Gravitational and electromagnetic fields near a de Sitter-like infinity

Pavel Krtouš, Jiří Podolský, and Jiří Bičák
Institute of Theoretical Physics, Faculty of Mathematics and Physics, Charles University in Prague, V Holešovičkách 2, 180 00 Prague 8, Czech Republic
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We present a charecterization of general gravitational and electromagnetic fields near de Sitter-like conformal infinity which supplements the standard peeling behaviour. This is based on an explicit evaluation of the dependence of the radiative component of the fields on the null direction from which infinity is approached. It is shown that the directional pattern of radiation has a universal character that is determined by the algebraic (Petrov) type of the spacetime. Specifically, the radiative field vanishes along directions opposite to principal null directions.

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A direct observation of gravitational waves will be properly understood only when it can be compared with reliable predictions supplied by numerical relativity. To make such predictions is difficult since, among other things, no rigorous statements are available which relate the properties of sufficiently general strong sources to the radiation fields produced. Only a few explicit radiative solutions of Einstein's equations are known which can be used as test beds for numerical codes (see e.g. [1, 2]), in particular, spacetimes representing "uniformly accelerated particles or black holes". These are the main inspiration for our present analysis of the general asymptotic properties of radiation. We concentrate on spacetimes with the positive cosmological constant Λ. The motivation for considering de Sitter-like universes arises, not only by their role in inflationary theories, but also by the fact that many *non-vacuum* cosmological models with $\Lambda>0$ (as suggested by recent observations) approach the de Sitter universe asymptotically in time ("cosmic no-hair conjecture") and hence have a de Sitter-like infinity.

We have recently constructed the test fields of uniformly accelerated charges in de Sitter spacetime (the Born solutions generalized for $\Lambda>0$) and investigated how their radiative properties depend on the way in which infinity is approached [3]. Somewhat surprisingly, we have found analogous results [4] in the case of the exact solution of the Einstein-Maxwell equations with $\Lambda>0$, namely the "charged C-metric" describing a pair of charged accelerated black holes in a de Sitter universe.

In the following, we shall demonstrate that the directional pattern of radiation near an infinity of de Sitter type has a universal character that is determined by the algebraic (Petrov) type of the spacetime. In spacetimes which are asymptotically de Sitter, there are two disjoint past and future (conformal) infinities $\mathcal{I}^-$ and $\mathcal{I}^+$, both spacelike [5, 6]. In such spacetimes both cosmological horizons and event horizons for geodesic observers occur and, consequently, advanced effects have to be present if the fields are smooth [7]. Curiously enough, with $\Lambda>0$ the global existence has been established of asymptotically simple vacuum solutions which differ on an arbitrary given Cauchy surface by a finite but sufficiently small amount from de Sitter data [8], while an analogous result for data close to Minkowski ($\Lambda=0$) is still under investigation [2, 8]. Thus, many vacuum asymptotically simple spacetimes with de Sitter-like $\mathcal{I}^+$ do exist. Assuming their existence, Penrose proved already in 1965 [5, 6] that both the gravitational and the electromagnetic fields satisfy the peeling-off property with respect to null geodesics reaching any point of $\mathcal{I}^+$. This means that along a null geodesic parametrized by an affine parameter η, the part of any spin s zero rest-mass field proportional to $\eta^{-(k+1)}$, $k=0,1,\ldots,2s$, has, in general, $2s-k$ coincident principal null directions. In particular, the part of the field that falls off as η^{-1} is a *radiation* ("null") *field.* The peeling-off property is easier to prove with $\Lambda>0$ than in asymptotically Minkowskian spacetimes when $\mathcal{I}^+$ is a null hypersurface [5]. With a spacelike $\mathcal{I}^+$, however, one can approach any point on $\mathcal{I}^+$ from infinitely many different null directions and, consequently, the radiation field becomes mixed up with other components of the field when the null geodesic is changed. This fact of the "origin dependence" of the radiation field in case of a spacelike $\mathcal{I}^+$ has been repeatedly emphasized by Penrose [5, 6]. Exactly this *directional radiation pattern*, i.e., the dependence of fields (with respect to appropriate tetrad) on the direction along which the null geodesic reaches a point on a spacelike $\mathcal{I}^+$, is analyzed in the present work.

