

# Astrophysics of gravitational wave sources

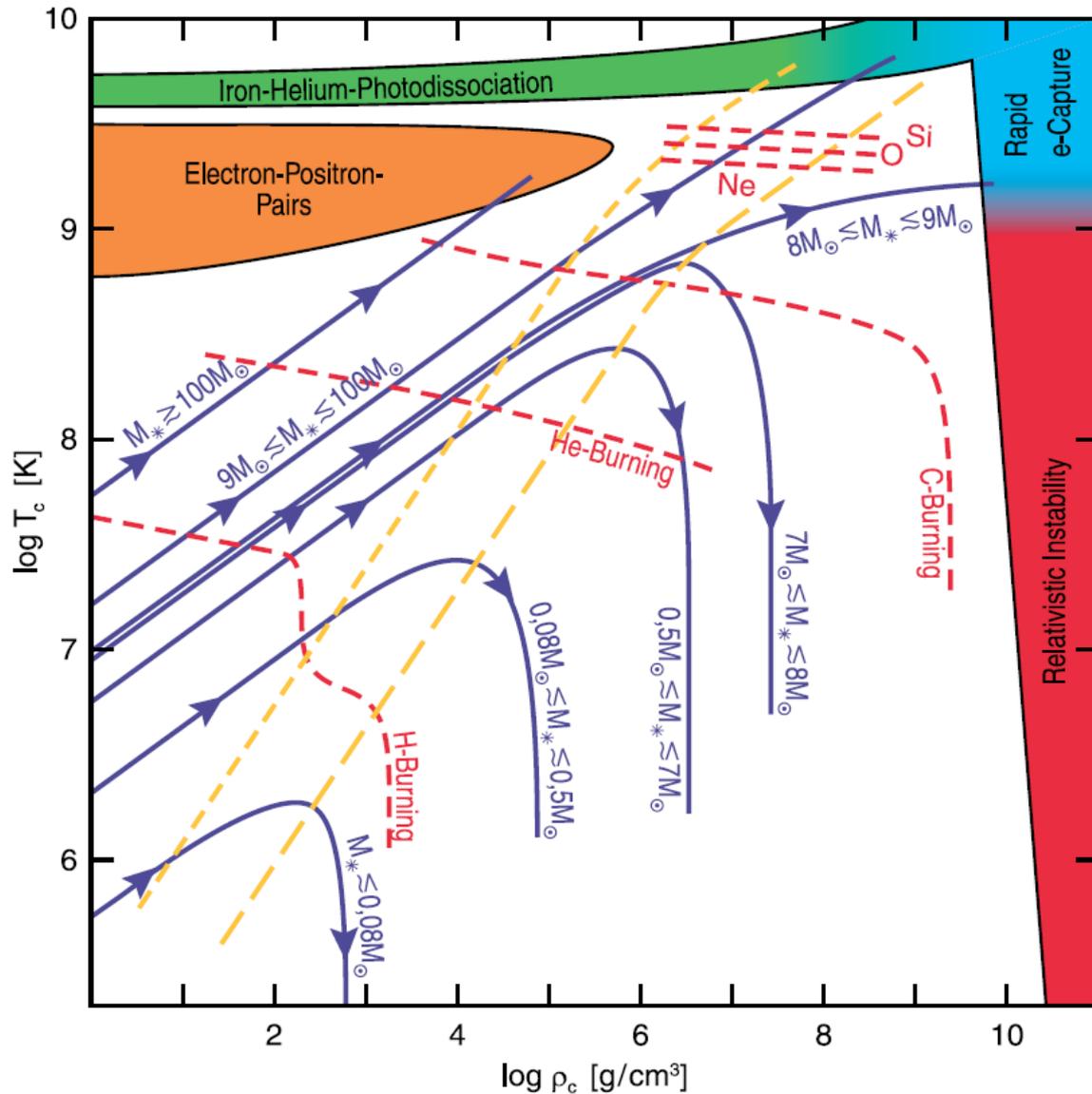
Lecture 11: Binary and multiple stars & their evolution

