CVIČENÍ QMI ... (2) .. 7. 10. 2020

úvodní diskuse:

$S_x = \begin{pmatrix} 0 & e^{-i\delta} \\ e^{i\delta} & 0 \end{pmatrix}$ - $e^{i\phi_1}|z+\rangle$, $e^{i\phi_2}|z-\rangle$

$$S_x = \begin{pmatrix} \langle +|S_x|+\rangle & \langle +|S_x|-\rangle \\ \langle -|S_x|+\rangle & \langle -|S_x|-\rangle \end{pmatrix} - \begin{pmatrix} 0 & ? \\ 1 & 0 \end{pmatrix}$$

$\langle z+|e^{-i\phi_1}$

$\boxed{e^{-i\phi_1 + i\phi_2}} = e^{i(\phi_2 - \phi_1)}$

$$S_y = \begin{pmatrix} 0 & \\ & 0 \end{pmatrix} - \begin{pmatrix} & -i \\ & \end{pmatrix}$$

Cvičení 2 ... $\boxed{U1}$

$$\hat{S}_n = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix} = \frac{\hbar}{2}\sigma_n$$

$\boxed{\pm\frac{\hbar}{2}\lambda}$

$\boxed{U2}$ $\det(\hat{\sigma}_n - \lambda \hat{I}) = 0$

$\hookrightarrow$ $-(\cos\theta - \lambda)(\cos\theta + \lambda) - \sin^2\theta = 0$

$\sin^2\theta + \cos^2\theta = 1$

$\lambda = \pm 1$ $\qquad \lambda^2 = 1$

$$\boxed{\sigma_n \begin{pmatrix} a \\ b \end{pmatrix} = 1 \begin{pmatrix} a \\ b \end{pmatrix}}$$

$\cos\theta\, a + \sin\theta\, e^{-i\varphi} b = a$

$(\cos\theta - 1)\, a + \sin\theta\, e^{-i\varphi} b = 0$

$|n+\rangle = \begin{pmatrix} \sin\theta\, e^{-i\varphi} \\ 1 - \cos\theta \end{pmatrix}$ ... S_n ... $+\frac{\hbar}{2}$

$\langle n+|n+\rangle = \sin^2\theta + 1 + \cos^2\theta - 2\cos\theta$

$= 2 - 2\cos\theta = 2(1 - \cos\theta)$

normování $|n+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\frac{\sin\theta\, e^{-i\varphi}}{\sqrt{1-\cos\theta}} \\ \sqrt{1-\cos\theta}\end{pmatrix}$ $\qquad \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$\sqrt{1-\cos} = \sqrt{2}\sin\frac{\theta}{2}$

$\sin\theta = \sin 2\cdot\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

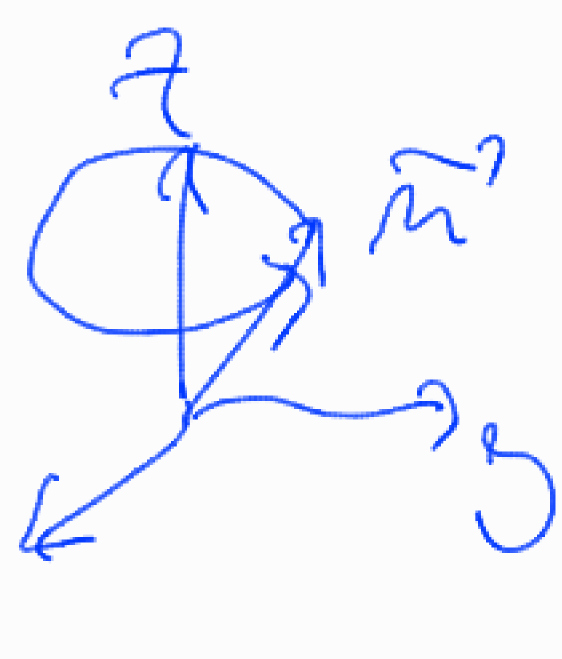
$$|n+\rangle = \frac{1}{\cancel{\sqrt{2}}}\begin{pmatrix}\frac{\cancel{2}\,\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}\, e^{-i\varphi}}{\cancel{\sqrt{2}}\,\cancel{\sin\frac{\theta}{2}}} \\ \cancel{\sqrt{2}}\sin\frac{\theta}{2}\end{pmatrix} = \begin{pmatrix}\cos\frac{\theta}{2}\, e^{-i\varphi} \\ \sin\frac{\theta}{2}\end{pmatrix}\cdot e^{i\alpha}$$

fázová konvence

$$\boxed{|n+\rangle = \begin{pmatrix}\cos\frac{\theta}{2}\cdot e^{-i\frac{\varphi}{2}} \\ \sin\frac{\theta}{2}\cdot e^{i\frac{\varphi}{2}}\end{pmatrix}} \cdot e^{i\pi}$$

$\cancel{\varphi \to \theta + 2\pi}$

φ



$\varphi \to \varphi + 2\pi$

$|n+\rangle \to -|n+\rangle$

$\langle n-|n+\rangle = 0$

$ab - ab = 0$

$$|n-\rangle = \begin{pmatrix}\sin\frac{\theta}{2}\, e^{-i\frac{\varphi}{2}} \\ -\cos\frac{\theta}{2}\, e^{i\frac{\varphi}{2}}\end{pmatrix}$$

$$\underset{n+}{\begin{pmatrix}a \\ b\end{pmatrix}} \xrightarrow{\perp} \underset{n-}{\begin{pmatrix}b^* \\ -a^*\end{pmatrix}}$$

03 a) $|\psi\rangle = |z+\rangle \equiv \begin{pmatrix}1\\0\end{pmatrix}$

$\hat{S}_n$ měření

$$p_{n+} = |\langle n+|\psi\rangle|^2 = |\langle z+|n+\rangle|^2 = \left(\cos\frac{\theta}{2}\right)^2 = \left(\sqrt{\frac{1+\cos\theta}{2}}\right)^2 = \frac{1+\cos\theta}{2}.$$

$$p_{n-} = |\langle n-|\psi\rangle|^2 = |\langle z+|n-\rangle|^2 = \left|\sin\frac{\theta}{2}\right|^2 = \left(\sqrt{\frac{1-\cos\theta}{2}}\right)^2 = \frac{1-\cos\theta}{2}.$$

$p_{n+} + p_{n-} = 1$

b) $|\psi\rangle = |x+\rangle$ měření $\hat{S}_n$?

$$= \frac{1}{\sqrt{2}}\left(|z+\rangle + |z-\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}.$$

$\sigma_x |x+\rangle = |x+\rangle$

$|n_+\rangle = \begin{pmatrix}a\\b\end{pmatrix}$

$$p_{n+} = |\langle n+|\psi\rangle|^2 = |\langle\psi|n+\rangle|^2$$

$$= \left|\frac{1}{\sqrt{2}}(1,1)\begin{pmatrix}a\\b\end{pmatrix}\right|^2 = \frac{1}{2}|a+b|^2$$

$$= \frac{1}{2}\left|\cos\frac{\theta}{2}e^{-i\frac{\varphi}{2}} + \sin\frac{\theta}{2}e^{i\frac{\varphi}{2}}\right|^2 =$$

$$\dots = \frac{1}{2}\left[1 + \sin\theta\cos\varphi\right] = \frac{1}{2}\left[1 + n_x\right]$$

$$\vec{n} = (\cos\varphi\sin\theta,\ \sin\varphi\sin\theta,\ \cos\theta) = (n_x, n_y, n_z)$$

$$p_{n-} = \dots = \frac{1}{2}\left[1 - n_x\right]$$