cos(x) + c \sin(x)}} dx$$

> int(1/sqrt(a+d\*cos(x-x0)),x);

$$\frac{2 \sqrt{\cos\left(\frac{1}{2}x - \frac{1}{2}x_0\right)^2} \operatorname{EllipticF}\left(\sin\left(\frac{1}{2}x - \frac{1}{2}x_0\right), \sqrt{2} \sqrt{\frac{d}{a+d}}\right) \sqrt{\frac{a-d+2d\cos\left(\frac{1}{2}x - \frac{1}{2}x_0\right)^2}{a+d}}}{\cos\left(\frac{1}{2}x - \frac{1}{2}x_0\right) \sqrt{a-d+2d\cos\left(\frac{1}{2}x - \frac{1}{2}x_0\right)^2}}$$

=====

**dsolve(F(f'(x),f(x),x)=0, f(x) )**

> dsolve( {diff(f(x),x)=x\*f(x)\*(1-f(x)), f(0)=3}, f(x) );

$$f(x) = \frac{1}{1 - \frac{2}{3} e^{(-1/2)x^2}}$$

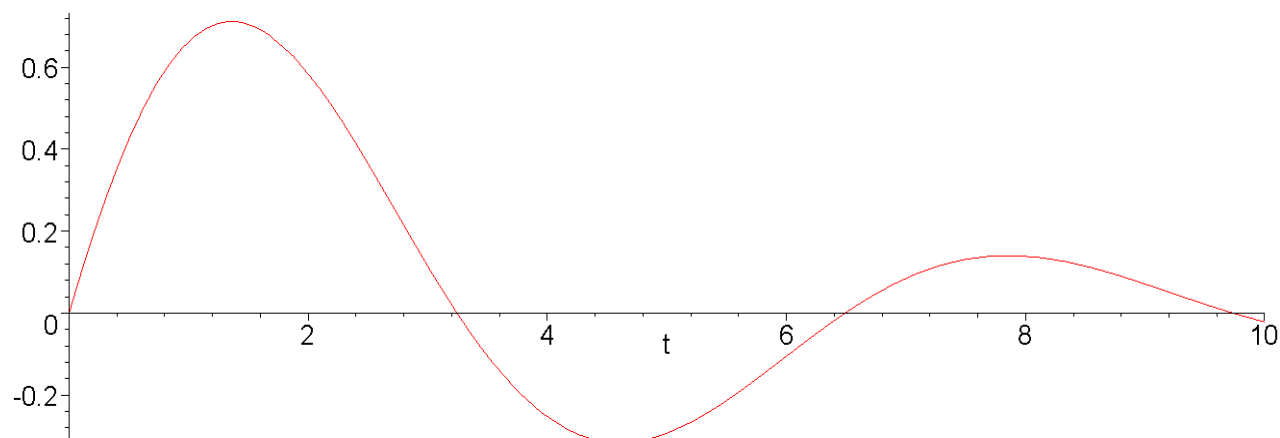
> rovnice:=diff(f(t),t,t)+b\*diff(f(t),t)+Omega^2\*f(t)=0;

> reseni:=dsolve({rovnice, f(0)=0, D(f)(0)=1}, f(t));

> plot( subs(A=1,b=0.5, Omega=1, omega=2, rhs(reseni)), t=0..10);

$$\text{rovnice} := \left( \frac{\partial^2}{\partial t^2} \hat{f}(t) \right) + b \left( \frac{\partial}{\partial t} \hat{f}(t) \right) + \Omega^2 \hat{f}(t) = 0$$

$$\text{reseni} := f(t) = \frac{e^{(-1/2(b-\sqrt{(b-2\Omega)(b+2\Omega)})t)}}{\sqrt{b^2-4\Omega^2}} - \frac{e^{(-1/2(b+\sqrt{(b-2\Omega)(b+2\Omega)})t)}}{\sqrt{b^2-4\Omega^2}}$$



> ddsolve:= ( F,f,x ) -> dsolve( subs( {f.2=diff(f(x),x,x), f.1=diff(f(x),x), f=f(x)}, F), f(x) );