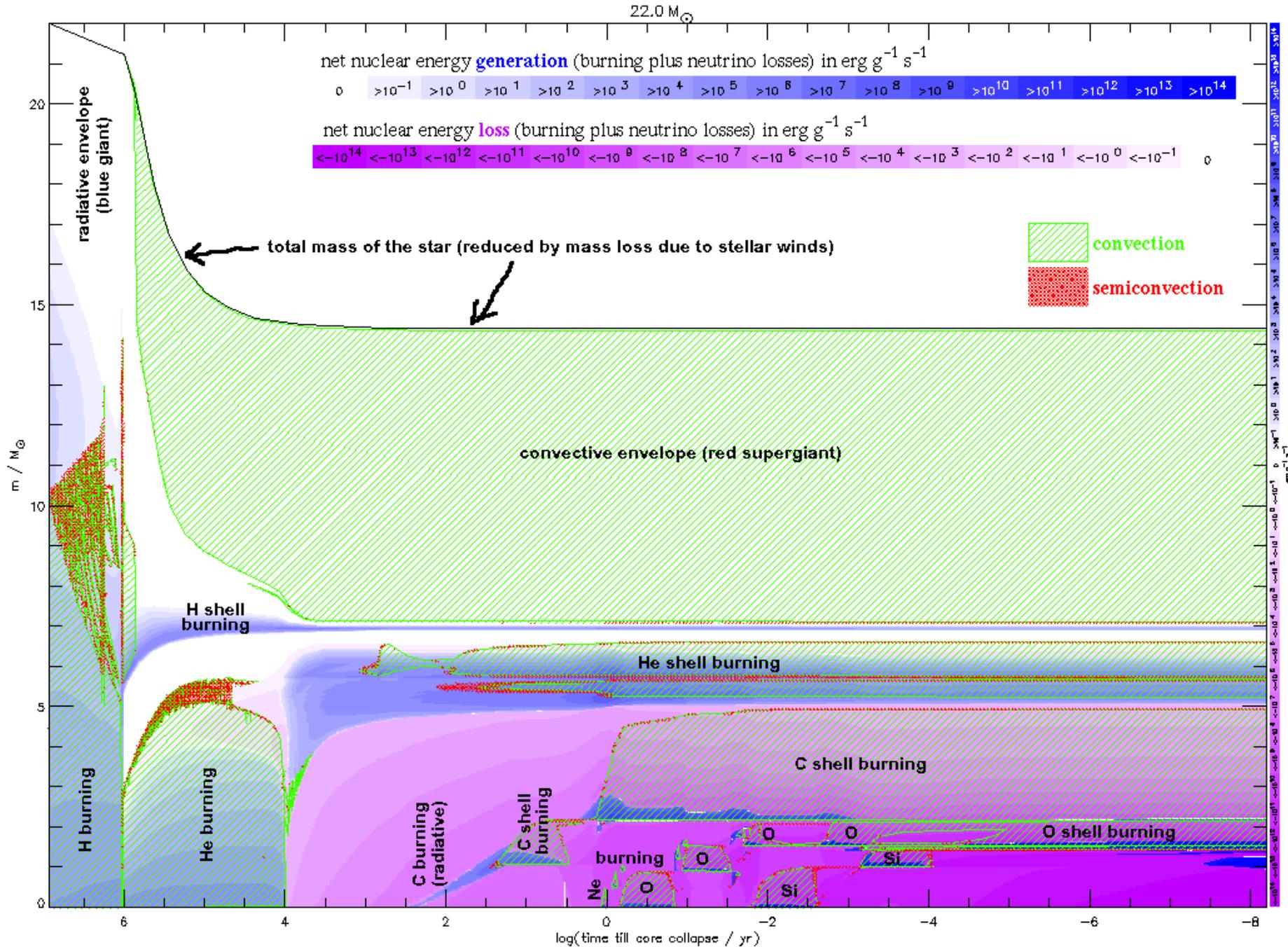
Ondřej Pejcha

ÚTF MFF UK

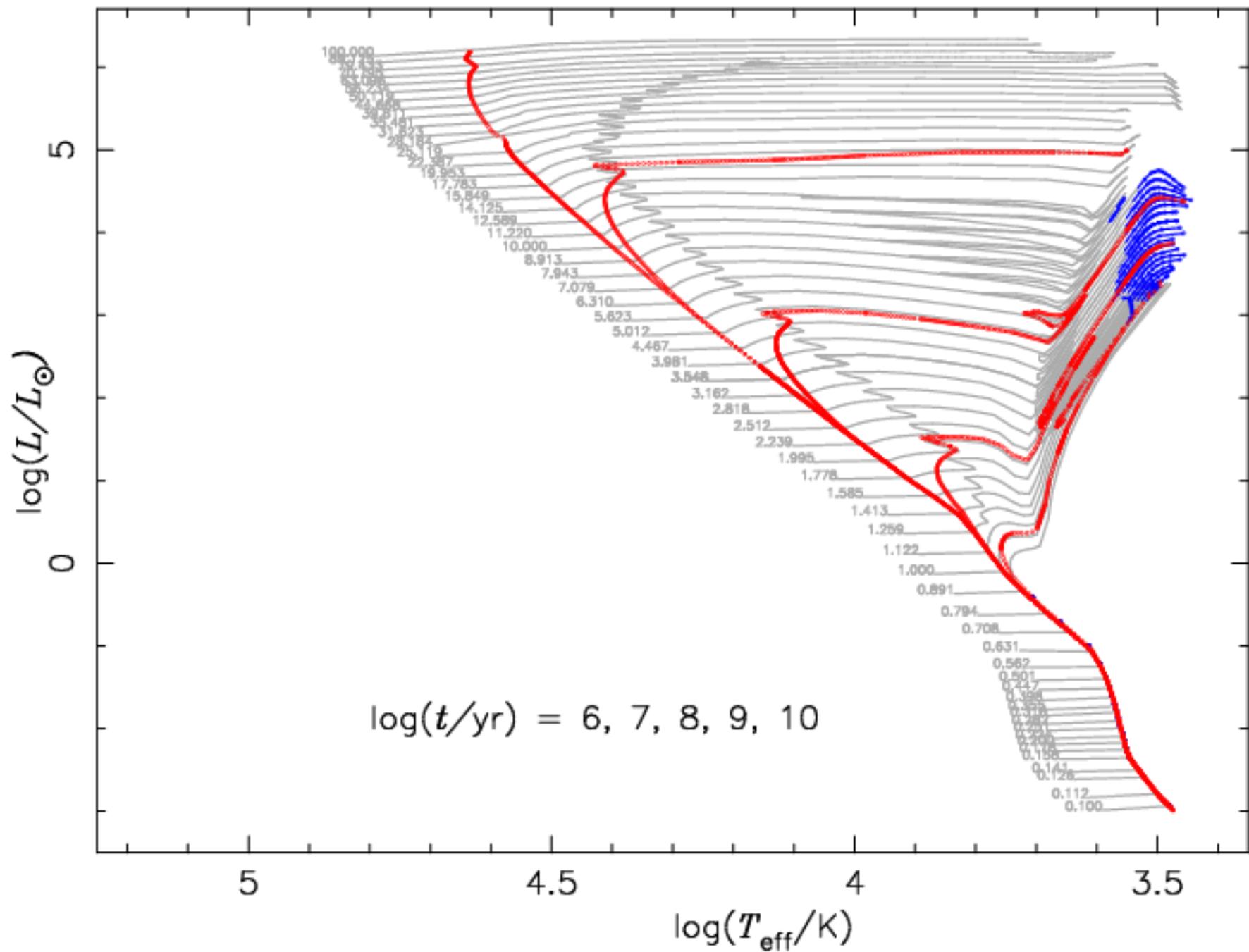
# Single star evolution before core-collapse

