Astrophysics of gravitational wave sources

Lecture 8: Lidov-Kozai and cluster dynamics

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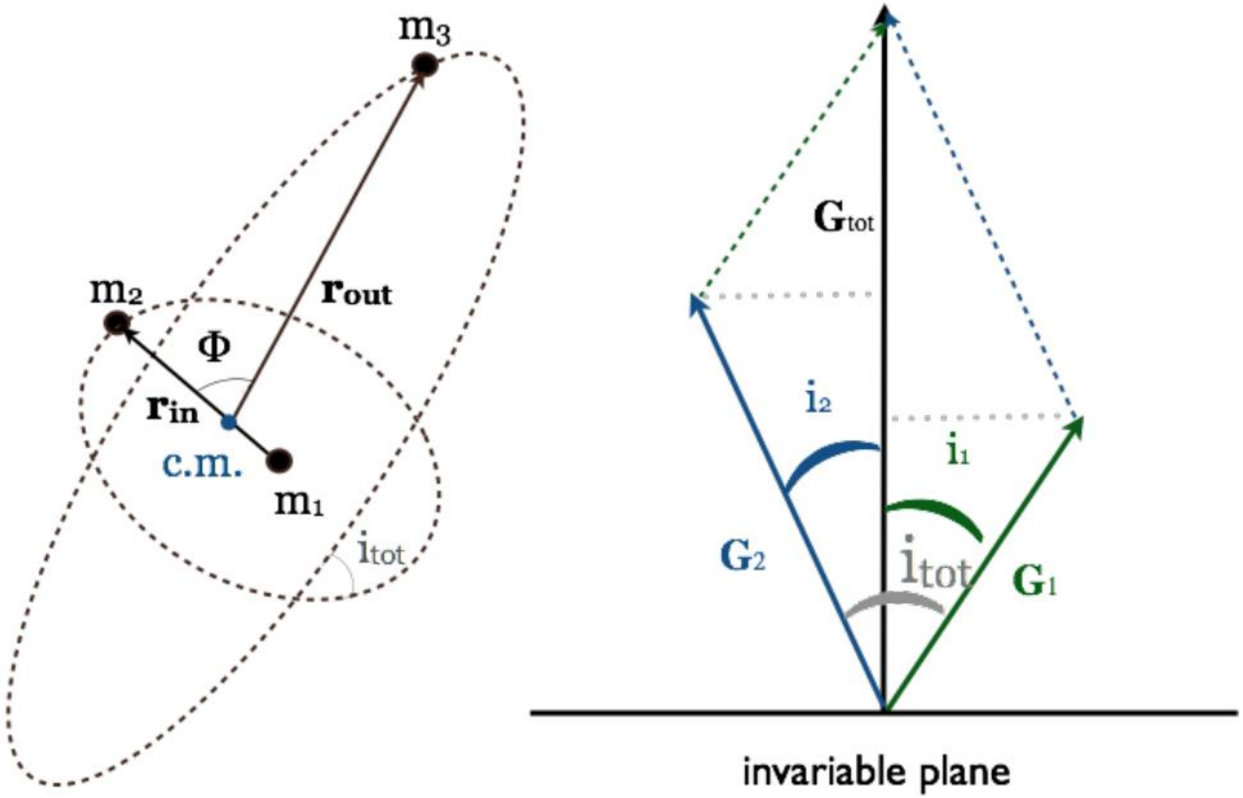
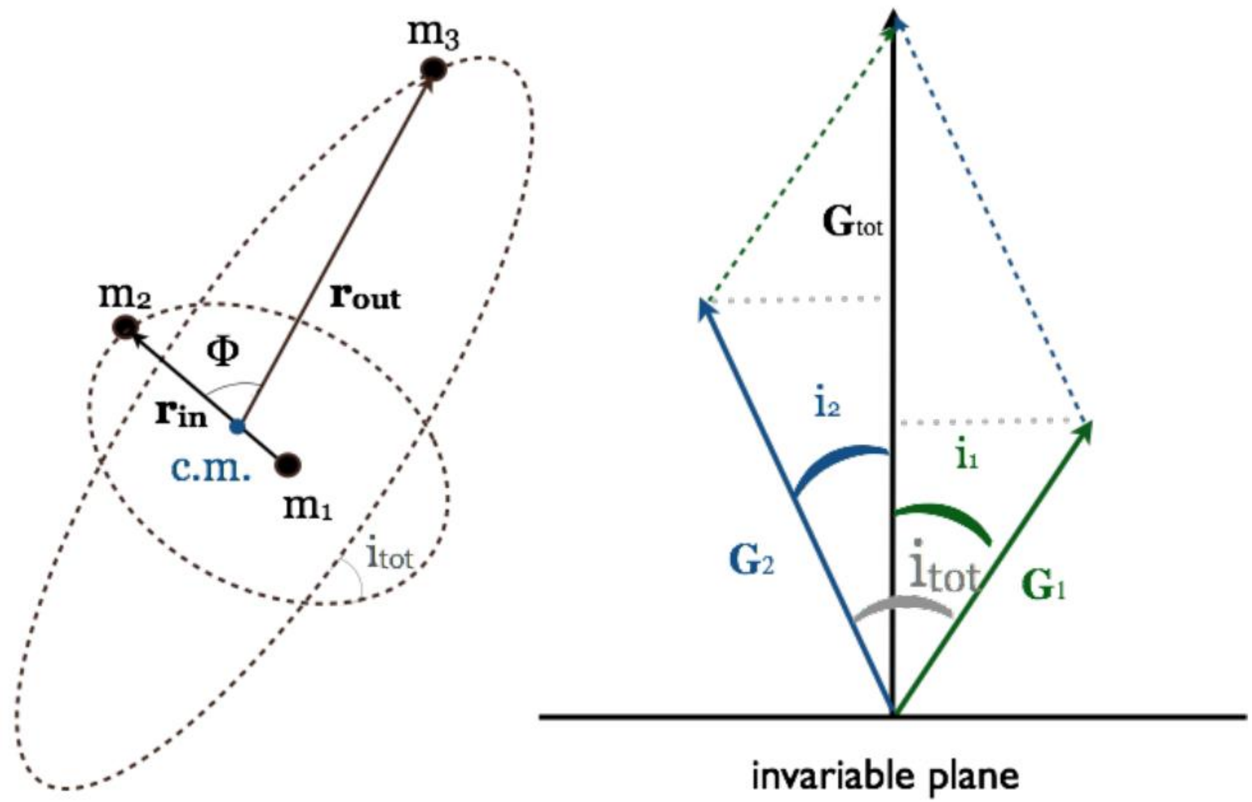


