

Table of Clebsch–Gordan coefficients

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This is a **table of Clebsch–Gordan coefficients** used for adding **angular momentum** values in **quantum mechanics**. The overall sign of the coefficients for each set of constant j_1, j_2, j is arbitrary to some degree and has been fixed according to the Condon–Shortley and Wigner sign convention as discussed by Baird and Biedenharn.^[1] Tables with the same sign convention may be found in the *Particle Data Group's Review of Particle Properties*^[2] and in online tables.^[3]

Formulation [[edit](#)]

The Clebsch–Gordan coefficients are the solutions to

$$|j_1, j_2; J, M\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} |j_1, m_1; j_2, m_2\rangle \langle j_1, j_2; m_1, m_2 | j_1, j_2; J, M\rangle$$

Explicitly:

$$\begin{aligned} & \langle j_1, j_2; m_1, m_2 | j_1, j_2; J, M\rangle \\ &= \delta_{M, m_1+m_2} \sqrt{\frac{(2J+1)(J+j_1-j_2)!(J-j_1+j_2)!(j_1+j_2-J)!}{(j_1+j_2+J+1)!}} \times \\ & \sqrt{(J+M)!(J-M)!(j_1-m_1)!(j_1+m_1)!(j_2-m_2)!(j_2+m_2)!} \times \\ & \sum_k \frac{(-1)^k}{k!(j_1+j_2-J-k)!(j_1-m_1-k)!(j_2+m_2-k)!(J-j_2+m_1+k)!(J-j_1-m_2+k)!}. \end{aligned}$$

The summation is extended over all integer k for which the argument of every factorial is nonnegative.^[4]

For brevity, solutions with $M < 0$ and $j_1 < j_2$ are omitted. They may be calculated using the simple relations

$$\langle j_1, j_2; m_1, m_2 | j_1, j_2; J, M\rangle = (-1)^{J-j_1-j_2} \langle j_1, j_2; -m_1, -m_2 | j_1, j_2; J, -M\rangle.$$

and

$$\langle j_1, j_2; m_1, m_2 | j_1, j_2; J, M\rangle = (-1)^{J-j_1-j_2} \langle j_2, j_1; m_2, m_1 | j_2, j_1; J, M\rangle.$$

$j_2 = 0$ [[edit](#)]

When $j_2 = 0$, the Clebsch–Gordan coefficients are given by $\delta_{j,j_1} \delta_{m,m_1}$.

$j_1 = \frac{1}{2}, j_2 = \frac{1}{2}$ [[edit](#)]

$m = 1$

	j	1
m_1, m_2		
$\frac{1}{2}, \frac{1}{2}$		1

$m = -1$

	j	1
m_1, m_2		
$-\frac{1}{2}, -\frac{1}{2}$		1

$m = 0$

	j	1	0
m_1, m_2			
$\frac{1}{2}, -\frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$-\frac{1}{2}, \frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	

$j_1 = 1, j_2 = \frac{1}{2}$ [[edit](#)]

$m = \frac{3}{2}$

	j	$\frac{3}{2}$
m_1, m_2		
$1, \frac{1}{2}$		1

$m = \frac{1}{2}$

	j	$\frac{3}{2}$	$\frac{1}{2}$
m_1, m_2			
$1, -\frac{1}{2}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$	
$0, \frac{1}{2}$	$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$	

$j_1 = \frac{3}{2}, j_2 = \frac{1}{2}$ [[edit](#)]

$m = 2$

	j	2
m_1, m_2		
$\frac{3}{2}, \frac{1}{2}$		1

$m = 1$

	j	2	1
m_1, m_2			
$\frac{3}{2}, -\frac{1}{2}$	$\frac{1}{2}$	$\sqrt{\frac{3}{4}}$	
$\frac{1}{2}, \frac{1}{2}$	$\sqrt{\frac{3}{4}}$	$-\frac{1}{2}$	

$m = 0$

	j	2	1
m_1, m_2			
$\frac{1}{2}, -\frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$-\frac{1}{2}, \frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	