Central temperature and pressure

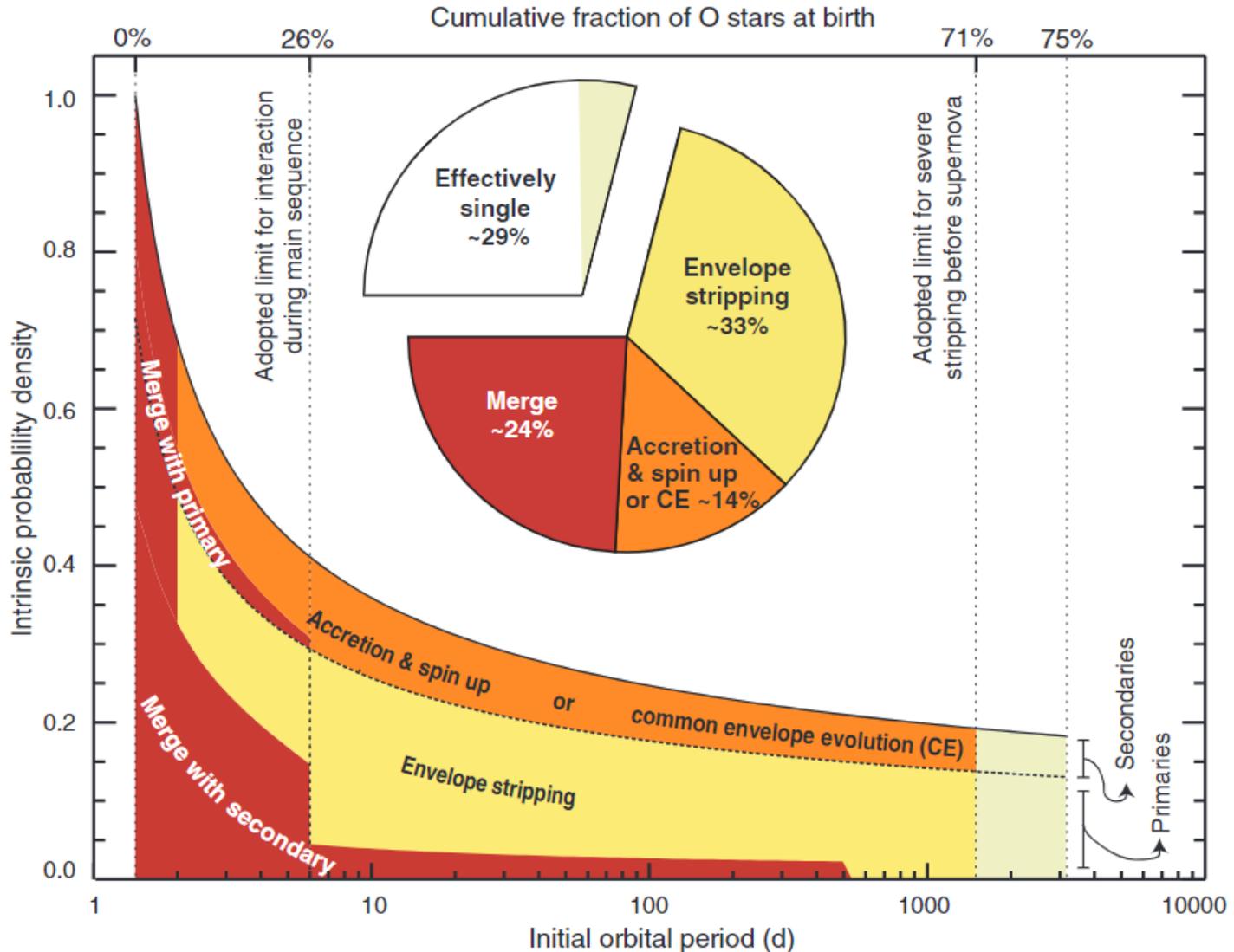




A. Heger website 2sn.org



# Binary star interaction dominates the evolution of massive stars



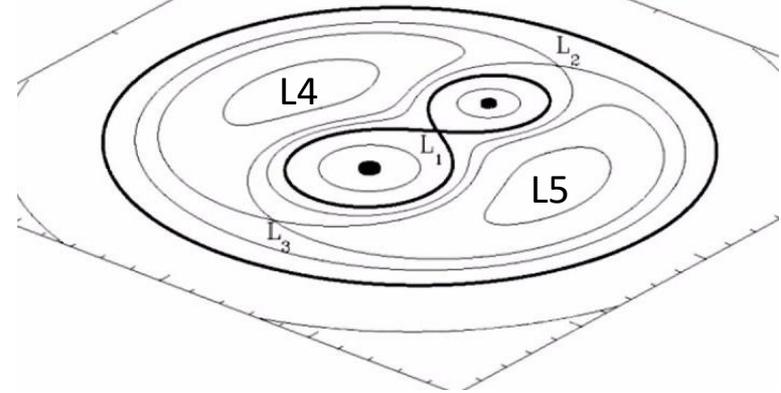
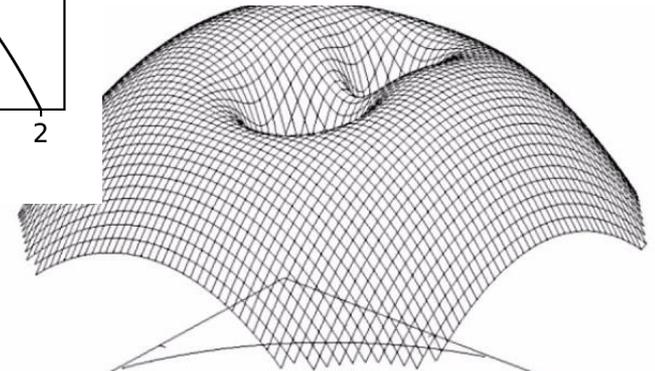
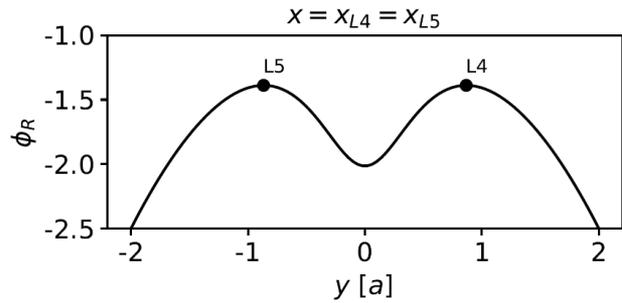
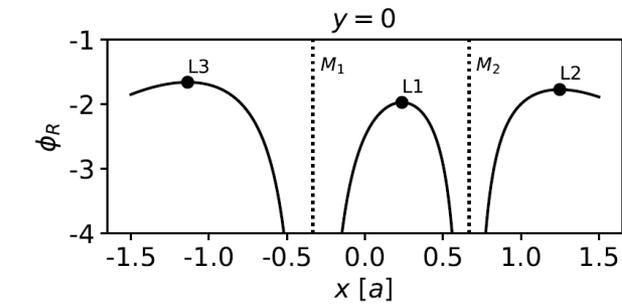
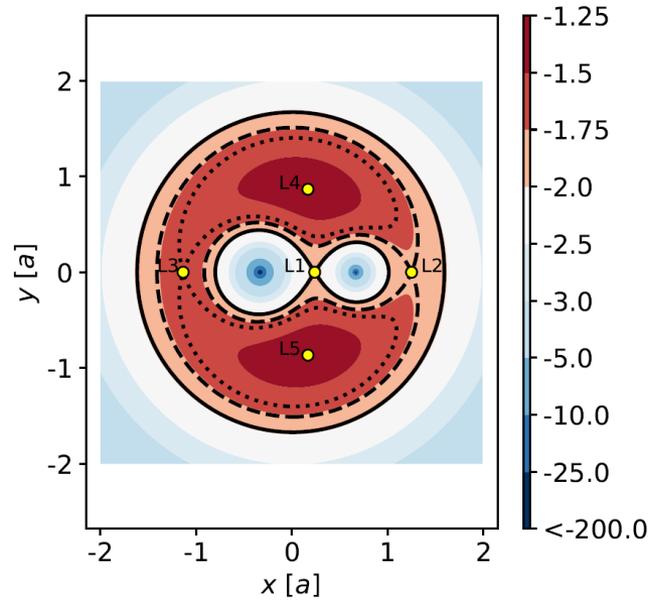
# Binary stars - Roche potential