The secular approximation (i.e., phase averaged, long term evolution) can be applied, where the interactions between two non-resonant orbits is equivalent to treating the two orbits as massive wires. Here, the line-density is inversely proportional to orbital velocity and the two orbits torque each other and exchange angular momentum, but not energy. Therefore the orbits can change shape and orientation (on timescales much longer than their orbital periods), but not semi-major axes of the orbits.

Naoz (2016)



$$\mathcal{H} = \frac{k^2 m_1 m_2}{2a_1} + \frac{k^2 m_3 (m_1 + m_2)}{2a_2} + \frac{k^2}{r_2} \sum_{n=2}^{n=\infty} \left(\frac{r_{\rm in}}{r_{\rm out}} \right)^n M_n P_n(\cos\Phi)$$

$$M_n = m_1 m_2 m_3 \frac{m_1^{n-1} - (-m_2)^{n-1}}{(m_1 + m_2)^n}$$

Naoz (2016)

$$\mathcal{H} = \frac{3}{8} k^2 \frac{m_1 m_3}{a_2} \left(\frac{a_1}{a_2}\right)^2 \frac{1}{(1-e_2^2)^{3/2}} F_{quad} \ ,$$

$$F_{\rm quad} = -\frac{e_1^2}{2} + \theta^2 + \frac{3}{2} e_1^2 \theta^2 + \frac{5}{2} e_1^2 (1-\theta^2) \cos(2\omega_1)$$