> ddsolve(y1=x\*y\*(1-y),y,x);

ddsolve(f2+f=0,f,x);

$$y(x) = \frac{1}{1 + e^{(-1/2)x^2}} \_C1$$

$$f(x) = \_C1 \cos(x) + \_C2 \sin(x)$$

> Order:=10:

dsolve( {x^2\*diff(f(x),x,x)+x\*diff(f(x),x)+(x^2-1)\*f(x)=0, f(0)=0, D(f)(0)=1/2}, f(x), type=series );

$$f(x) = \frac{1}{2}x - \frac{1}{16}x^3 + \frac{1}{384}x^5 - \frac{1}{18432}x^7 + \frac{1}{1474560}x^9 + O(x^{10})$$

> series(BesselJ(1,x),x);

$$\frac{1}{2}x - \frac{1}{16}x^3 + \frac{1}{384}x^5 - \frac{1}{18432}x^7 + \frac{1}{1474560}x^9 + O(x^{10})$$

---

## value(f)

```
> F:=Diff(sin(x),x);
value(F);
```

$$F := \frac{\partial}{\partial x} \sin(x)$$

$$\cos(x)$$

---

## eval(f,x=a),limit(f,x=a)

```
> 'diff(sin(x),x)';
```

$$\frac{\partial}{\partial x} \sin(x)$$

```
> eval(%);
```

$$\cos(x)$$

```
> eval(cos(x),x=Pi);
```

$$-1$$

```
> eval(x^a+y^b,{a=2,b=3});
```

$$x^2 + y^3$$

```
> limit(sin(x)/x,x=0);
```

$$1$$

---

## Unevaluated expressions, 'expr'

```
> # 3D Laplace(x/r)
simplify(sum('diff(x1/sqrt(x1^2+x2^2+x3^2),x.i,x.i)',i=1..3));
```

$$-2 \frac{x^I}{(x^I + x^2 + x^3)^{(3/2)}}$$

A special case of unevaluation is used to unassign a name

```
> x:=2;
```

$$x := 2$$

```
> diff(sin(x),x);
```

```
Error, wrong number (or type) of parameters in function diff
```

```
> x:='x';
```

$$x := x$$

```
> diff(sin(x),x);
```

$$\cos(x)$$

---

## expand(f)

```
> expand((x+y)^9);
```

$$x^9 + 9xyx^8 + 36x^7y^2 + 84x^6y^3 + 126x^5y^4 + 126x^4y^5 + 84x^3y^6 + 36x^2y^7 + 9xy^8 + y^9$$

```
> expand(exp(ln(cos(x+y))+a));
```

$$e^a \cos(x) \cos(y) - e^a \sin(x) \sin(y)$$

```
> expand(BesselJ(4,t));
```

$$48 \frac{\text{BesselJ}(1,t)}{t^3} - 24 \frac{\text{BesselJ}(0,t)}{t^2} - 8 \frac{\text{BesselJ}(1,t)}{t} + \text{BesselJ}(0,t)$$

---

## normal(f); simplify(f); factor(f)

```
> F:=1/(x+1)+3/(x+2)+3/(x+3)+1/(x+4);
```

```
F = simplify(F);
```

$$\frac{1}{x+1} + 3\frac{1}{x+2} + 3\frac{1}{x+3} + \frac{1}{x+4} = 2\frac{4x^3 + 30x^2 + 68x + 45}{(x+1)(x+2)(x+3)(x+4)}$$

> factor(F);

$$2\frac{(2x+5)(2x^2+10x+9)}{(x+1)(x+2)(x+3)(x+4)}$$

> simplify(diff(1/(x^2-1), x\$5));

$$-240\frac{x(3x^4+10x^2+3)}{(x^2-1)^6}$$

> simplify(diff(1/(1-2\*m/r+Q^2/r^2), r\$3));  
factor(%);

