Accelerating black holes in anti-de Sitter universe *

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Abstract

A physical interpretation of the C-metric with a negative cosmological constant Λ is suggested. Using a convenient coordinate system it is demonstrated that this class of exact solutions of Einstein's equations describes uniformly accelerating (possibly charged) black holes in anti-de Sitter universe. Main differences from the analogous de Sitter case are emphasised.

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1 Introduction

The well-known C-metric [1], [2] is an important explicit representative of a class of spacetimes with boost-rotation symmetry [3]. It was shown already in 1970 by Kinnersley and Walker [4] that this describe a pair of uniformly accelerating black holes of masses m and possible charges e in Minkowski background. The acceleration A is caused by conical singularities which may be interpreted either as a strut between the two black holes or two semi-infinite strings connecting them to infinity. Geometrical and asymptotic properties were subsequently studied in [5], [6]. Bonnor [7] found a transformation of the C-metric into the boost-rotational canonical form. However, the explicit metric functions are somewhat complicated and depend on specific ranges of the initial "static" coordinates [7]-[10]. Recently the limit of unbounded acceleration $A \to \infty$ was investigated [11]. It has been demonstrated that such limit of the C-metric is identical to the solution which represents a spherical impulsive gravitational wave generated by a snapping string (or an expanding strut) [12]-[17].

Interestingly, there exists a more general class of the C-metric solutions presented by Plebanski and Demianski [18] which admits a non-vanishing value of a cosmological constant Λ (plus possibly a rotational parameter, see also [19]-[22]). These solutions have already been used for investigation of the pair creation of black holes in de Sitter [23] and anti-de Sitter backgrounds [24]. It is also natural to expect that in the limit $A \to \infty$ this generalised C-metric generates a specific spherical impulsive gravitational wave in the (anti-)de Sitter universe which may be interesting in the context of quantum and string theories. However, physical and global properties of the spacetimes [18] with $\Lambda \neq 0$ have not yet been investigated thoroughly even at the classical level. Only the first steps have been done. Using a suitable coordinate system adapted to the motion of uniformly accelerating particles in de Sitter space the physical meaning of the parameters appearing in solutions [18] could be determined [25]. On the other hand, the most recent paper by Bičák and Krtouš [26] on accelerated

*Dedicated to my academic teacher Prof. J. Bičák on the occasion of his 60^{th} birthday.

test sources in de Sitter spacetime may be of a great help for understanding the global structure of these solutions.

It is the purpose of this short contribution to provide a basic physical interpretation of the above family of exact solutions with a *negative* value of the cosmological constant Λ as uniformly accelerating black holes in anti-de Sitter universe. Although this is a simple extension of our previous results presented in [25], we shall demonstrate that there are some interesting new features and fundamental differences from the analogous de Sitter case.

2 Uniformly accelerating observers in anti-de Sitter space

Anti-de Sitter universe is the (maximally symmetric) spacetime of constant negative curvature [27]. It can be understood as a 4-dimensional hyperboloid

$$-Z_0^2 + Z_1^2 + Z_{23}^2 - Z_4^2 = -a^2 \,, \tag{1}$$

where $a = \sqrt{-3/\Lambda}$, in a 5-dimensional Minkowski space

$$\mathrm{d}s^2 = -\mathrm{d}Z_0^2 + \mathrm{d}Z_1^2 + \mathrm{d}Z_{23}^2 + Z_{23}^2\,\mathrm{d}\Phi^2 - \mathrm{d}Z_4^2 \,, \tag{2}$$

as shown in Fig. 1. We have introduced here for later convenience the coordinates $Z_{23} \in [0,\infty)$ and $\Phi \in [0, 2\pi)$ by $Z_2 = Z_{23}\cos\Phi$ and $Z_3 = Z_{23}\sin\Phi$, instead of the standard Cartesian coordinates Z_2, Z_3. With the parameterisation