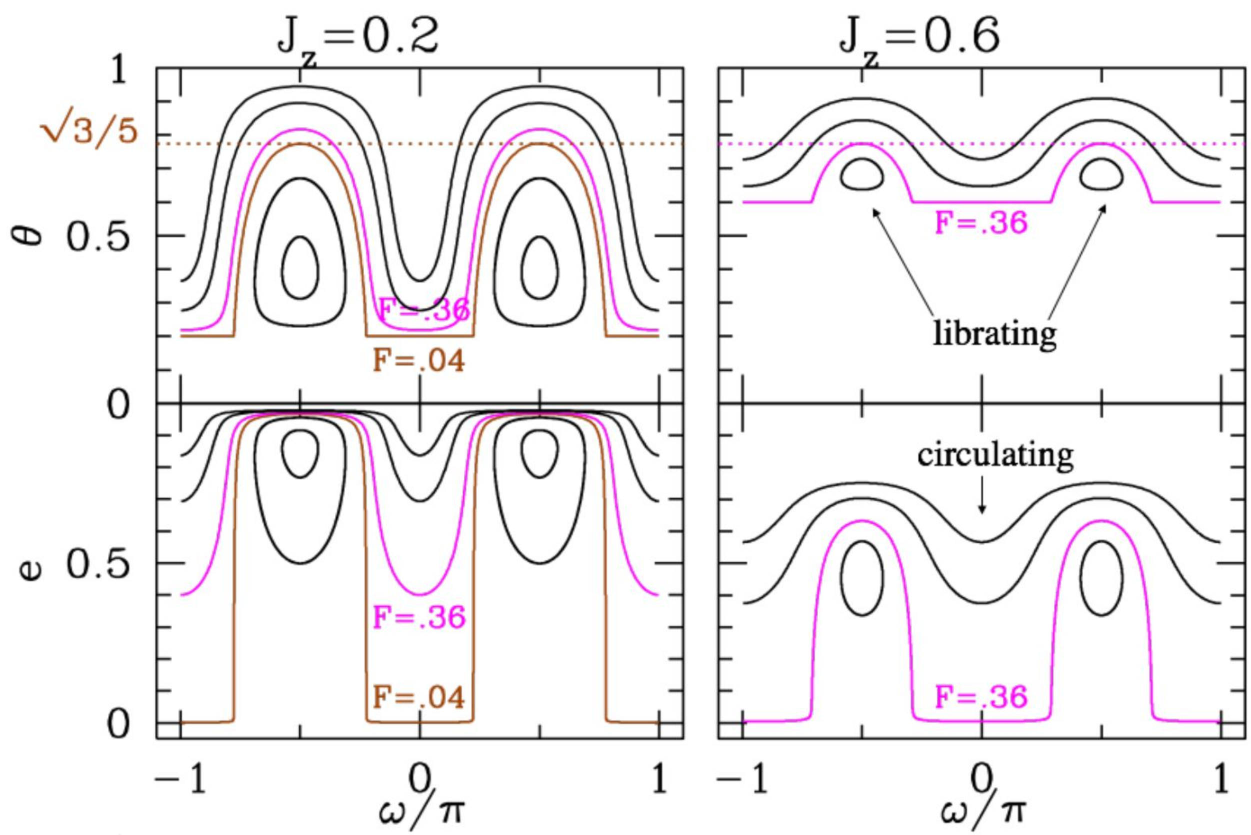
$$j_{z,1} = \sqrt{1-e_1^2} \cos i_{\rm tot} = {\rm Const.}$$