$j_1 = 2, j_2 = \frac{1}{2}$ [[edit](#)]

$m = \frac{5}{2}$

	j	$\frac{5}{2}$
m_1, m_2		
$2, \frac{1}{2}$		1

$m = \frac{3}{2}$

	j	$\frac{5}{2}$	$\frac{3}{2}$
m_1, m_2			
$2, -\frac{1}{2}$	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$	
$1, \frac{1}{2}$	$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$	

$m = \frac{1}{2}$

	j	$\frac{5}{2}$	$\frac{3}{2}$
m_1, m_2			
$1, -\frac{1}{2}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$	
$0, \frac{1}{2}$	$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$	

$$j_1 = 2, j_2 = 1 \quad [\text{edit}]$$

$$m = 3$$

m_1, m_2	j	3
2, 1		1

$$m = 2$$

m_1, m_2	j	3	2
2, 0		$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$
1, 1		$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$

$$m = 1$$

m_1, m_2	j	3	2	1
2, -1		$\sqrt{\frac{1}{15}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{3}{5}}$
1, 0		$\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{3}{10}}$
0, 1		$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{10}}$

$$m = 0$$

m_1, m_2	j	3	2	1
1, -1		$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$
0, 0		$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$
-1, 1		$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$

$$j_1 = \frac{3}{2}, j_2 = \frac{3}{2} \quad [\text{edit}]$$

$$m = 3$$

m_1, m_2	j	3
$\frac{3}{2}, \frac{3}{2}$		1

$$m = 2$$

m_1, m_2	j	3	2
$\frac{3}{2}, \frac{1}{2}$		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$\frac{1}{2}, \frac{3}{2}$		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$$m = 1$$

m_1, m_2	j	3	2	1
$\frac{3}{2}, -\frac{1}{2}$		$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$
$\frac{1}{2}, \frac{1}{2}$		$\sqrt{\frac{3}{5}}$	0	$-\sqrt{\frac{2}{5}}$
$-\frac{1}{2}, \frac{3}{2}$		$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{3}{10}}$

$$m = 0$$

m_1, m_2	j	3	2	1	0
$\frac{3}{2}, -\frac{3}{2}$		$\sqrt{\frac{1}{20}}$	$\frac{1}{2}$	$\sqrt{\frac{9}{20}}$	$\frac{1}{2}$
$\frac{1}{2}, -\frac{1}{2}$		$\sqrt{\frac{9}{20}}$	$\frac{1}{2}$	$-\sqrt{\frac{1}{20}}$	$-\frac{1}{2}$
$-\frac{1}{2}, \frac{1}{2}$		$\sqrt{\frac{9}{20}}$	$-\frac{1}{2}$	$-\sqrt{\frac{1}{20}}$	$\frac{1}{2}$
$-\frac{3}{2}, \frac{3}{2}$		$\sqrt{\frac{1}{20}}$	$-\frac{1}{2}$	$\sqrt{\frac{9}{20}}$	$-\frac{1}{2}$

$$j_1 = \frac{3}{2}, j_2 = 1 \quad [\text{edit}]$$

$$m = \frac{5}{2}$$

m_1, m_2	j	$\frac{5}{2}$
$\frac{3}{2}, 1$		1

$$m = \frac{3}{2}$$

m_1, m_2	j	$\frac{5}{2}$	$\frac{3}{2}$
$\frac{3}{2}, 0$		$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$
$\frac{1}{2}, 1$		$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$

$$m = \frac{1}{2}$$

m_1, m_2	j	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
$\frac{3}{2}, -1$		$\sqrt{\frac{1}{10}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{2}}$
$\frac{1}{2}, 0$		$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{1}{15}}$	$-\sqrt{\frac{1}{3}}$
$-\frac{1}{2}, 1$		$\sqrt{\frac{3}{10}}$	$-\sqrt{\frac{8}{15}}$	$\sqrt{\frac{1}{6}}$