$$\begin{aligned} Z_0 &= a\,\frac{\sin(T/a)}{\cos\chi}\,, \qquad & Z_4 &= a\,\frac{\cos(T/a)}{\cos\chi}\,, \\ Z_1 &= a\,\tan\chi\,\cos\Theta\,, & Z_{23} &= a\,\tan\chi\,\sin\Theta\,, \end{aligned} \tag{3}$$

where $T \in (-\infty, +\infty)$, $\chi \in [0, \frac{\pi}{2})$, $\Theta \in [0, \pi]$, we obtain the well-known metric in global coordinates

$$\mathrm{d}s^2 = \frac{a^2}{\cos^2\chi}\left[-\mathrm{d}\,(T/a)^2 + \mathrm{d}\chi^2 + \sin^2\chi\,(\mathrm{d}\Theta^2 + \sin^2\Theta\,\mathrm{d}\Phi^2)\right] . \tag{4}$$

This immediately gives the familiar Penrose diagram for the anti-de Sitter space (see Fig. 1) in which the boundary $\chi = \frac{\pi}{2}$ represents the conformal infinity $\mathcal{J}$ for null and spacelike geodesics [27].

Let us consider the timelike worldlines in (4) of the form

$$T = \tau\,\cos\chi_0\,, \quad \chi = \chi_0\,, \quad \Theta = \Theta_0\,, \quad \Phi = \Phi_0\,, \tag{5}$$

where τ is the proper time, and χ_0, Θ_0, Φ_0 are constants. The corresponding 4-velocity is $u^\mu = (\cos\chi_0, 0, 0, 0)$, and the 4-acceleration is $\dot{u}^\mu \equiv u^\mu{}_{;\nu}u^\nu = (0, a^{-2}\sin\chi_0\cos\chi_0, 0, 0)$, with the constant modulus

$$|A| \equiv |\,\dot{u}^\mu| = \frac{\sin\chi_0}{a}\,. \tag{6}$$

Moreover, due to the relation $u_\mu\dot{u}^\mu = 0$, this constant value of $|A|$ is identical to the modulus of the 3-acceleration measured in the natural local frame orthogonal to the observer's 4-velocity u^μ. The family of worldlines (5) thus represents the motion of *uniformly accelerating observers in the anti-de Sitter universe*. Their trajectories are given by simple vertical lines in the Penrose diagram shown in Fig. 1. For $\chi_0 = 0$ the worldline is a geodesic with zero acceleration. With growing values of χ_0 the acceleration of the corresponding observers given by (5) uniformly grows. Interestingly, there exists the *maximal possible value* A_{max},

$$|A| \le A_{max} \equiv \sqrt{-\tfrac{1}{3}\Lambda}\,, \tag{7}$$

of uniform acceleration of the privileged observers (5) in the anti-de Sitter space which is reached in the limit $\chi_0 = \frac{\pi}{2}$, i.e. for those moving "along the conformal infinity". This is completely different from the situation in the de Sitter universe, in which case the acceleration A of analogous observers may be unbounded, see e.g. [23], [25], [26]. Note that such infinite acceleration corresponds to (null) observers in the de Siter universe which move along the cosmological horizon. This is absent in the anti-de Sitter space.

In the 5-dimensional formalism the uniformly accelerated trajectories (5) are also privileged. It immediately follows from (3) that these are given by constant values of Z_1 and Z_{23}. Therefore, the trajectories are just simple closed timelike loops of the radius $a/\cos\chi_0$ around the anti-de Sitter hyperboloid indicated in Fig. 1.