$$\theta = \cos i_{\rm tot}$$

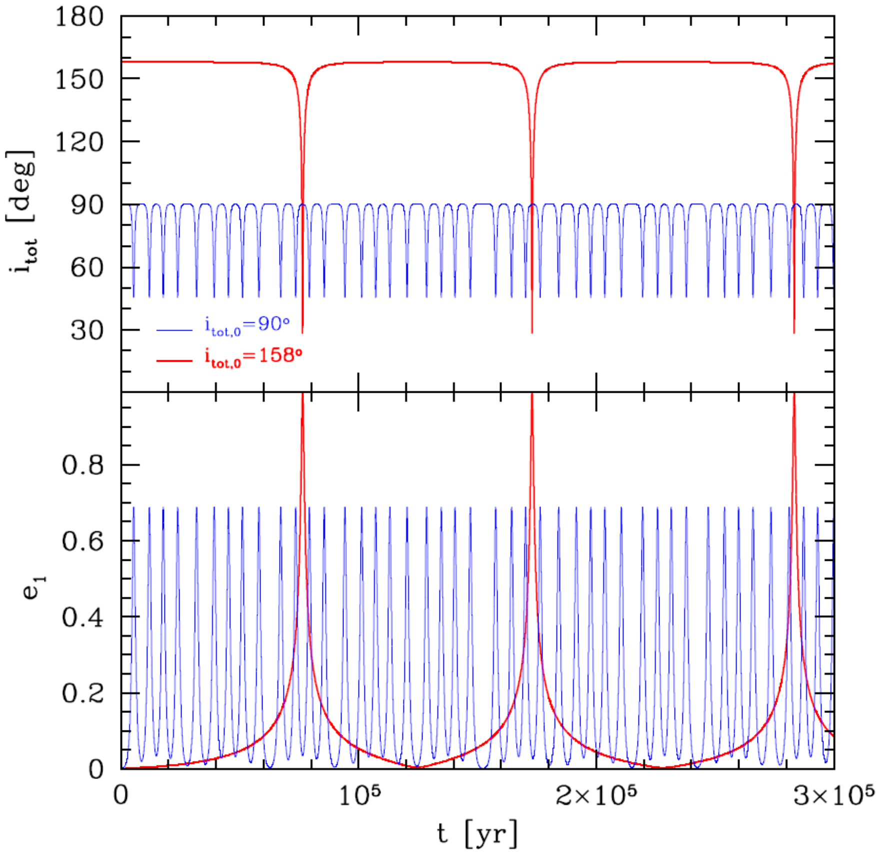
arguments of periastron, ω_1 and ω_2

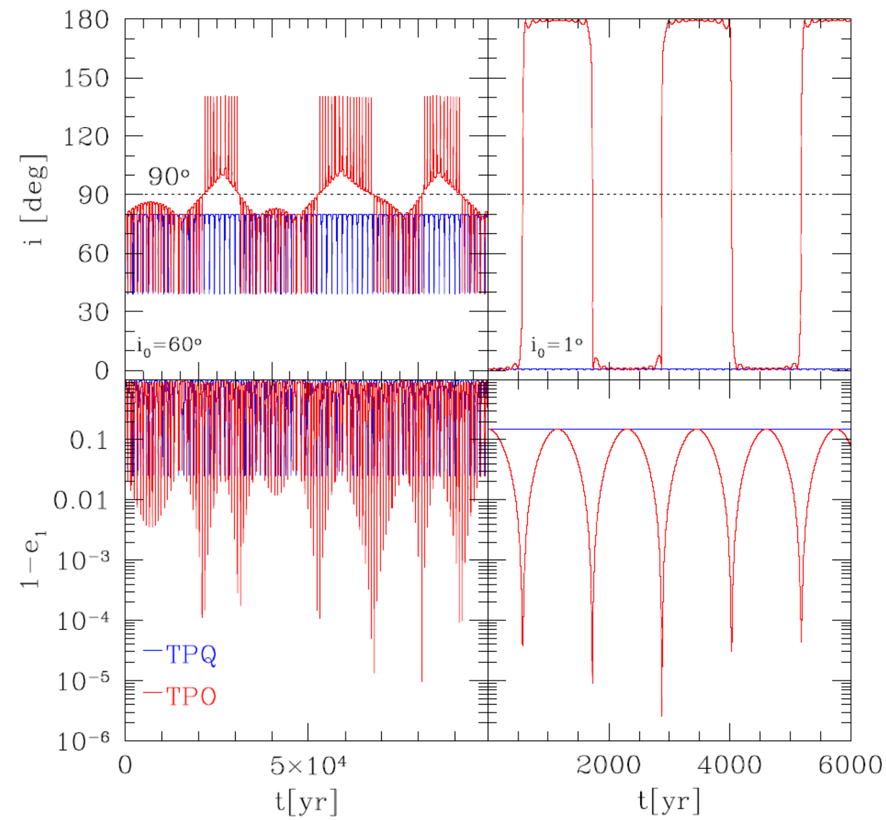
$$j_{z,1} = \sqrt{1-e^2_{1,max/min}} \cos i_{1,min/max} = \sqrt{1-e^2_{1,0}} \cos i_{1,0}$$