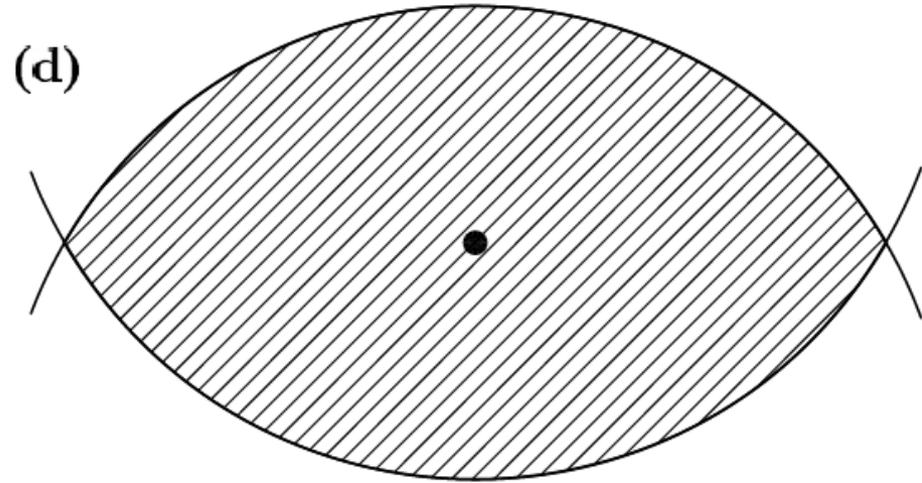
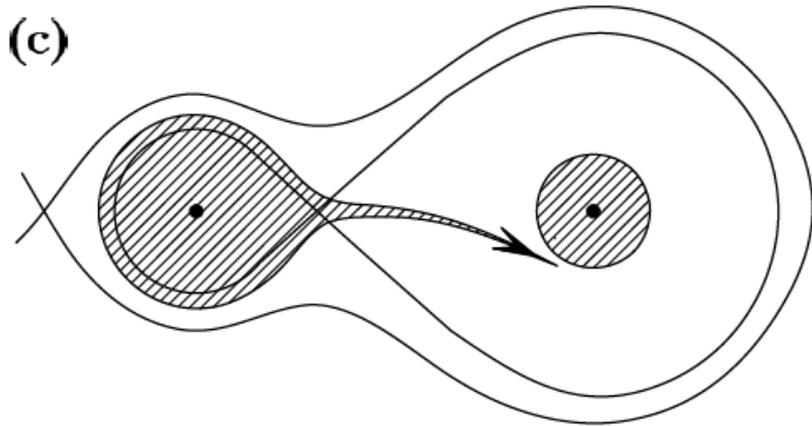
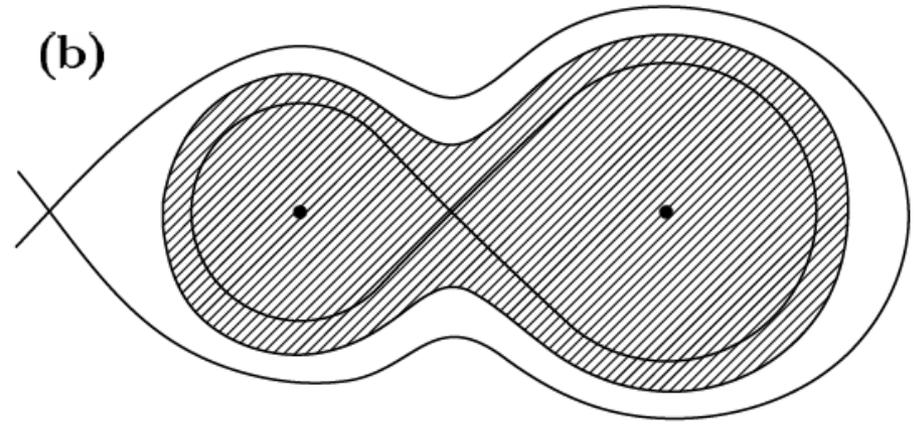
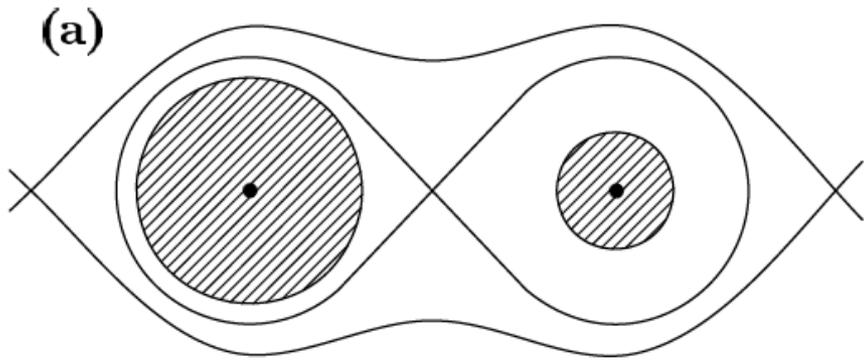
$$\ddot{\mathbf{r}} = -\nabla\phi_R(\mathbf{r}) - 2\boldsymbol{\omega} \times \dot{\mathbf{r}},$$

$$\phi_R(\mathbf{r}) = -\frac{GM_1}{|\mathbf{r} - \mathbf{r}_1|} - \frac{GM_2}{|\mathbf{r} - \mathbf{r}_2|} + \frac{1}{2}|\boldsymbol{\omega} \times \mathbf{r}|^2$$

$$\omega^2 = \frac{GM}{a^3} = \left(\frac{2\pi}{T}\right)^2 \quad q = \frac{M_2}{M_1} \quad \mu = \frac{M_2}{M_1 + M_2} = \frac{q}{1 + q}$$

# Roche potential





# Mass-radius relations of stars, evolutionary timescales

ZAMS:

$$\frac{R}{R_{\odot}} \simeq \left( \frac{M}{M_{\odot}} \right)^{0.7} \quad (\text{Polytrope with } \gamma = 5/3 \text{ has adiabatic } R \sim M^{-1/3})$$