We may now introduce a new coordinate system which is well adapted to description of uniformly accelerating test point sources in the anti-de Sitter space. This analogue of the accelerated coordinates for the de Sitter spacetime [25] is given by the following parameterisation of the hyperboloid (1),

$$\begin{aligned}
Z_0 &= \frac{\sqrt{a^2+r^2}\,\sin(T/a)}{\sqrt{1-a^2A^2}+A\,r\cos\theta}\,, & Z_4 &= \frac{\sqrt{a^2+r^2}\,\cos(T/a)}{\sqrt{1-a^2A^2}+A\,r\cos\theta}\,,\\
Z_1 &= \frac{\sqrt{1-a^2A^2}\;r\cos\theta-a^2A}{\sqrt{1-a^2A^2}+A\,r\cos\theta}\,, & Z_{23} &= \frac{r\sin\theta}{\sqrt{1-a^2A^2}+A\,r\cos\theta}\,,
\end{aligned} \tag{8}$$

where $r\in[0,\infty)$, $T\in(-\infty,\infty)$, and $\theta\in[0,\pi]$. In these coordinates, the anti-de Sitter space takes the form

$$\mathrm{d}s^2 = \frac{1}{[\sqrt{1-a^2A^2}+A\,r\cos\theta\,]^2}\left\{-\left(1+r^2/a^2\right)\mathrm{d}T^2+\frac{\mathrm{d}r^2}{1+r^2/a^2}+r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\Phi^2)\right\}. \tag{9}$$

In the case $A=0$ this static metric reduces to the standard form (4) if we perform obvious transformations $r=a\tan\chi$, and $\theta=\Theta$. Moreover, the metric (9) is conformal to this for a general value of the acceleration $A<A_{max}$. The explicit transformation between (4) and (9) is obtained by comparing (3) with (8),

$$\begin{aligned}
\cos\chi &= \frac{\sqrt{1-a^2A^2}+A\,r\cos\theta}{\sqrt{1+r^2/a^2}}\,,\\
a\tan\chi\sin\Theta &= \frac{r\sin\theta}{\sqrt{1-a^2A^2}+A\,r\cos\theta}\,,\\
a\tan\chi\cos\Theta &= \frac{\sqrt{1-a^2A^2}\;r\cos\theta-a^2A}{\sqrt{1-a^2A^2}+A\,r\cos\theta}\,,
\end{aligned} \tag{10}$$

which relate χ,Θ to r,θ (the coordinates T and Φ have the same meaning in both the metrics). It is obvious from the first relation in (10) that the origin $r=0$ of the coordinates (9) corresponds to $\cos\chi_0=\sqrt{1-a^2A^2}$, i.e. $\sin\chi_0=a|A|$. From (6) we immediately conclude that the parameter A in the metric (9) is exactly the value of the acceleration of the corresponding observer. Therefore, *the origin* $r=0$ *of the coordinates in* (9) *is accelerating in anti-de Sitter universe with uniform acceleration* A. It also follows from (10) that the coordinate singularity $r=0$ is located at $\Theta_0=0$ when $A<0$, whereas it is located at $\Theta_0=\pi$ when $A>0$. Considering (8) we finally observe that the trajectory of uniformly accelerating origin $r=0$ corresponds to a uniform motion of a single point around the anti-de Sitter hyperboloid. This closed trajectory is a circle of radius $a/\sqrt{1-a^2A^2}$, which is given by an intersection of the hyperboloid (1) with the plane having the constant values $Z_1=-a^2A/\sqrt{1-a^2A^2}$ and $Z_{23}=0$, see Fig. 1.

3 Physical interpretation of the C-metric with $\Lambda < 0$

The Plebanski–Demianski solution for the C-metric with a cosmological constant can be written in the form [18]

$$\mathrm{d}s^2 = \frac{1}{(p+q)^2}\left(\frac{\mathrm{d}p^2}{\mathcal{P}} + \frac{\mathrm{d}q^2}{\mathcal{Q}} + \mathcal{P}\,\mathrm{d}\sigma^2 - \mathcal{Q}\,\mathrm{d}\tau^2\right) , \tag{11}$$

where

$$\begin{aligned} \mathcal{P}(p) &= A^2 - p^2 + 2m\,p^3 - e^2p^4 , \\ \mathcal{Q}(q) &= \frac{1}{a^2} - A^2 + q^2 + 2m\,q^3 + e^2q^4 . \end{aligned} \tag{12}$$