$$e_{max} = \sqrt{1 - \frac{5}{3} \cos^2 i_0}$$



Naoz (2016)

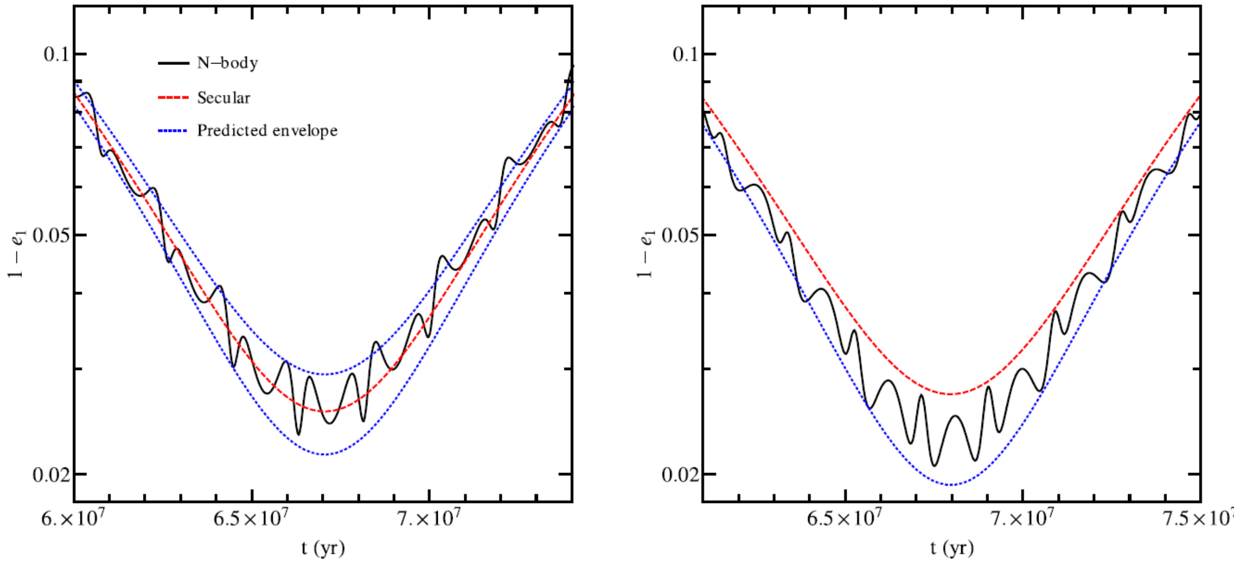




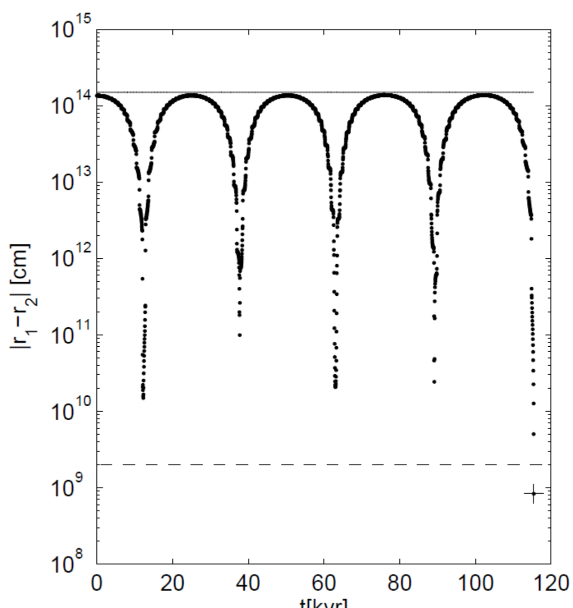
$$t_{\rm quad} \sim \frac{16}{15}\frac{a_2^3(1-e_2^2)^{3/2}\sqrt{m_1+m_2}}{a_1^{3/2}m_3k}$$
$$= \frac{16}{30\pi}\frac{m_1+m_2+m_3}{m_3}\frac{P_2^2}{P_1}(1-e_2^2)^{3/2}$$