$$j_1 = 2, j_2 = \frac{3}{2} \quad [\text{edit}]$$

$$m = \frac{7}{2}$$

m_1, m_2	j	$\frac{7}{2}$
$2, \frac{3}{2}$		1

$$m = \frac{5}{2}$$

m_1, m_2	j	$\frac{7}{2}$	$\frac{5}{2}$
$2, \frac{1}{2}$		$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{4}{7}}$
$1, \frac{3}{2}$		$\sqrt{\frac{4}{7}}$	$-\sqrt{\frac{3}{7}}$

$$m = \frac{3}{2}$$

m_1, m_2	j	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{3}{2}$
$2, -\frac{1}{2}$		$\sqrt{\frac{1}{7}}$	$\sqrt{\frac{16}{35}}$	$\sqrt{\frac{2}{5}}$
$1, \frac{1}{2}$		$\sqrt{\frac{4}{7}}$	$\sqrt{\frac{1}{35}}$	$-\sqrt{\frac{2}{5}}$
$0, \frac{3}{2}$		$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{18}{35}}$	$\sqrt{\frac{1}{5}}$

$$m = \frac{1}{2}$$

m_1, m_2	j	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
$2, -\frac{3}{2}$		$\sqrt{\frac{1}{35}}$	$\sqrt{\frac{6}{35}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{5}}$
$1, -\frac{1}{2}$		$\sqrt{\frac{12}{35}}$	$\sqrt{\frac{5}{14}}$	0	$-\sqrt{\frac{3}{10}}$
$0, \frac{1}{2}$		$\sqrt{\frac{18}{35}}$	$-\sqrt{\frac{3}{35}}$	$-\sqrt{\frac{1}{5}}$	$\sqrt{\frac{1}{5}}$
$-1, \frac{3}{2}$		$\sqrt{\frac{4}{35}}$	$-\sqrt{\frac{27}{70}}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{1}{10}}$

$$j_1 = 1, j_2 = 1 \quad [\text{edit}]$$

$$m = 2$$

m_1, m_2	j	2
1, 1		1

$$m = 1$$

m_1, m_2	j	2	1
1, 0		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
0, 1		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$$m = 0$$

m_1, m_2	j	2	1	0
1, -1		$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$
0, 0		$\sqrt{\frac{2}{3}}$	0	$-\sqrt{\frac{1}{3}}$
-1, 1		$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$

$$j_1 = 2, j_2 = 2 \quad [\text{edit}]$$

$$m = 4$$

m_1, m_2	j	4
2, 2		1

$$m = 3$$

m_1, m_2	j	4	3
2, 1		$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
1, 2		$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$$m = 2$$

m_1, m_2	j	4	3	2
2, 0		$\sqrt{\frac{3}{14}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$
1, 1		$\sqrt{\frac{4}{7}}$	0	$-\sqrt{\frac{3}{7}}$
0, 2		$\sqrt{\frac{3}{14}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{2}{7}}$

$$m = 1$$

m_1, m_2	j	4	3	2	1
2, -1		$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{5}}$
1, 0		$\sqrt{\frac{3}{7}}$	$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$
0, 1		$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{14}}$	$\sqrt{\frac{3}{10}}$
-1, 2		$\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{3}{10}}$	$\sqrt{\frac{3}{7}}$	$-\sqrt{\frac{1}{5}}$

$$m = 0$$

m_1, m_2	j	4	3	2	1	0
2, -2		$\sqrt{\frac{1}{70}}$	$\sqrt{\frac{1}{10}}$	$\sqrt{\frac{2}{7}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{5}}$
1, -1		$\sqrt{\frac{8}{35}}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{14}}$	$-\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{1}{5}}$
0, 0		$\sqrt{\frac{18}{35}}$	0	$-\sqrt{\frac{2}{7}}$	0	$\sqrt{\frac{1}{5}}$
-1, 1		$\sqrt{\frac{8}{35}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{14}}$	$\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{1}{5}}$
-2, 2		$\sqrt{\frac{1}{70}}$	$-\sqrt{\frac{1}{10}}$	$\sqrt{\frac{2}{7}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{1}{5}}$