SPACETIME INFINITY, FIELDS AND TETRADS

Following general formalism [5, 6], a spacetime $\mathcal{M}$ with physical metric $\boldsymbol{g}$ can be embedded into a larger *conformal manifold* $\tilde{\mathcal{M}}$ with *conformal metric* $\tilde{\boldsymbol{g}}$ related to $\boldsymbol{g}$ by $\tilde{\boldsymbol{g}}=\omega^2\boldsymbol{g}$. Here, the conformal factor ω, negative in $\mathcal{M}$, vanishes on the boundary of $\mathcal{M}$ in $\tilde{\mathcal{M}}$ called conformal infinity $\mathcal{I}$. It is *spacelike* if the gradient $\mathbf{d}\omega$ on $\mathcal{I}$ has timelike character. This may be either future infinity $\mathcal{I}^+$ or past infinity $\mathcal{I}^-$. Near $\mathcal{I}^+$ we decompose $\tilde{\boldsymbol{g}}$ into a spatial 3-metric ${}^{\mathcal{I}}\tilde{\boldsymbol{g}}$ tangent to $\mathcal{I}^+$ and a part orthogonal to $\mathcal{I}^+$

$$\boldsymbol{g}=\omega^{-2}(-\tilde{N}^2\,\mathbf{d}\omega^2+{}^{\mathcal{I}}\tilde{\boldsymbol{g}})\,. \tag{1}$$

The conformal lapse function $\tilde{N}$ can be chosen to be constant on $\mathcal{I}^+$, e.g., equal to $\ell = \sqrt{3/\Lambda}$. The form (1) allows us to define a timelike unit vector $\mathbf{n}$ normal to $\mathcal{I}^+$,

$$\mathbf{n}^\mu = \omega^{-1} \tilde{N}\, \boldsymbol{g}^{\mu\nu}\, \mathbf{d}_\nu \omega \ . \tag{2}$$

Next, we denote the vectors of an *orthonormal* tetrad as $\mathbf{t}$, $\mathbf{q}$, $\mathbf{r}$, $\mathbf{s}$, where $\mathbf{t}$ is a unit timelike vector and the remaining three are unit spacelike vectors. With this tetrad we associate a *null* tetrad $\mathbf{k}$, $\mathbf{l}$, $\mathbf{m}$, $\bar{\mathbf{m}}$, such that $\mathbf{k}\cdot\mathbf{l} = -1$, $\mathbf{m}\cdot\bar{\mathbf{m}} = 1$,

$$\begin{aligned} \mathbf{k} &= \tfrac{1}{\sqrt{2}}(\mathbf{t}+\mathbf{q}) \ , & \mathbf{l} &= \tfrac{1}{\sqrt{2}}(\mathbf{t}-\mathbf{q}) \ , \\ \mathbf{m} &= \tfrac{1}{\sqrt{2}}(\mathbf{r}-i\,\mathbf{s}) \ , & \bar{\mathbf{m}} &= \tfrac{1}{\sqrt{2}}(\mathbf{r}+i\,\mathbf{s}) \ . \end{aligned} \tag{3}$$

Various specific tetrads introduced below will be distinguished by an additional label in subscript.

As usual, we parametrize the Weyl tensor $\mathbf{C}_{\alpha\beta\gamma\delta}$ (representing the gravitational field) by 5 complex coefficients

$$\begin{gathered} \Psi_0 = \mathbf{C}_{\alpha\beta\gamma\delta}\, \mathbf{k}^\alpha\, \mathbf{m}^\beta\, \mathbf{k}^\gamma\, \mathbf{m}^\delta \ , \quad \Psi_1 = \mathbf{C}_{\alpha\beta\gamma\delta}\, \mathbf{k}^\alpha\, \mathbf{l}^\beta\, \mathbf{k}^\gamma\, \mathbf{m}^\delta \ , \\ \Psi_2 = -\mathbf{C}_{\alpha\beta\gamma\delta}\, \mathbf{k}^\alpha\, \mathbf{m}^\beta\, \mathbf{l}^\gamma\, \bar{\mathbf{m}}^\delta \ , \\ \Psi_3 = \mathbf{C}_{\alpha\beta\gamma\delta}\, \mathbf{l}^\alpha\, \mathbf{k}^\beta\, \mathbf{l}^\gamma\, \bar{\mathbf{m}}^\delta \ , \quad \Psi_4 = \mathbf{C}_{\alpha\beta\gamma\delta}\, \mathbf{l}^\alpha\, \bar{\mathbf{m}}^\beta\, \mathbf{l}^\gamma\, \bar{\mathbf{m}}^\delta \ , \end{gathered} \tag{4}$$