Naoz (2016)

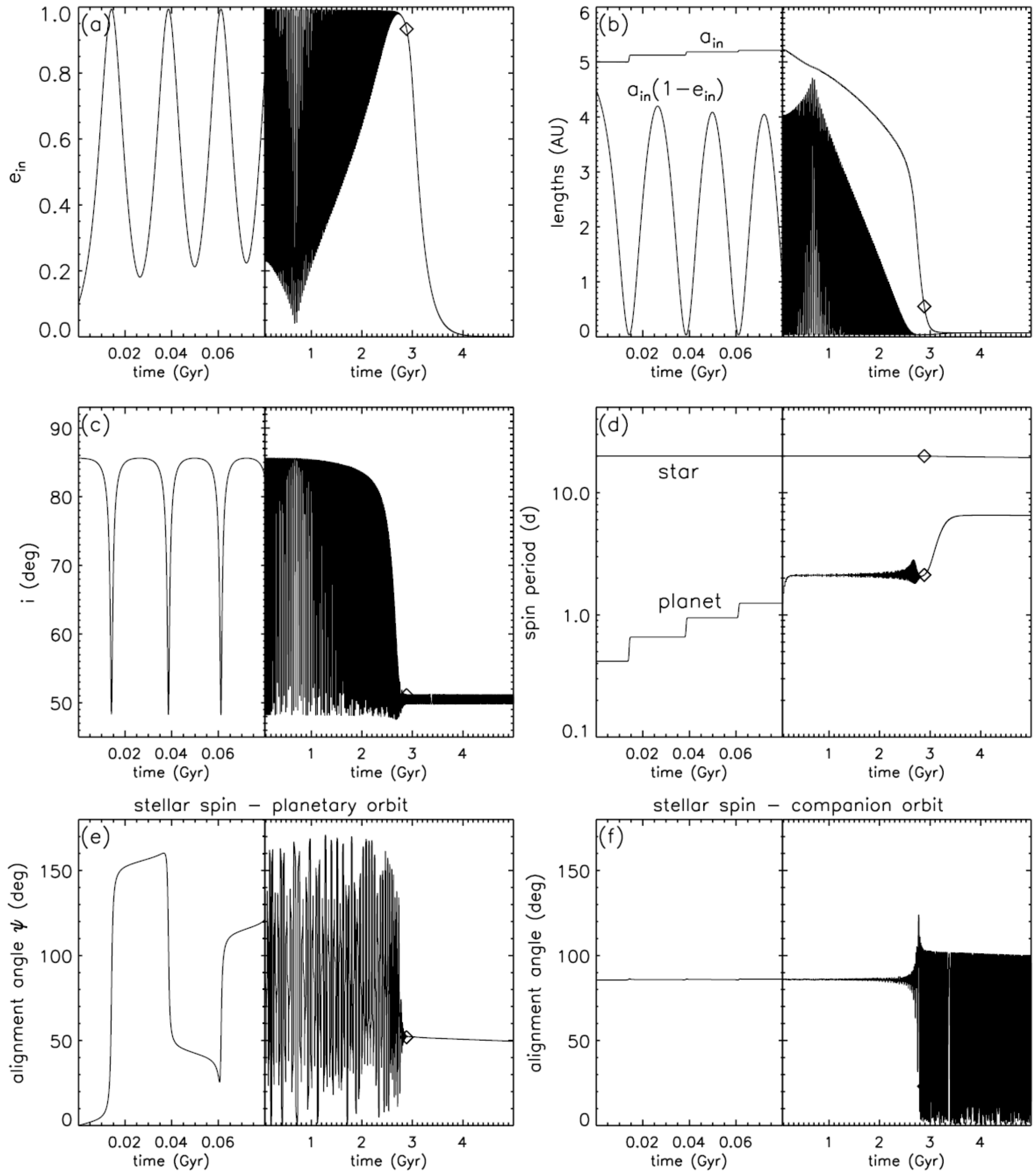
Non-secular effects



$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64}{5} \frac{G^3 \mu M^2}{c^5 a^3 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right)$$ Antognini et al. (2014)