Note that we do not consider the possible rotation here. Also, we have used the coordinate freedom to remove the linear terms in (12), and to set the coefficients of the quadratic terms to unity. In order to maintain the spacetime signature, it is necessary that $\mathcal{P} > 0$ which places a restriction on the range of p. However, there is no restriction on the sign of $\mathcal{Q}$ which may describe both static and non-static regions, separated by horizons localised on $\mathcal{Q} = 0$. Using the standard "Descartes sign rule" we obtain for $m > 0$ and $|A| < A_{max}$ that the polynomial $\mathcal{P}(p)$ given by (12) has at most *four* real roots p_i (three positive and one negative). On the other hand, the polynomial $\mathcal{Q}(q)$ has at most *two* real (necessarily negative) roots q_i. There are thus four possible static spacetime regions for all permitted values of p and q, as illustrated in Fig. 2 by the shaded areas. These are bounded by horizons q_i and "coordinate" singularities p_i. In the following we shall concentrate on the spacetime for which $p_1 < p < p_2$ and $p+q < 0$ (note that $p+q = 0$ corresponds to the spacetime boundary on which the conformal factor in (11) becomes unbounded, whereas $q = -\infty$ represents a curvature singularity). This covers one non-static plus two static regions separated by inner and outer black hole horizons, $q = q_1$ and $q = q_2$, respectively. Again, this is different from the de Sitter case for which *both* the polynomials $\mathcal{P}$ and $\mathcal{Q}$ may have up to four real roots so that an additional cosmological horizon (on $q = q_3 < 0$) is present.

Let us perform the coordinate transformation of the metric (11) given by

$$\begin{aligned} &T = \sqrt{1-a^2A^2}\,\tau , \qquad r = -\frac{\sqrt{1-a^2A^2}}{q} , \\ &\theta = \int_{\zeta_1} \frac{\mathrm{d}\zeta}{\sqrt{1-\zeta^2+2mA\zeta^3-e^2A^2\zeta^4}} , \qquad \Phi = \frac{A}{c}\,\sigma , \end{aligned} \tag{13}$$

where $\zeta \equiv p/A$, and c is a constant. We obtain

$$\mathrm{d}s^2 = \frac{1}{[\sqrt{1-a^2A^2} - A\,r\,\zeta(\theta)\,]^2}\left\{ -F(r)\,\mathrm{d}T^2 + \frac{\mathrm{d}r^2}{F(r)} + r^2\Big(\mathrm{d}\theta^2 + G^2(\theta)\,c^2\mathrm{d}\Phi^2\Big)\right\} , \tag{14}$$

in which

$$\begin{aligned} F(r) &= 1 + \frac{r^2}{a^2} - \sqrt{1-a^2A^2}\,\frac{2m}{r} + (1-a^2A^2)\,\frac{e^2}{r^2} , \\ G^2(\theta) &= 1 - \zeta^2(\theta) + 2mA\,\zeta^3(\theta) - e^2A^2\,\zeta^4(\theta) , \end{aligned} \tag{15}$$

and $\zeta(\theta)$ is the inverse function of $\theta(\zeta)$ given by the integral in (13). It may be seen that, either when $A = 0$ or when both $m = 0$ and $e = 0$, we obtain $\zeta(\theta) = -\cos\theta$ so that $G(\theta) = \sin\theta$. Otherwise these can be expressed in terms of Jacobian elliptic functions [25].

It is obvious that for $A = 0$, $c = 1$ the metric (14) exactly reduces to the familiar form of the Reissner–Nordstrøm–anti-de Sitter black hole solution in which the parameters m and e have the usual interpretation and the curvature singularity is located at $r = 0$. Moreover, when $A \neq 0$, $c = 1$

and $m = 0 = e$, the line element (14) is *identical* to (9) in which $r = 0$ corresponds to a uniformly accelerating single point in the anti-de Sitter background. When m and e are small, the solution (14) can naturally be regarded as a perturbation of (9). The metric (14) can thus be interpreted as describing *a charged black hole which is uniformly accelerating in anti-de Sitter universe.*