and the electromagnetic field $\mathbf{F}_{\alpha\beta}$ by 3 complex coefficients

$$\begin{gathered} \Phi_0 = \mathbf{F}_{\alpha\beta}\, \mathbf{k}^\alpha\, \mathbf{m}^\beta \ , \quad \Phi_2 = \mathbf{F}_{\alpha\beta}\, \bar{\mathbf{m}}^\alpha\, \mathbf{l}^\beta \ , \\ \Phi_1 = \tfrac{1}{2}\, \mathbf{F}_{\alpha\beta} \left(\mathbf{k}^\alpha\, \mathbf{l}^\beta - \mathbf{m}^\alpha\, \bar{\mathbf{m}}^\beta\right) \ . \end{gathered} \tag{5}$$

These simply transform under special Lorentz transformations — null rotations with $\mathbf{k}$ or $\mathbf{l}$ fixed, boosts in $\mathbf{k}$-$\mathbf{l}$ plane, and spatial rotations in $\mathbf{m}$-$\bar{\mathbf{m}}$ plane. For instance, under a null rotation with $\mathbf{l}$ fixed, parametrized by $K \in \mathbb{C}$:

$$\begin{gathered} \mathbf{l} = \mathbf{l}_\mathrm{o} \ , \quad \mathbf{k} = \mathbf{k}_\mathrm{o} + \bar{K}\mathbf{m}_\mathrm{o} + K\bar{\mathbf{m}}_\mathrm{o} + K\bar{K}\,\mathbf{l}_\mathrm{o} \ , \\ \mathbf{m} = \mathbf{m}_\mathrm{o} + K\,\mathbf{l}_\mathrm{o} \ , \quad \bar{\mathbf{m}} = \bar{\mathbf{m}}_\mathrm{o} + \bar{K}\,\mathbf{l}_\mathrm{o} \ , \end{gathered} \tag{6}$$

$$\begin{aligned} \Psi_0 &= K^4\Psi_4^\mathrm{o} + 4K^3\Psi_3^\mathrm{o} + 6K^2\Psi_2^\mathrm{o} + 4K\,\Psi_1^\mathrm{o} + \Psi_0^\mathrm{o} \ , \\ \Phi_0 &= K^2\Phi_2^\mathrm{o} + 2K\,\Phi_1^\mathrm{o} + \Phi_0^\mathrm{o} \ . \end{aligned} \tag{7}$$

Similarly, null rotations with $\mathbf{k}$ fixed can be parametrized by $L \in \mathbb{C}$. For boosts in $\mathbf{k}$-$\mathbf{l}$ plane, cf. Eqs. (12), (13). For details and notation see, e.g., Ref. [4, 9].

Our goal is to investigate the field components in an appropriate interpretation tetrad parallelly transported along all null geodesics $z(\eta)$ which terminate at $\mathcal{I}^+$ at a point P_+. A geodesic reaches $\mathcal{I}^+$ at an infinite value of the affine parameter η. The conformal factor ω and lapse $\tilde{N}$ can be expanded along the geodesic in powers of $1/\eta$:

$$\omega \approx \omega_* \eta^{-1} + \dots \ , \quad \tilde{N} \approx \tilde{N}_+ + \dots \ . \tag{8}$$

The value $\tilde{N}_+ = \tilde{N}|_{P_+}$ is the same for all geodesics ending at point P_+. We require that the approach of geodesics to $\mathcal{I}^+$ is "comparable", independent of their *direction*, so we assume also ω_* to be constant. This is equivalent to fixing the energy $E_\mathrm{o} = -\mathbf{p}\cdot\mathbf{n}$ ($\mathbf{p} = \frac{Dz}{d\eta}$ being 4-momentum) at a given small value of ω, i.e., at given proximity from the conformal infinity [4].