Katz & Dong (2012)



Fabrycky & Tremaine (2007)

Cluster dynamics

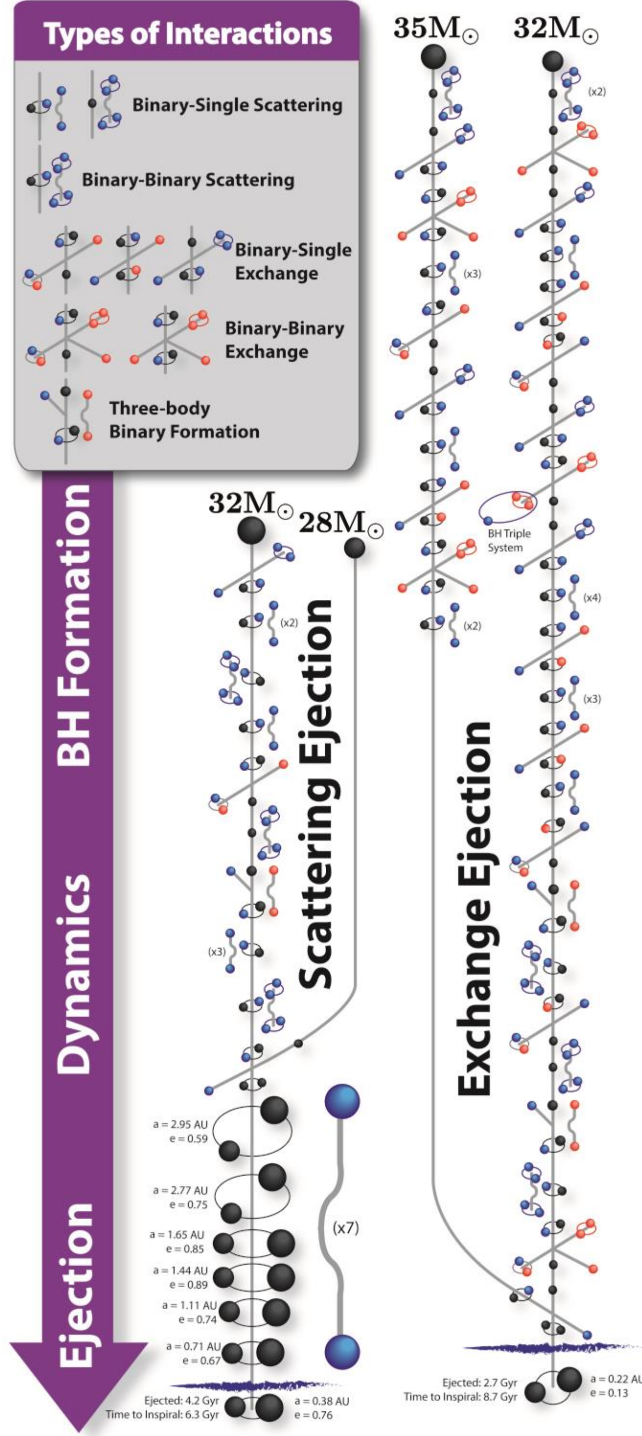
$$t_{\rm relax} \simeq \frac{N}{8 \ln \Lambda} t_{\rm dyn}$$