$$-12\frac{-mQ^4+2Q^4r+m r^4-2Q^2r^3}{(r^2-2mr+Q^2)^4}$$

$$-12\frac{(r-Q)(r+Q)(m r^2-2Q^2r+m Q^2)}{(r^2-2mr+Q^2)^4}$$

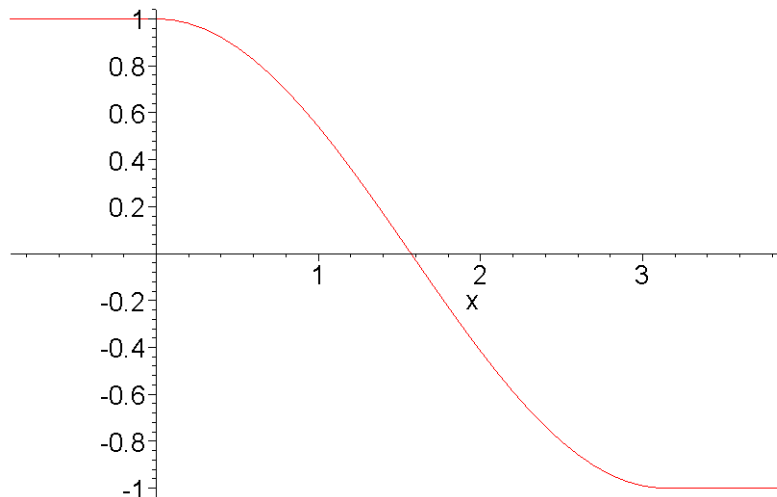
> normal(sin(x)^4-cos(x)^4);  
simplify(sin(x)^4-cos(x)^4);  
factor(sin(x)^4-cos(x)^4);

$$\frac{\sin(x)^4 - \cos(x)^4}{1 - 2\cos(x)^2}$$

$$-(\cos(x) - \sin(x))(\cos(x) + \sin(x))(\cos(x)^2 + \sin(x)^2)$$

piecewise(...)

> plot(piecewise(x<0,1,x<Pi,cos(x),-1),x=-0.9..3.9);



> diff(piecewise(x<0,1,x<Pi,cos(x),-1),x,x);

$$\begin{cases} 0 & x < 0 \\ \text{undefined} & x = 0 \\ -\cos(x) & x < \pi \\ \text{undefined} & x = \pi \\ 0 & \pi < x \end{cases}$$

series(f)

> series(Si(x),x,11);

$$x - \frac{1}{18}x^3 + \frac{1}{600}x^5 - \frac{1}{35280}x^7 + \frac{1}{3265920}x^9 + O(x^{11})$$

> int(series(sin(x)/x,x,11),x);

$$x - \frac{1}{18}x^3 + \frac{1}{600}x^5 - \frac{1}{35280}x^7 + \frac{1}{3265920}x^9 + O(x^{11})$$

> simplify(series(sin(x)/x,x,7)^2+series(cos(x)/x,x,7)^2);  
series(series(sin(x)/x,x,19)^2+series(cos(x)/x,x,19)^2,x,19);

$$\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{5040}x^6 + O(x^8)\right)^2 + \left(x^{-1} - \frac{1}{2}x + \frac{1}{24}x^3 - \frac{1}{720}x^5 + O(x^7)\right)^2$$

$$x^{-2} + O(x^{17})$$

> `lprint(series(sin(x)/x,x,9)^2);`

`(series(1-1/6*x^2+1/120*x^4-1/5040*x^6+1/362880*x^8+O(x^10),x,10))^2`

> `Order:=8;`

`KR:=E=epsilon*sin(E)+M;`

$$KR := E = \varepsilon \sin(E) + M$$

> `RKR:=solve(KR,E);`

$$RKR := \text{RootOf}(\_Z - \varepsilon \sin(\_Z) - M)$$

> `RRKR:=series(RKR,epsilon);`

$$RRKR := M + \sin(M) \varepsilon + \cos(M) \sin(M) \varepsilon^2 + \frac{1}{2} \sin(M) (2 \cos(M)^2 - \sin(M)^2) \varepsilon^3 +$$

$$\frac{1}{3} \cos(M) \sin(M) (3 \cos(M)^2 - 5 \sin(M)^2) \varepsilon^4 + \frac{1}{24} \sin(M) (24 \cos(M)^4 - 88 \cos(M)^2 \sin(M)^2 + 13 \sin(M)^4) \varepsilon^5 +$$

$$\frac{1}{15} \cos(M) \sin(M) (15 \cos(M)^4 - 100 \cos(M)^2 \sin(M)^2 + 47 \sin(M)^4) \varepsilon^6 +$$

$$\frac{1}{720} \sin(M) (-541 \sin(M)^6 + 720 \cos(M)^6 - 7800 \cos(M)^4 \sin(M)^2 + 7746 \sin(M)^4 \cos(M)^2) \varepsilon^7 + O(\varepsilon^8)$$