To define an *interpretation null tetrad* $\mathbf{k}_\mathrm{i}, \mathbf{l}_\mathrm{i}, \mathbf{m}_\mathrm{i}, \bar{\mathbf{m}}_\mathrm{i}$ we have to specify it in a comparable way for all geodesics along different directions. The geodesics reach the same point P_+ and we prescribe its form there. We require the null vector $\mathbf{k}_\mathrm{i}$ to be proportional to the tangent vector of the geodesic,

$$\mathbf{k}_\mathrm{i} = \frac{1}{\sqrt{2}\tilde{N}_+}\, \frac{Dz}{d\eta} \ ; \tag{9}$$

the factor is again chosen independent of the direction. The null vector $\mathbf{l}_\mathrm{i}$ is fixed by $\mathbf{k}_\mathrm{i}\cdot\mathbf{l}_\mathrm{i} = -1$ and by the requirement that the normal vector $\mathbf{n}$ belongs to the $\mathbf{k}_\mathrm{i}$-$\mathbf{l}_\mathrm{i}$ plane. The vectors $\mathbf{m}_\mathrm{i}$, $\bar{\mathbf{m}}_\mathrm{i}$ (or, $\mathbf{r}_\mathrm{i}$, $\mathbf{s}_\mathrm{i}$) cannot be specified canonically — they will be chosen by Eqs. (12), (16).

Now, the projection of $\mathbf{k}_\mathrm{i}$ on the normal $\mathbf{n}$ is

$$-\mathbf{k}_\mathrm{i}\cdot\mathbf{n} \approx \tfrac{1}{\sqrt{2}}\,\eta^{-1} \ , \tag{10}$$

so that, as $\eta \to \infty$, the interpretation tetrad is "infinitely boosted" with respect to an observer with 4-velocity $\mathbf{n}$. To see this explicitly, we introduce an auxiliary tetrad $\mathbf{t}_\mathrm{r}$, $\mathbf{q}_\mathrm{r}$, $\mathbf{r}_\mathrm{r}$, $\mathbf{s}_\mathrm{r}$ adapted to the conformal infinity, $\mathbf{t}_\mathrm{r} = \mathbf{n}$, with $\mathbf{q}_\mathrm{r}$ oriented along the spatial direction of the geodesic,

$$\mathbf{q}_\mathrm{r} \propto \mathbf{k}_\mathrm{i}^\perp = (\mathbf{k}_\mathrm{i}\cdot\mathbf{n})\,\mathbf{n} + \mathbf{k}_\mathrm{i} \ , \tag{11}$$

and we choose the remaining spatial vectors $\mathbf{r}_\mathrm{r}, \mathbf{s}_\mathrm{r}$ to coincide with those of the interpretation tetrad. Using Eqs. (9), (10) and the definition of $\mathbf{l}_\mathrm{i}$, we get

$$\begin{aligned} \mathbf{k}_\mathrm{i} &= B_\mathrm{i}\,\mathbf{k}_\mathrm{r} = \eta^{-1}\tfrac{1}{\sqrt{2}}\,(\mathbf{n}+\mathbf{q}_\mathrm{r}) \ , & \mathbf{m}_\mathrm{i} &= \mathbf{m}_\mathrm{r} \ , \\ \mathbf{l}_\mathrm{i} &= B_\mathrm{i}^{-1}\,\mathbf{l}_\mathrm{r} = \ \ \eta\,\tfrac{1}{\sqrt{2}}\,(\mathbf{n}-\mathbf{q}_\mathrm{r}) \ , & \bar{\mathbf{m}}_\mathrm{i} &= \bar{\mathbf{m}}_\mathrm{r} \ , \end{aligned} \tag{12}$$

$B_\mathrm{i} = 1/\eta$ being the boost parameter. Under such a boost the field components (4), (5) transform as [4, 9]

$$\Psi^\mathrm{i}_j = B_\mathrm{i}^{2-j}\,\Psi^\mathrm{r}_j \ , \quad \Phi^\mathrm{i}_j = B_\mathrm{i}^{1-j}\,\Phi^\mathrm{r}_j \ . \tag{13}$$

Together with behavior (22) of the field components in any tetrad adapted to infinity we thus obtain the peeling-off property.

DIRECTIONAL PATTERN OF RADIATION

We shall now derive the directional dependence of radiation near a point P_+ at $\mathcal{I}^+$. It is necessary to parametrize the direction of the null geodesic reaching P_+. This can be done with respect to a suitable *reference tetrad* $\mathbf{t}_\mathrm{o}$, $\mathbf{q}_\mathrm{o}$, $\mathbf{r}_\mathrm{o}$, $\mathbf{s}_\mathrm{o}$, with the time vector $\mathbf{t}_\mathrm{o}$ adapted to the conformal infinity, $\mathbf{t}_\mathrm{o} = \mathbf{n}$, and spatial directions chosen arbitrarily. It is convenient to choose them in accordance with the spacetime geometry. Privileged choices will be discussed later (cf. Fig. 1).