$$\frac{L}{L_{\odot}} \simeq \left( \frac{M}{M_{\odot}} \right)^{3.8}$$

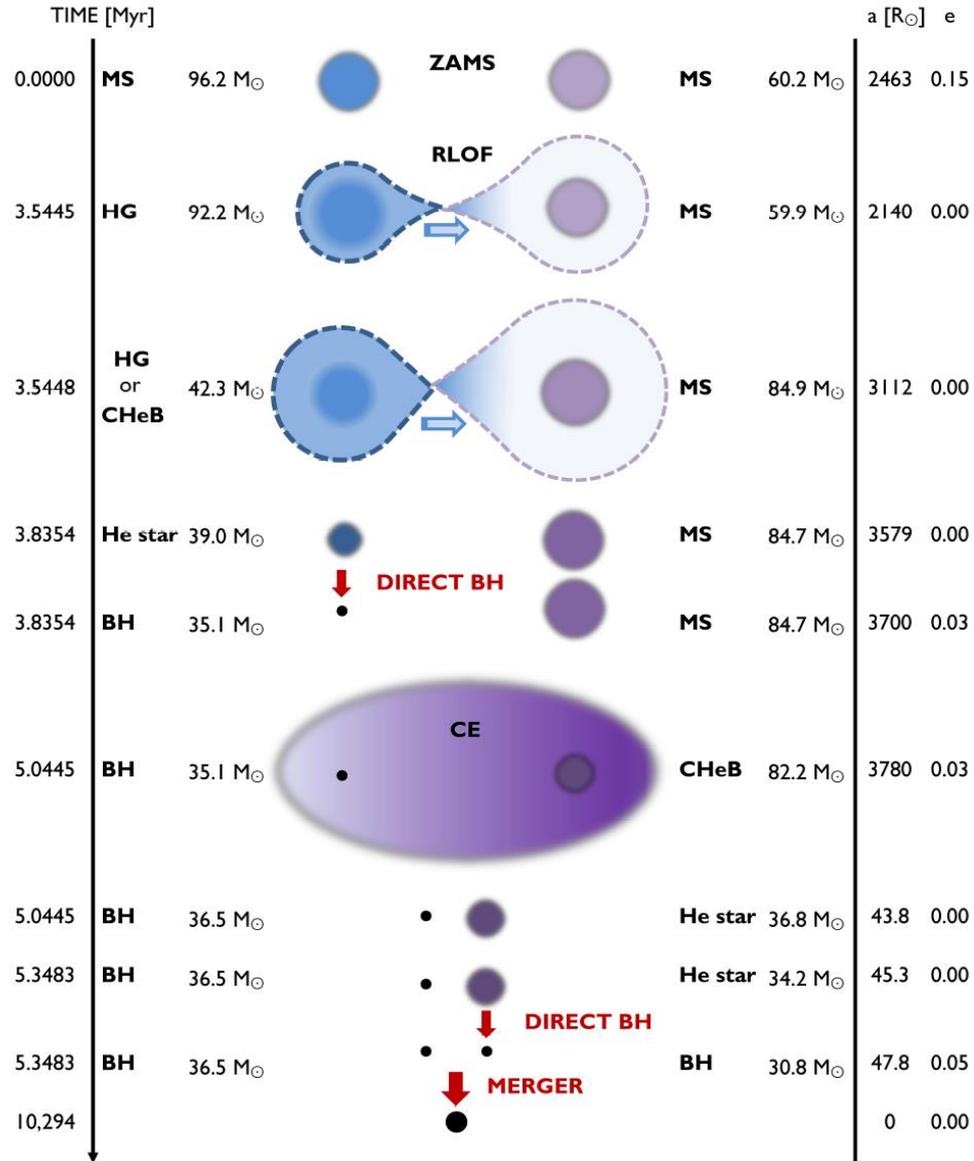
$$\tau_{\text{dyn}} = \frac{R}{c_s} \approx 0.04 \left( \frac{M_{\odot}}{M} \right)^{1/2} \left( \frac{R}{R_{\odot}} \right)^{3/2} \text{ day}$$

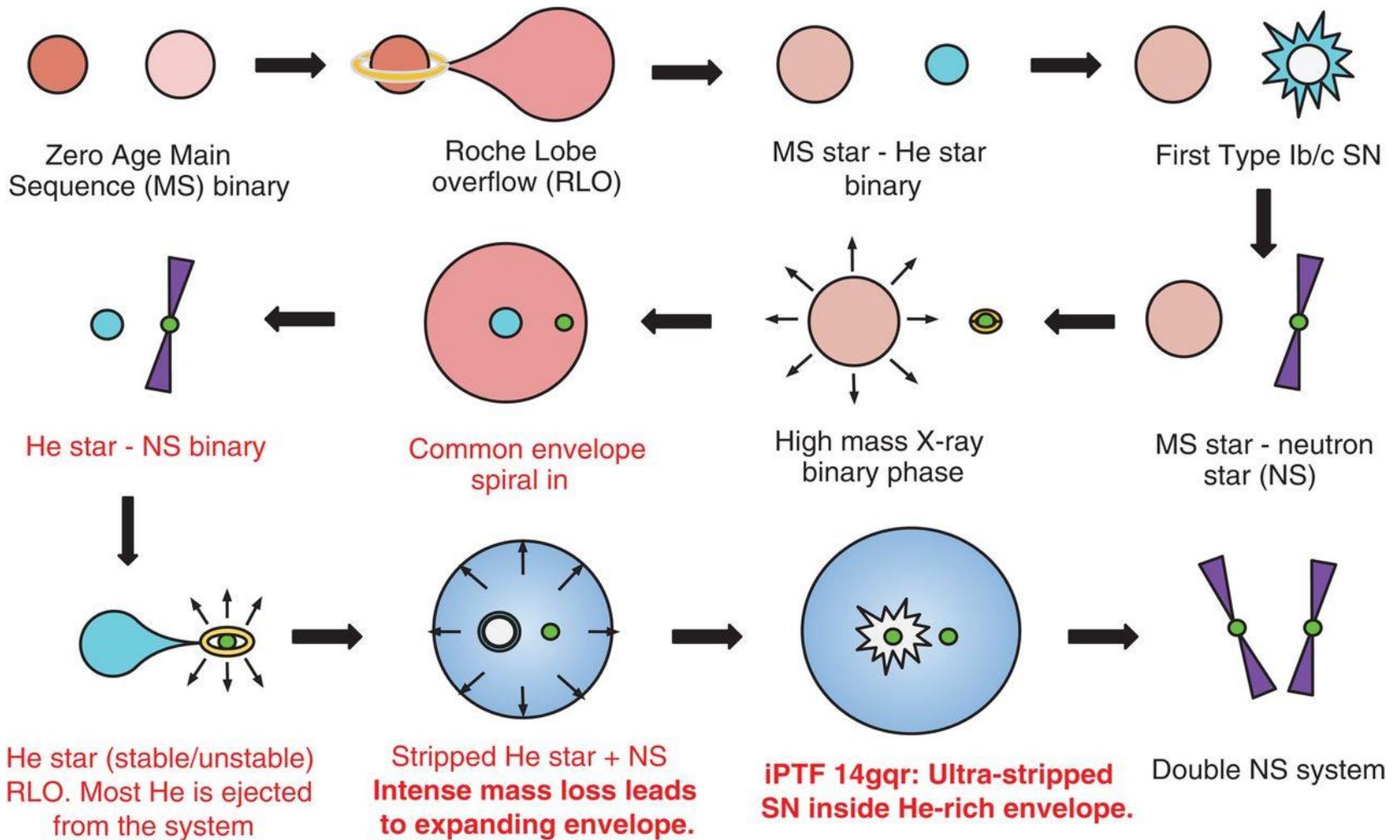
$$\tau_{\text{KH}} = \frac{E_{\text{th}}}{L} \approx \frac{GM^2}{2RL} \approx 1.5 \times 10^7 \left( \frac{M}{M_{\odot}} \right)^2 \frac{R_{\odot}}{R} \frac{L_{\odot}}{L} \text{ yr}$$

$$\tau_{\text{nuc}} = 0.007 \frac{M_{\text{core}} c^2}{L} \approx 10^{10} \frac{M}{M_{\odot}} \frac{L_{\odot}}{L} \text{ yr}$$

# Roche lobe overflow

- Nuclear, thermal, dynamical timescale
- Conservative vs nonconservative evolution
- Nonconservative processes:
  - Stellar wind
  - Magnetic braking
  - Gravitational radiation
  - Tidal friction
  - Secular dynamics
  - (supernova, common envelope, cluster dynamics)





Logarithmic change of the Roche lobe radius with orbital angular momentum and total mass fixed:

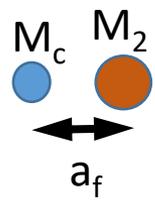
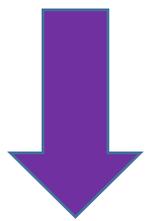
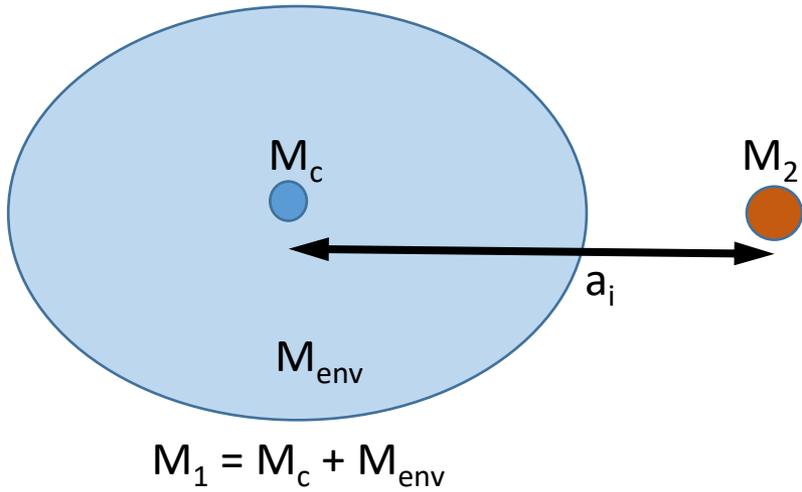
$$R'_L \equiv \frac{d \log R_L}{d \log M_1} = (1 + q) \cdot \left( \frac{d \log R_L/a}{d \log q} + \frac{d \log a}{d \log q} \right) \\ \approx 2.13q - 1.67, \quad 0 < q \lesssim 50;$$

Polytrope with  $\gamma = 5/3$  has adiabatic  $R \sim M^{-1/3}$ , which leads to critical  $q$

Dynamical instability -> total energy  
conserved

(Eggleton's book)

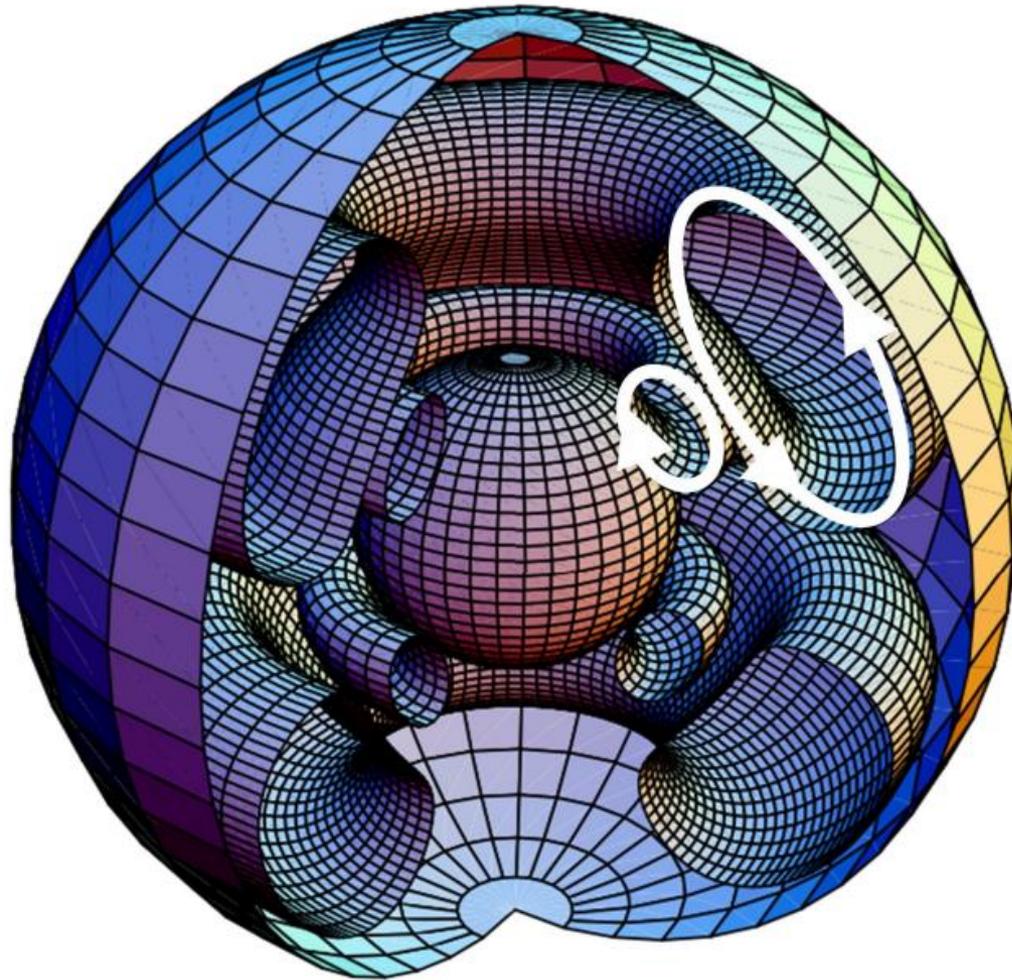
# Common envelope - calculation of energy balance