> `factor(simplify(RRKR));`

$$M + \sin(M) \varepsilon + \cos(M) \sin(M) \varepsilon^2 + \frac{1}{2} \sin(M) (3 \cos(M)^2 - 1) \varepsilon^3 + \frac{1}{3} \cos(M) \sin(M) (8 \cos(M)^2 - 5) \varepsilon^4 +$$

$$\frac{1}{24} \sin(M) (125 \cos(M)^4 - 114 \cos(M)^2 + 13) \varepsilon^5 + \frac{1}{15} \cos(M) \sin(M) (162 \cos(M)^4 - 194 \cos(M)^2 + 47) \varepsilon^6 +$$

$$\frac{1}{720} \sin(M) (16807 \cos(M)^6 - 24915 \cos(M)^4 + 9369 \cos(M)^2 - 541) \varepsilon^7 + O(\varepsilon^8)$$

> `RRR:=convert(combine(%),polynom);`

$$RRR := M + \sin(M) \varepsilon + \frac{1}{2} \sin(2M) \varepsilon^2 + \left(\frac{3}{8} \sin(3M) - \frac{1}{8} \sin(M)\right) \varepsilon^3 + \left(\frac{1}{3} \sin(4M) - \frac{1}{6} \sin(2M)\right) \varepsilon^4$$

$$+ \left(\frac{125}{384} \sin(5M) - \frac{27}{128} \sin(3M) + \frac{1}{192} \sin(M)\right) \varepsilon^5 + \left(\frac{27}{80} \sin(6M) + \frac{1}{48} \sin(2M) - \frac{4}{15} \sin(4M)\right) \varepsilon^6$$

$$+ \left(\frac{16807}{46080} \sin(7M) - \frac{3125}{9216} \sin(5M) + \frac{243}{5120} \sin(3M) - \frac{1}{9216} \sin(M)\right) \varepsilon^7$$

>

>

## =====

## Funkce

> `KepE2 := (M,epsilon)->M+sin(M)*epsilon+1/2*sin(2*M)*epsilon^2;`

$$KepE2 := (M, \varepsilon) \rightarrow M + \sin(M) \varepsilon + \frac{1}{2} \sin(2M) \varepsilon^2$$

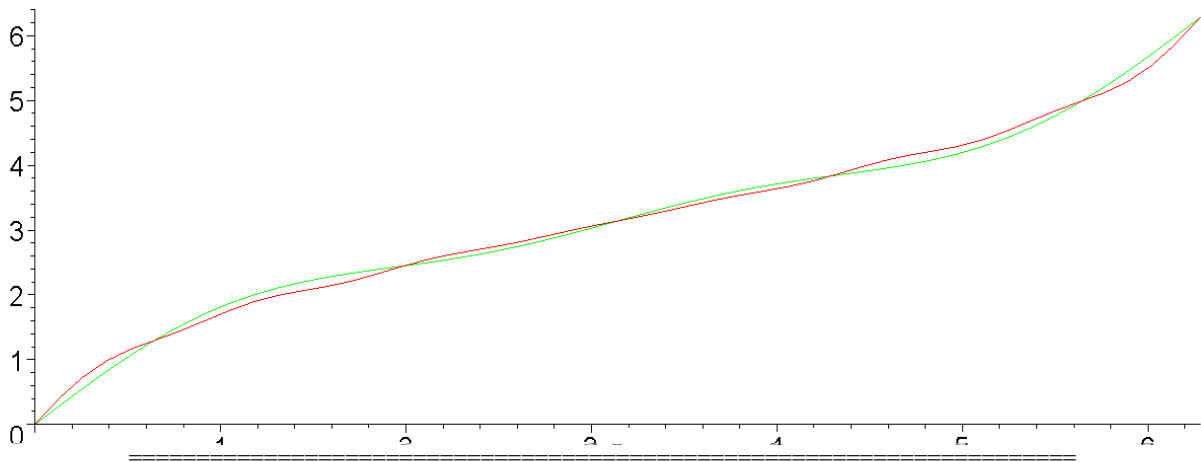
> `KepE7 := unapply(RRR,M,epsilon);`

$$KepE7 := (M, \varepsilon) \rightarrow M + \sin(M) \varepsilon + \frac{1}{2} \sin(2M) \varepsilon^2 + \left(\frac{3}{8} \sin(3M) - \frac{1}{8} \sin(M)\right) \varepsilon^3 + \left(\frac{1}{3} \sin(4M) - \frac{1}{6} \sin(2M)\right) \varepsilon^4$$