The unit spatial direction $\mathbf{q}_\mathrm{r}$ of a general null geodesic near $\mathcal{I}^+$ can be expressed in terms of *spherical angles* θ, ϕ, with respect to the reference tetrad,

$$\mathbf{q}_\mathrm{r} = \cos\theta\;\mathbf{q}_\mathrm{o} + \sin\theta\cos\phi\;\mathbf{r}_\mathrm{o} + \sin\theta\sin\phi\;\mathbf{s}_\mathrm{o} \ . \tag{14}$$

It is useful to introduce the *stereographic representation* of the angles θ, ϕ,

$$R = \tan(\theta/2)\, \exp(-i\phi)\ . \tag{15}$$

Then the null rotation (6) with $K = R$ transforms $\mathbf{k}_{\rm o}$ into $\mathbf{k}$ with its spatial direction $\mathbf{k}^{\perp} \propto \mathbf{q}_{\rm r}$ specified by θ, ϕ.

Now, the interpretation tetrad is related by boost (12) to the tetrad $\mathbf{t}_{\rm r}$, $\mathbf{q}_{\rm r}$, $\mathbf{r}_{\rm r}$, $\mathbf{s}_{\rm r}$, which is a spatial rotation of the reference tetrad. If we choose

$$\begin{aligned} \mathbf{r}_{\rm r} &= -\sin\theta\cos\phi\,\left(\mathbf{q}_{\rm o} + \tan\tfrac{\theta}{2}\,(\cos\phi\,\mathbf{r}_{\rm o} + \sin\phi\,\mathbf{s}_{\rm o})\right) + \mathbf{r}_{\rm o}\ ,\\ \mathbf{s}_{\rm r} &= -\sin\theta\sin\phi\,\left(\mathbf{q}_{\rm o} + \tan\tfrac{\theta}{2}\,(\cos\phi\,\mathbf{r}_{\rm o} + \sin\phi\,\mathbf{s}_{\rm o})\right) + \mathbf{s}_{\rm o}\ , \end{aligned} \tag{16}$$

the spatial rotation is a composition of the null rotations with $\mathbf{l}$ fixed, $\mathbf{k}$ fixed and the boost given by parameters $K = R$, $L = -R/(1+|R|^2)$, $B = (1+|R|^2)^{-1}$. This decomposition into elementary Lorentz transformations enables us to calculate the field components in the interpretation tetrad. We start with $\Psi_j^{\rm o}$, or $\Phi_j^{\rm o}$ in the reference tetrad and we characterize these in terms of algebraically privileged *principal null directions* (PNDs).

Principal null directions of the gravitational (or electromagnetic) field are null directions $\mathbf{k}$ such that $\Psi_0 = 0$ (or $\Phi_0 = 0$) in a null tetrad $\mathbf{k}, \mathbf{l}, \mathbf{m}, \bar{\mathbf{m}}$ (a choice of $\mathbf{l}, \mathbf{m}, \bar{\mathbf{m}}$ is irrelevant). In the tetrad related to the reference tetrad by null rotation (6), such a condition for Ψ_0 takes the form of a quartic (or quadratic for Φ_0) equation for K, cf. Eqs. (7). The roots $K = R_n$, $n = 1,2,3,4$ (or $K = R_n^{\rm E}$, $n = 1,2$) of this equation thus parametrize PNDs $\mathbf{k}_n$ (or $\mathbf{k}_n^{\rm E}$). As follows from the note after Eq. (15), the angles θ_n, ϕ_n of these PNDs are related to R_n exactly by Eq. (15).