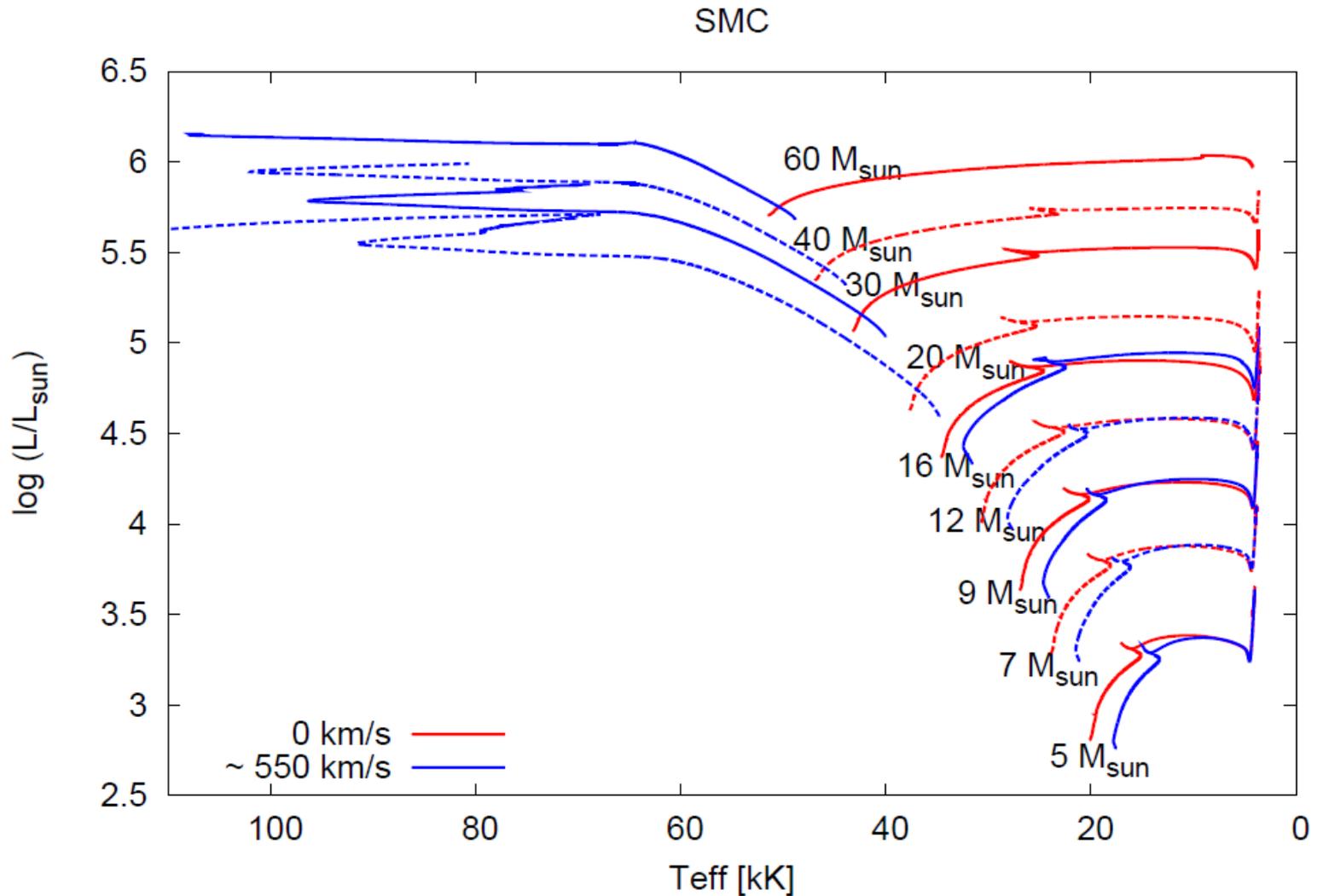
# Chemically-homogeneous evolution

What drives evolutionary expansion of stars?

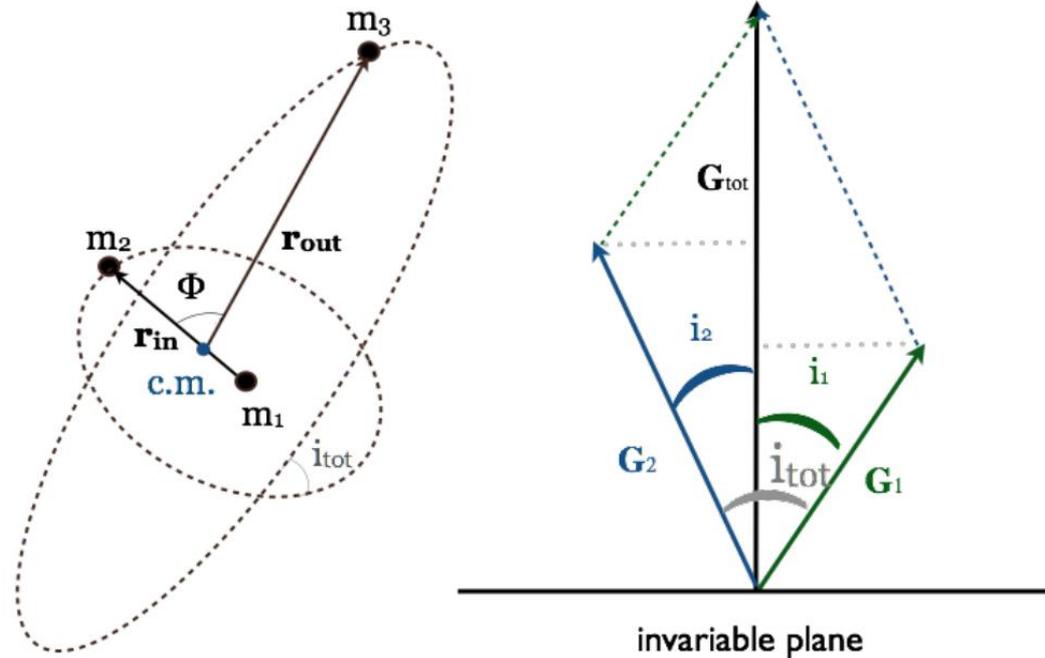
# Rotation in stars



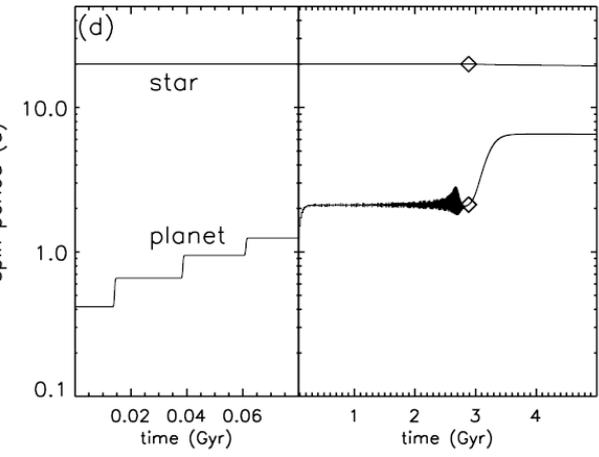
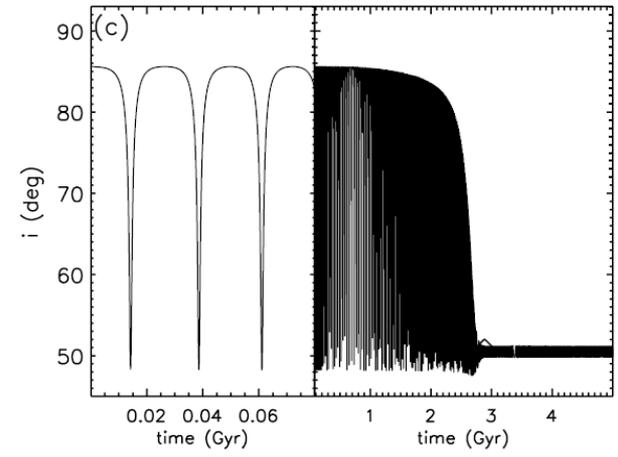
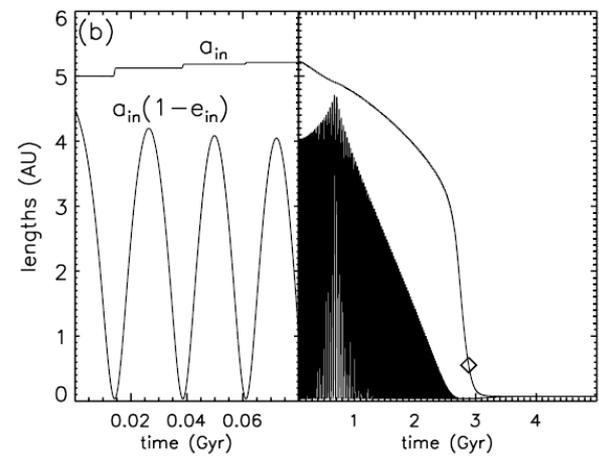
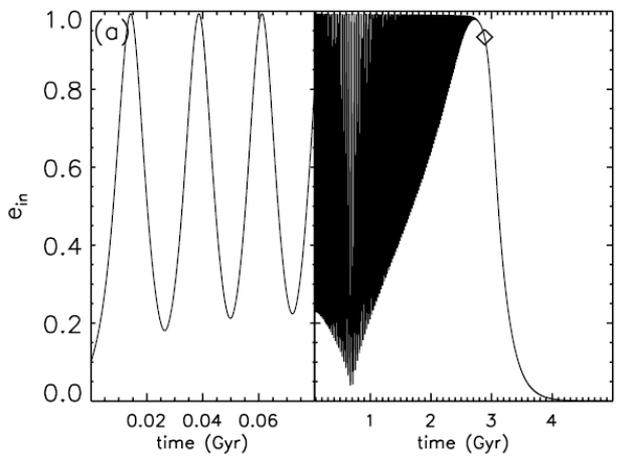
# Chemically-homogeneous evolution