$$+ \left(\frac{125}{384} \sin(5M) - \frac{27}{128} \sin(3M) + \frac{1}{192} \sin(M)\right) \varepsilon^5 + \left(\frac{27}{80} \sin(6M) + \frac{1}{48} \sin(2M) - \frac{4}{15} \sin(4M)\right) \varepsilon^6$$

$$+ \left(\frac{16807}{46080} \sin(7M) - \frac{3125}{9216} \sin(5M) + \frac{243}{5120} \sin(3M) - \frac{1}{9216} \sin(M)\right) \varepsilon^7$$

> `plot({KepE2(x,0.7),KepE7(x,0.7)},x=0..2*Pi);`



## with(linalg)

```

> restart;
with(linalg);
Warning, new definition for norm
Warning, new definition for trace
[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle,
augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion,
concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues,
eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim,
gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod,
intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor,
minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector,
rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector,
sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian]
> matrix([ [1,0],[0,-1] ]) &* matrix([ [0,1],[1,0] ]) - matrix([ [0,1],[1,0]
])&*matrix([ [1,0],[0,-1] ]);
M1:=evalm(%);

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \&* \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \&* \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

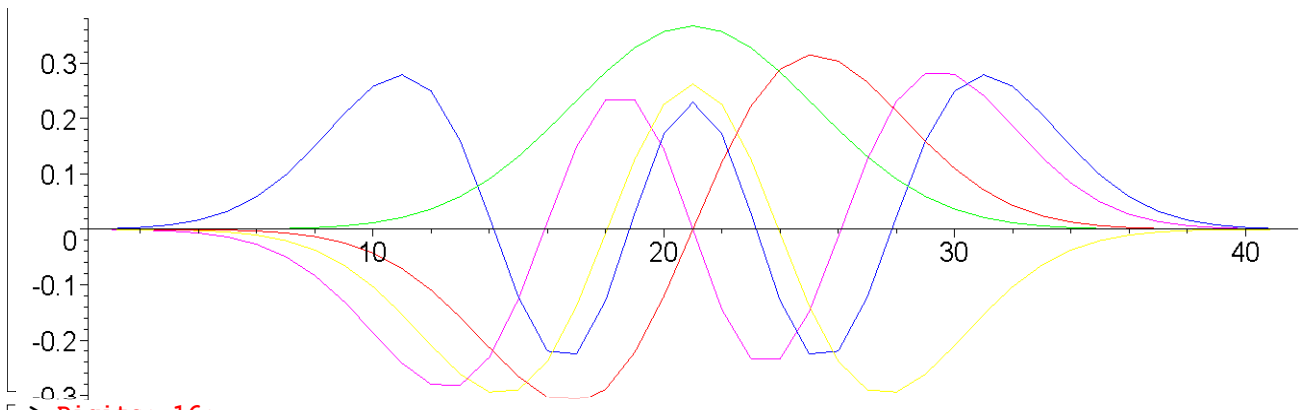

$$M1 := \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

> eigenvectors(M1);
[2 I, 1, {[1, I]}, [-2 I, 1, {[1, -I]}]
> evalm(exponential(M1));

$$\begin{bmatrix} \cos(2) & \sin(2) \\ -\sin(2) & \cos(2) \end{bmatrix}$$

>
> N:=41;A:=5.0;P:=(N+1)/2/A^2;
N:=41
A:=5.0
> V:=diag(seq(1/2/P*(i-(N+1)/2)^2,i=1..N));
> T:=band([-P/2,P,-P/2],N);
>
> evalm(V);
> H:=evalm(T+V);
> VH:=eigenvectors(H);
> eigensort:=(x,y)->evalb(x[1]<y[1]);
VH2:=sort([VH],eigensort);
eigensort := (x, y) -> evalb(x1 < y1)
> plot({seq([seq([i,op(VH2[j],3)][i]],i=1..N)],j=1..5)});

```



```
> Digits:=16;
plot({seq(interp([seq(A*2/(N+1)*(i-(N+1)/2),i=1..N)],op(VH2[j,3]),z),j=1..5)},z=-4..4)
;
```