In a generic situation we have $\Psi_4^{\rm o} \neq 0$ (or $\Phi_2^{\rm o} \neq 0$) and we can express the remaining components of the Weyl (or electromagnetic) tensor in terms of roots R_n (or $R_n^{\rm E}$)

$$\begin{aligned} \Psi_3^{\rm o} &= -\tfrac{1}{4}\,\Psi_4^{\rm o}\,(R_1 + R_2 + R_3 + R_4)\ ,\\ \Psi_2^{\rm o} &= \ \ \tfrac{1}{6}\,\Psi_4^{\rm o}\,(R_1R_2 + R_1R_3 + R_1R_4 + \\ &\qquad\qquad R_2R_3 + R_2R_4 + R_3R_4)\ ,\\ \Psi_1^{\rm o} &= -\tfrac{1}{4}\,\Psi_4^{\rm o}\,(R_1R_2R_3 + R_1R_2R_4 + \\ &\qquad\qquad R_1R_3R_4 + R_2R_3R_4)\ ,\\ \Psi_0^{\rm o} &= \ \ \Psi_4^{\rm o}\,R_1R_2R_3R_4\ , \end{aligned} \tag{17}$$

$$\Phi_1^{\rm o} = -\tfrac{1}{2}\,\Phi_2^{\rm o}\,(R_1^{\rm E} + R_2^{\rm E})\ ,\quad \Phi_0^{\rm o} = \Phi_2^{\rm o}\,R_1^{\rm E}R_2^{\rm E}\ . \tag{18}$$

Transforming these to tetrad $\mathbf{k}_{\rm r}$, $\mathbf{l}_{\rm r}$, $\mathbf{m}_{\rm r}$, $\bar{\mathbf{m}}_{\rm r}$ we obtain

$$\begin{aligned} \Psi_4^{\rm r} = \Psi_4^{\rm o}\,&\left(1+|R|^2\right)^{-2}\\ &\times\left(1-\tfrac{R_1}{R_{\rm a}}\right)\left(1-\tfrac{R_2}{R_{\rm a}}\right)\left(1-\tfrac{R_3}{R_{\rm a}}\right)\left(1-\tfrac{R_4}{R_{\rm a}}\right)\ , \end{aligned} \tag{19}$$

$$\Phi_2^{\rm r} = \Phi_2^{\rm o}\left(1+|R|^2\right)^{-1}\left(1-\tfrac{R_1^{\rm E}}{R_{\rm a}}\right)\left(1-\tfrac{R_2^{\rm E}}{R_{\rm a}}\right)\ . \tag{20}$$

Here, the complex number $R_{\rm a}$,

$$R_{\rm a} = -\bar{R}^{-1} = -\cot(\theta/2)\,\exp(-i\phi)\ , \tag{21}$$

characterizes a spatial direction *opposite* to the direction given by R, i.e., the *antipodal* direction with $\theta_{\rm a} = \pi - \theta$ and $\phi_{\rm a} = \phi + \pi$. Finally, we express the leading term of the field components in the interpretation tetrad. The freedom in the choice of the vectors $\mathbf{m}_{\rm i}$, $\bar{\mathbf{m}}_{\rm i}$ changes just a phase of the field components, so only their modulus has a physical meaning. It is also known [5, 6] that as a consequence of field equations, field components in a reference tetrad near $\mathcal{I}^+$ behave as

$$\Psi_j^{\rm o} \approx \Psi_{j*}^{\rm o}\,\eta^{-3}\ ,\quad \Phi_j^{\rm o} \approx \Phi_{j*}^{\rm o}\,\eta^{-2}\ . \tag{22}$$

Combining Eqs. (13) and (19), (20), we thus obtain

$$\begin{aligned} \left|\Psi_4^{\rm i}\right| \approx |\Psi_{4*}^{\rm o}|\,&\eta^{-1}\,\cos^4(\theta/2)\\ &\times\left|1-\tfrac{R_1}{R_{\rm a}}\right|\left|1-\tfrac{R_2}{R_{\rm a}}\right|\left|1-\tfrac{R_3}{R_{\rm a}}\right|\left|1-\tfrac{R_4}{R_{\rm a}}\right|\ , \end{aligned} \tag{23}$$

$$\left|\Phi_2^{\rm i}\right| \approx |\Phi_{2*}^{\rm o}|\,\eta^{-1}\,\cos^2(\theta/2)\,\left|1-\tfrac{R_1^{\rm E}}{R_{\rm a}}\right|\left|1-\tfrac{R_2^{\rm E}}{R_{\rm a}}\right|\ . \tag{24}$$

DISCUSSION

These expressions characterize the asymptotic behavior of fields near de Sitter-like infinity. In a general spacetime there are *four* spatial directions along which the radiative component of the gravitational field $\Psi_4^{\rm i}$ vanishes, namely directions $R_{\rm a} = R_n$, $n = 1,2,3,4$ (or *two* such directions for electromagnetic field $\Phi_2^{\rm i}$). In fact, the spatial parts of these directions, $-\mathbf{k}_n^{\perp}$, are exactly *opposite to the projections of the principal null directions* $\mathbf{k}_n^{\perp}$ *onto* $\mathcal{I}^+$.