# von Zeipel-Lidov-Kozai

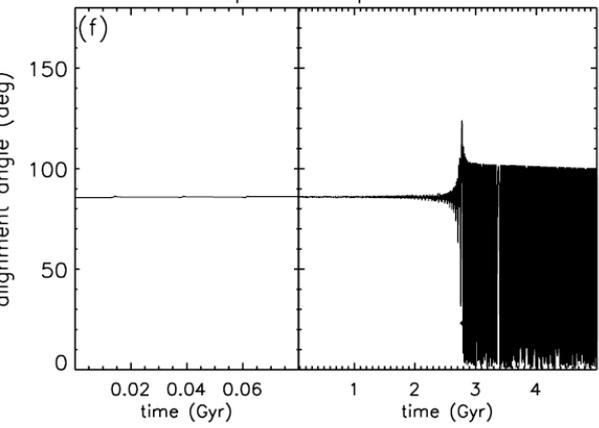
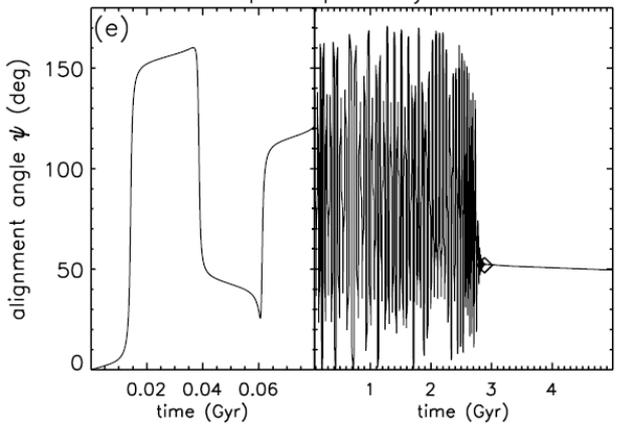


The secular approximation (i.e., phase averaged, long term evolution) can be applied, where the interactions between two non-resonant orbits is equivalent to treating the two orbits as massive wires. Here, the line-density is inversely proportional to orbital velocity and the two orbits torque each other and exchange angular momentum, but not energy. Therefore the orbits can change shape and orientation (on timescales much longer than their orbital periods), but not semi-major axes of the orbits.



stellar spin – planetary orbit

stellar spin – companion orbit



Fabrycky &  
Tremaine  
(2007)

# Cluster dynamics

$$t_{\text{relax}} \simeq \frac{N}{8 \ln \Lambda} t_{\text{dyn}}$$

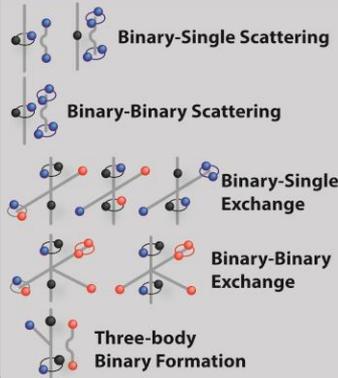
$$t_{\text{dyn}} \simeq (G\bar{\rho})^{-1/2}$$

$$\ln \Lambda = \ln(b_{\text{max}}/b_{\text{min}})$$

$$b_{\text{min}} = 2Gm/\sigma^2$$



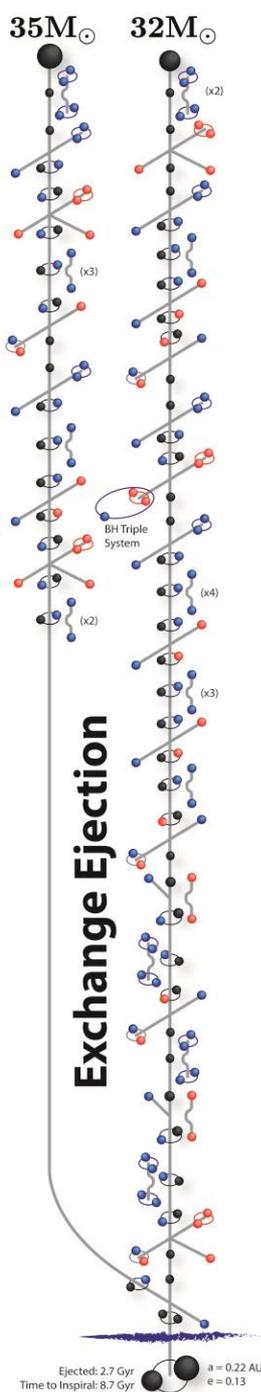
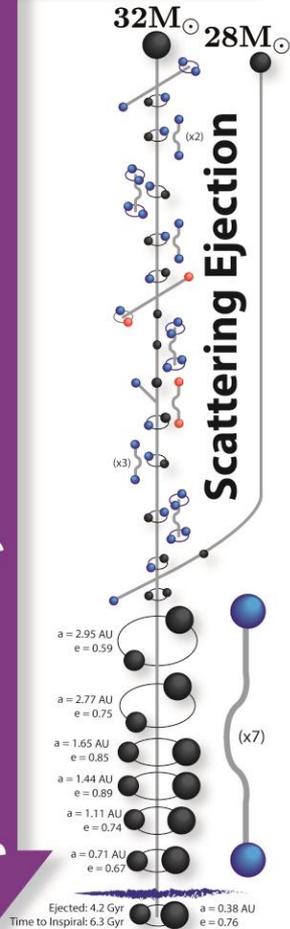
# Types of Interactions



BH Formation

Dynamics

Ejection



# Cool videos

- <https://www.youtube.com/watch?v=qJMom80Qdc8>
- <https://www.youtube.com/watch?v=3L3iSWRCtuA>
- <https://ciera.northwestern.edu/gallery/>
- Also worth: Adler planetarium