The spacetimes given by the metric (14) explicitly possess the boost and rotation symmetries corresponding to the Killing vectors ∂_T and ∂_Φ. There are Killing horizons where the norm of the Killing vector ∂_T vanishes. These occur on $F = 0$ and separate static and non-static regions of the spacetimes. Generally, for $r > 0$, $m > 0$ there are at most two real roots of $F(r)$ which correspond to the familiar inner and outer black hole horizons in the Reissner–Nordstrøm–anti-de Sitter spacetimes, see e.g. [28]-[30]. Their specific geometrical properties depend not only on the parameters m, e, and Λ, but also on the acceleration A. Contrary to the de Sitter background there is no cosmological horizon. Cases representing uniformly accelerating extreme black holes or naked singularities in anti-de Sitter universe are also described by (14) for specific ranges of the parameters such that the two possible roots of F are repeated or absent.

For $A \neq 0$ and a general choice of the parameters m, e, Λ, the complete global structure of the spacetime is complicated. Of course, for small m and e, the solutions (14) can be considered as perturbations of anti-de Sitter universe illustrated in Fig. 1. However, these pictures now are only schematic since $r = 0$ is not a single "test" point but a curvature singularity, and the black hole horizons also occur. For vanishing acceleration A, the conformal diagrams of the corresponding (spherically symmetric) spacetimes are already known [28]-[30]. Interestingly, these diagrams also describe the global structure of the complete spacetime (14) *in the "equatorial" plane* $\zeta(\theta) = 0$ orthogonal to the direction of acceleration, even in the case when $A \neq 0$. On this plane, $r = \infty$ corresponds to the anti-de Sitter conformal infinity $\mathcal{J}$ which is represented by the boundary $\chi = \frac{\pi}{2}$ in Fig. 1.

We finally clarify the character of the singularities at $\zeta_1 = p_1/A$ and $\zeta_2 = p_2/A$ which are the roots of $G(\zeta) = 0$. It follows from the integral in (13) that ζ_1 corresponds to $\theta = 0$, whereas ζ_2 to $\theta = 2K$, in which K is the "quarter period", i.e. the complete elliptic integral of the first kind related to (13). (When either $A = 0$ or $m = 0 = e$ we obtain $K = \frac{\pi}{2}$.) Therefore, the range of the "angular" coordinates in the metric (14) is $\theta \in [0, 2K]$, $\Phi \in [0, 2\pi)$.

As in the de Sitter case [25] we may compare the circumference of a small circle around the pole $\theta = 0$ with its radius (for any fixed value of r and T). We find that in general there is a deficit angle

$$\delta_1 = 2\pi \left[1 - \lim_{\theta \to 0} \frac{c\, G(\theta)}{\theta} \right] = 2\pi \left[1 - c\, G'(0) \right] \ , \tag{16}$$

where $G'(0) = -\zeta_1 + 3mA\,\zeta_1^2 - 2e^2A^2\,\zeta_1^3$, which is finite and independent of r and T. Thus, the singularity at ζ_1 represents a *cosmic string of constant tension* along the semi-axis $\theta = 0$. Of course, for any value of the physical parameters m, e, A and Λ, this can always be made regular by putting $c = 1/G'(0)$. In particular, if $A = 0$ (the case of spherically symmetric Reissner–Nordstrøm–anti-de Sitter spacetime), or if m and e are both zero (the anti-de Sitter universe in accelerating coordinates), the semi-axis $\theta = 0$ is regular when $c = 1$ because $G = \sin\theta$ in these cases.