In algebraically special spacetimes, some PNDs coincide and Eq. (23) simplifies. Moreover, it is always possible to choose the "canonical" reference tetrad: (i) the vector $\mathbf{q}_{\rm o}$ oriented along the spatial projection of the *degenerate* (multiple) PND, say $\mathbf{k}_4^{\perp}$ (i.e. $\mathbf{k}_{\rm o} = \mathbf{k}_4$); (ii) the $\mathbf{q}_{\rm o}$-$\mathbf{r}_{\rm o}$ plane oriented so that it contains the spatial projection of one of the remaining PNDs (for type N spacetimes this choice is arbitrary). Using such a reference tetrad, the degenerate PND $\mathbf{k}_4^{\perp}$ is given by $\theta_4 = 0$, i.e., $R_4 = 0$ (cf. Eqs. (14), (15)), whereas one of the remaining PNDs, say $\mathbf{k}_1^{\perp}$, has $\phi_1 = 0$, i.e. $R_1 = \tan\frac{\theta_1}{2}$ is a real constant.

Thus, for Petrov type N spacetimes (with quadruple PND) $R_1 = R_2 = R_3 = R_4 = 0$, so the asymptotic behavior of gravitational field (23) becomes

$$\left|\Psi_4^{\rm i}\right| = |\Psi_{4*}^{\rm o}|\,\eta^{-1}\,\cos^4\tfrac{\theta}{2}\ . \tag{25}$$

The corresponding directional pattern of radiation is illustrated in Fig. 1(N). It is axisymmetric, with maximum value at $\theta = 0$ along the spatial projection of the quadruple PND onto $\mathcal{I}^+$. Along the opposite direction, $\theta = \pi$, the field vanishes.

In the Petrov type III spacetimes, $R_1 = \tan\frac{\theta_1}{2}$, $R_2 = R_3 = R_4 = 0$, so (23) implies

$$\left|\Psi_4^{\rm i}\right| = |\Psi_{4*}^{\rm o}|\,\eta^{-1}\,\cos^4\tfrac{\theta}{2}\,\left|1 + \tan\tfrac{\theta_1}{2}\,\tan\tfrac{\theta}{2}\,e^{i\phi}\right|\ . \tag{26}$$

The directional pattern of radiation is shown in Fig. 1(III). The field vanishes along $\theta = \pi$ and along $\theta = \pi - \theta_1$, $\phi = \pi$ which are spatial directions opposite to the PNDs.