$$t_{\rm dyn} \simeq (G\bar{\rho})^{-1/2}$$

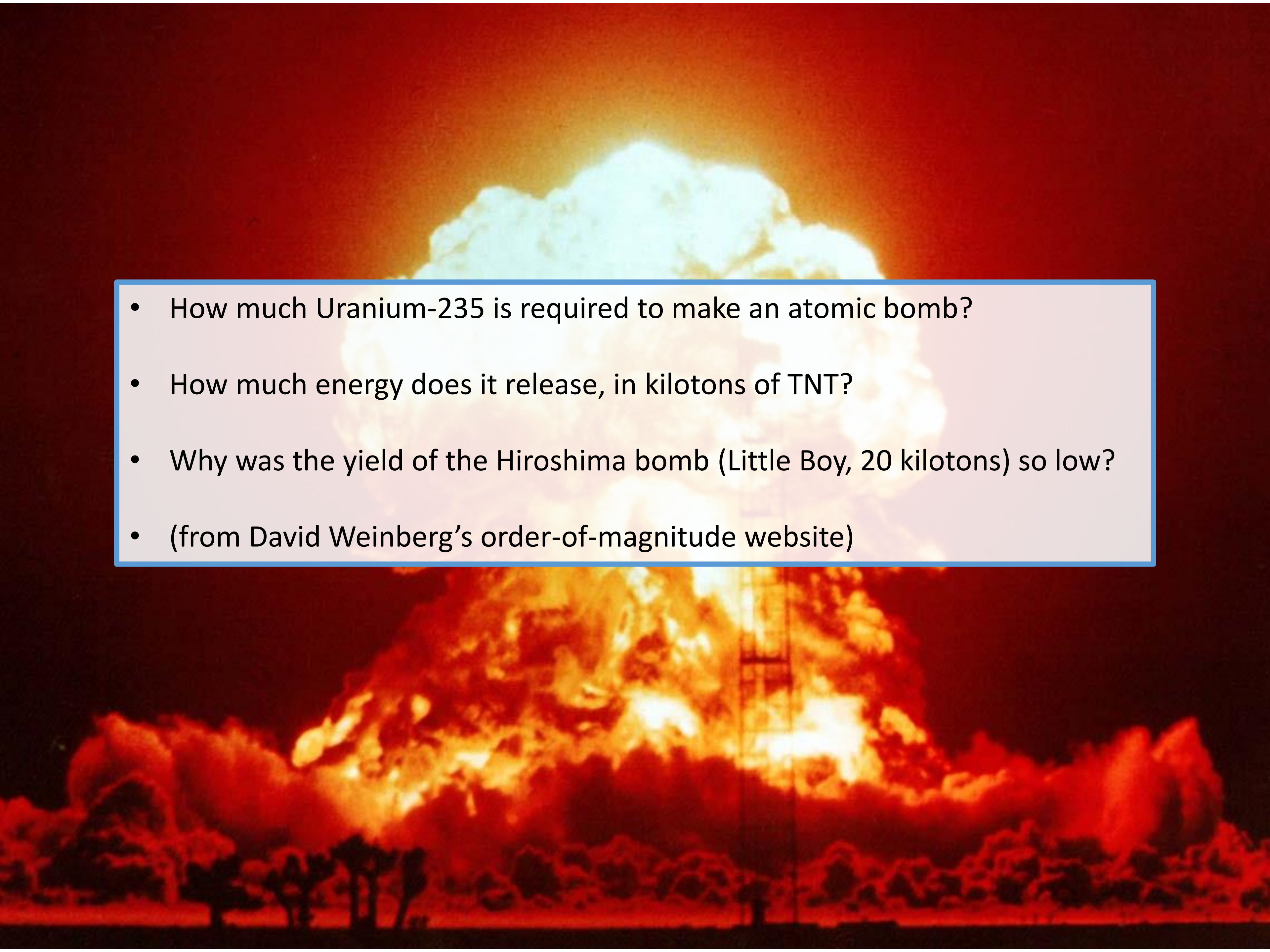
$$\ln \Lambda = \ln(b_{\rm max}/b_{\rm min})$$

$$b_{\rm min} = 2Gm/\sigma^2$$

Dehnen & Read (2011)



Rodriguez et al. (2016)



- How much Uranium-235 is required to make an atomic bomb?
- How much energy does it release, in kilotons of TNT?
- Why was the yield of the Hiroshima bomb (Little Boy, 20 kilotons) so low?
- (from David Weinberg’s order-of-magnitude website)