The deficit angle of the second cosmic string which extends in the opposite direction $\theta = 2K$ is

$$\delta_2 = 2\pi \left[1 - \lim_{\theta \to 2K} \frac{c\, G(\theta)}{2K - \theta} \right] = 2\pi \left[1 + c\, G'(2K) \right] \ , \tag{17}$$

where $G'(2K) = -\zeta_2 + 3mA\,\zeta_2^2 - 2e^2A^2\,\zeta_2^3$. Again, the string along the semi-axis $\theta = 2K$ can be removed by setting $c = -1/G'(2K)$. However, it is not possible in general to remove the strings in both directions simultaneously unless $G'(0) = -G'(2K)$. This condition can only be satisfied for $m = 0 = e$ or $A = 0$ (in which case $\zeta_2 = 1 = -\zeta_1$) with $c = 1$. Thus, uniformly accelerating black hole in anti-de Sitter universe must necessarily be connected to at least one cosmic string localised at

$\theta = 0$ and/or $\theta = 2K$, which may be considered to "cause" the acceleration (see also [24]). This is in accordance with the same result obtained in Minkowski [4] or de Sitter [23], [25] space. There are no (non-trivial) self-accelerating black holes in the backgrounds of constant curvature.

However, there are fundamental geometrical differences between the cases with positive and negative value of the cosmological constant Λ. In the de Sitter case, there are *two* uniformly accelerating black holes connected each other by finite string(s). In the presence of both strings these make a complete *circular loop* around the whole closed de Sitter universe. In the anti-de Sitter background, there is just a *single* uniformly accelerating black hole attached to *semi-infinite* open string(s). These have a "hyperboloidal" character and connect the black hole to the anti-de Sitter conformal infinity $\mathcal{J}$. Indeed, if we treat the solution (14) for sufficiently small m and e as a perturbation of the anti-de Sitter hyperboloid (1), the first string is localised along $\theta = 0$ whereas the second along $\theta = \pi$. Considering (8) we observe that both these cases correspond to $Z_{23} = 0$. From (1) it follows that the geometry of such strings in anti-de Sitter universe is given by a hyperboloidal 2-surface $(Z_0^2 + Z_4^2) - Z_1^2 = a^2$. The position of the strings at a given time is indicated in Fig. 1 by the wavy line. The first semi-infinite string localised on $\theta = 0$ connects the uniformly accelerated source $r = 0$ to the infinity $\mathcal{J}$ on the "left", $Z_1 > -a^2A/\sqrt{1-a^2A^2}$, while the second string on $\theta = \pi$ extends from the source in the opposite direction to the "right", $Z_1 < -a^2A/\sqrt{1-a^2A^2}$.

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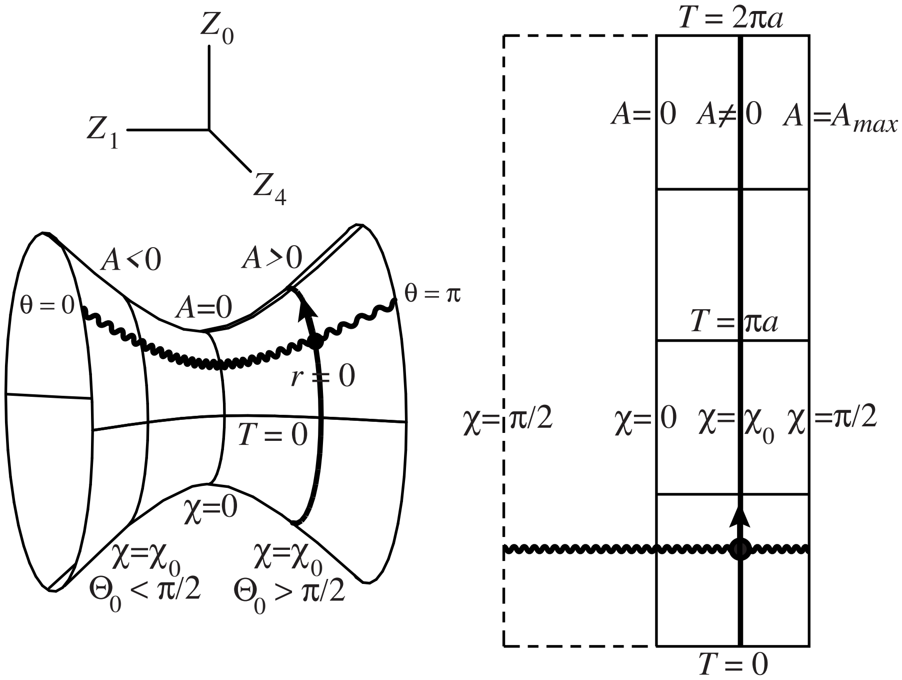