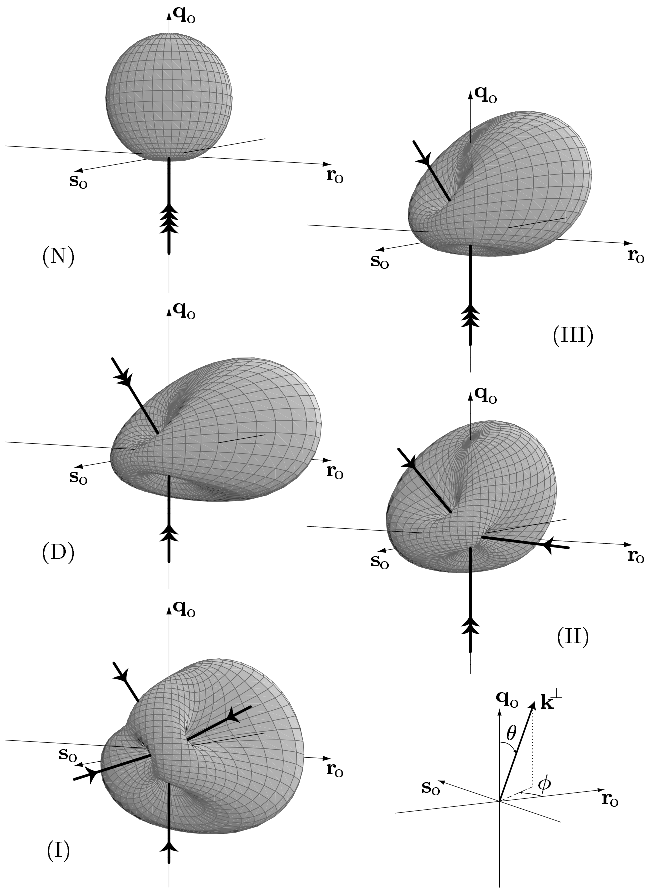

FIG. 1: Specific directional patterns of radiation for spacetimes of Petrov types N, III, D, II and I. Directions in the diagrams represent all spatial directions tangent to $\mathcal{I}^+$. For each type, the radiative component $\left|\Psi_4^{\mathrm{i}}\right|$ along a null geodesic is depicted in the corresponding spatial direction $\mathbf{k}^{\perp}$ parametrized by spherical angles θ, ϕ. [Degenerate] principal null directions (PNDs) are indicated by [multiple] bold arrows. Thick lines represent spatial directions (opposite to PNDs) along which the radiation vanishes.

The type D spacetimes admit two double degenerate PNDs, $R_1 = R_2 = \tan\frac{\theta_1}{2}$ and $R_3 = R_4 = 0$. The gravitational field near $\mathcal{I}^+$ thus takes the form

$$\left|\Psi_4^{\mathrm{i}}\right| = \left|\Psi_{4*}^{\mathrm{o}}\right| \eta^{-1} \cos^4\tfrac{\theta}{2} \left|1 + \tan\tfrac{\theta_1}{2} \tan\tfrac{\theta}{2}\, e^{i\phi}\right|^2 , \quad (27)$$

with two planes of symmetry, see Fig. 1(D). This directional dependence agrees with the radiation pattern for the C-metric with $\Lambda > 0$ derived recently [4].

For Petrov type II spacetimes only two PNDs coincide so that $R_1 = \tan\frac{\theta_1}{2}$, $R_2 = \tan\frac{\theta_2}{2}\exp(-i\phi_2)$, $R_3 = R_4 = 0$. The directional pattern of radiation is in Fig. 1(II). Finally, in case of algebraically general spacetimes one needs five real parameters to characterize the directional dependence Fig. 1(I) of the gravitational field.

An analogous discussion can be presented for the electromagnetic field. The square of expression (24) is, in fact, the Poynting vector with respect to the interpretation tetrad, $|\mathbf{S}_{\mathrm{i}}| \approx \frac{1}{4\pi}\left|\Phi_2^{\mathrm{i}}\right|^2$. If the two PNDs coincide ($R_1^{\mathrm{E}} = R_2^{\mathrm{E}} = 0$) the directional dependence of the Poynting vector at $\mathcal{I}^+$ is the same as in Eq. (25) (Fig. 1(N)); if they differ ($R_1^{\mathrm{E}} = \tan\frac{\theta_1}{2}$, $R_2^{\mathrm{E}} = 0$), the angular dependence of $|\mathbf{S}_{\mathrm{i}}|$ is given by Eq. (27) (Fig. 1(D)). The latter has already been obtained for the test field of uniformly accelerated charges in de Sitter spacetime [3].

To summarize: it is well-known that for *null* $\mathcal{I}^+$ the null direction $\mathbf{l}_{\mathrm{i}}$, which is complementary to the tangent vector $\mathbf{k}_{\mathrm{i}}$ of the null geodesic $z(\eta)$ reaching $\mathcal{I}^+$, is tangent to $\mathcal{I}^+$ and does not depend on the choice of $z(\eta)$. This is not the case when $\mathcal{I}^+$ is spacelike. The radiation field (η^{-1} term) is thus "less invariant" [6]. We have shown that the dependence of this field on the choice of $z(\eta)$ has a universal character that is determined by the algebraic (Petrov) type of the fields. In particular, we have proved that the radiation vanishes along directions opposite to PNDs. Near de Sitter-like $\mathcal{I}^+$ the radiative component of the fields generated by *any* source is nonvanishing in a *generic* direction. Thus, unlike in asymptotically flat spacetimes, the absence of η^{-1} component cannot be used to distinguish nonradiative sources. For a de Sitter-like infinity, the radiative component reflects not only properties of the sources but also their "kinematic" relation to an observer at infinity. Intuitively, near spacelike $\mathcal{I}^+$ our observer is, in general, moving "nonradially" from sources and thus measures "infinitely boosted" fields. Some important questions, such as how energy is radiated away in asymptotically de-Sitter spacetimes, still remain open. Since vacuum energy seems to be dominant in our universe, these appear to be of considerable interest.

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