Figure 1: The anti-de Sitter space represented as a hyperboloid for the section $Z_{23} = 0$ (left), and the corresponding Penrose conformal diagram (right). The trajectories of observers with uniform acceleration A given by $\chi = \chi_0$ (i.e., $Z_1 = -a^2 A\sqrt{1 - a^2A^2}$, or $r = 0$) are indicated. The possible two semi-infinite cosmic strings localised on $Z_{23} = 0$ extend from the observer in opposite directions $\theta = 0$ and $\theta = \pi$.

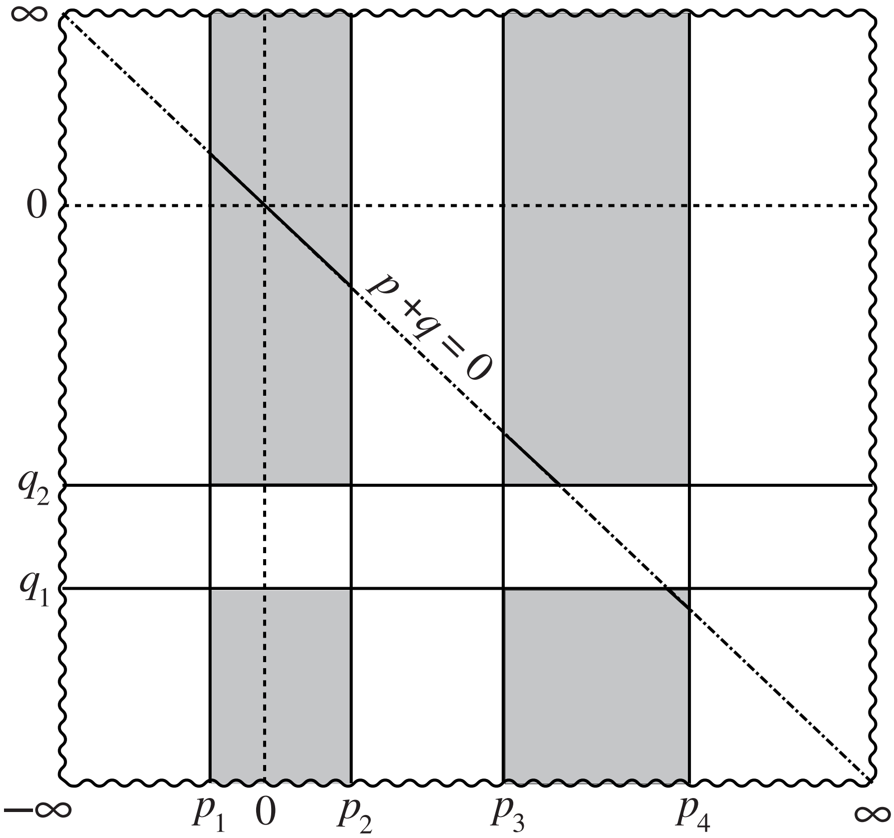

Figure 2: In the full ranges of p and q, physical spacetimes only occur when $p \in (p_1, p_2)$, or $p \in (p_3, p_4)$, where p_i are possible real roots of the polynomial $\mathcal{P}$. There exist at most two real (necessarily negative) roots q_1 and q_2 of the polynomial $\mathcal{Q}$. Generically, there are thus four static spacetime regions which are indicated by the shaded areas. We concentrate here on the spacetime spanned by $p_1 < p < p_2$ and $q < -p$ between the curvature singularity $q = -\infty$ and the conformal infinity $p + q = 0$. This contains one non-static and two static regions which are separated by inner $(q = q_1)$ and outer $(q = q_2)$ black hole horizons.