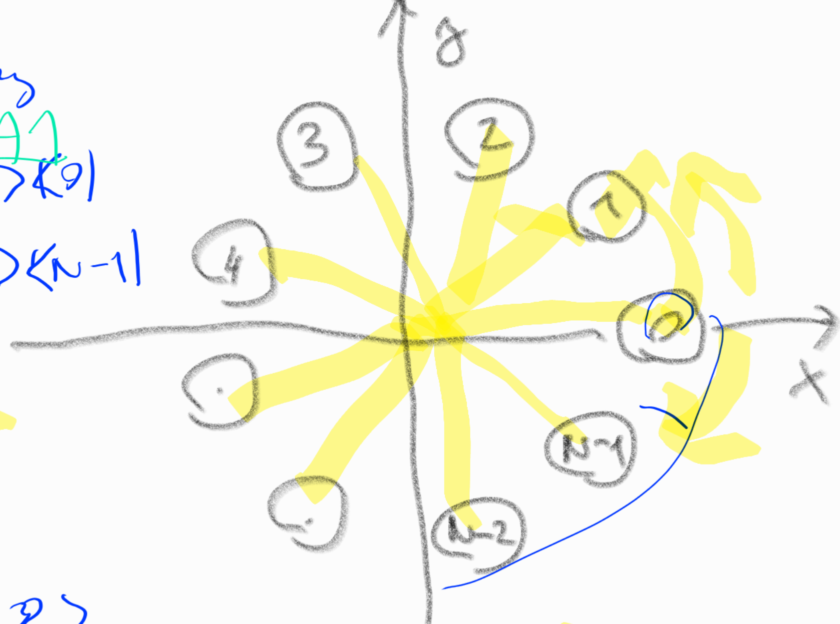
QMI - Cvičení 4: Q-tečky

ÚLOHA 1

$$\boxed{\hat R = \sum_{m'=0}^{N-1} |m'+1\rangle\langle m'|}$$

$|1\rangle\langle 0|$

$|0\rangle\langle N-1|$



$$\hat R|m\rangle = \sum_{m'} |m'+1\rangle \underbrace{\langle m'|m\rangle}_{\delta_{mm'}}$$

$$\hat R|m\rangle = |m+1\rangle$$

$$\hat R(\hat R|m\rangle) = \hat R|m+1\rangle = |m+2\rangle$$

$$\boxed{\hat R^k|m\rangle = |m+k\rangle} \quad k>0, \; k<0$$

$$\boxed{\hat R^{-1}\cdot \hat R = I}$$

$$(R^{-1}R)|m\rangle = |m\rangle$$

$$\boxed{R^{-1}|m+1\rangle = |m\rangle} \quad \forall m \qquad n = m-1$$

$m-1+1$

$$|m\rangle = |N+m\rangle$$

$$R^{-1}|m\rangle = |m-1\rangle$$

R je unitární? $\quad R^+ = \sum_n |n\rangle\langle n+1|$

$$R^+R = RR^+ = I = \sum_n |n\rangle\langle n| \qquad I|n\rangle = |n\rangle$$

$$\sum_m |m\rangle\langle m+1| \sum_n |n+1\rangle\langle n| = \sum_{mn} |m\rangle\langle m+1|n+1\rangle\langle n|$$

$\delta_{m+1,n+1}$

$$= \sum_{mn} |m\rangle\langle n| \underbrace{\delta_{m+1,n+1}}_{\delta_{mn}} = \sum_n |n\rangle\langle n| = I \qquad \begin{cases} m+1 = n+1 \\ m = n \end{cases}$$

$$[R, R^+] = I - I = 0$$

(vlastní č. a v.)

$$\hat R|\psi\rangle = \lambda|\psi\rangle \qquad |\psi\rangle = \mathcal{N}\sum_n e^{ikn}|n\rangle$$

e^{ix}

$$\lambda = e^{\frac{i\pi(l+1)}{N}} \qquad l \qquad \boxed{\lambda^N = 1^N}$$

$$\begin{pmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ 0 & & \ddots & 1 \\ 1 & & & 0 \end{pmatrix}$$

$$\hat R|\psi\rangle = \sum_m |m+1\rangle\langle m| \sum_n e^{ikn}|n\rangle \qquad n = n'-1$$

$$= \sum_{mn} e^{ikn}|m+1\rangle \underbrace{\langle m|n\rangle}_{\delta_{mn}} = \sum_n e^{ikn}\underbrace{|n+1\rangle}_{n'}$$

$$= \sum_{n'} e^{ik(n'-1)}|n'\rangle = e^{-ik}\underbrace{\sum_n e^{ikn}|n\rangle}_{|\psi\rangle}$$

$$\hat R|\psi\rangle = \underbrace{e^{-ik}}_{\lambda}|\psi\rangle \qquad \lambda = e^{-ik}$$

$$\psi_0 = \psi_N \qquad e^{ikN} = e^{ik0} = 1$$

$$(\lambda)^N = 1 \qquad \boxed{k_\ell = \frac{2\pi \ell}{N}}$$

$$|\psi\rangle = \begin{pmatrix} e^0 \\ e^{ik} \\ e^{2ik} \\ \vdots \\ e^{(N-1)ik} \end{pmatrix} ; \quad e^{ikn} \qquad \boxed{\ell \to \ell+N} \qquad \boxed{\ell = 0,1,\dots,N-1}$$

$$e^{ikn + \frac{2\pi}{N}\cdot N i} \qquad 2\pi i \qquad \ell = N \quad \ell = 0$$

závěr: vl. v. $|\psi_\ell\rangle = \eta \sum_n e^{ik_\ell n}|n\rangle$

vl. č. ... $\lambda = e^{-ik_\ell}$ $\qquad \ell = 0,1,\dots,N-1$

$$\boxed{\langle\psi_\ell|\psi_{\ell'}\rangle = \delta_{\ell\ell'}} \qquad \lambda = e^{\,}$$

$$\|\psi_\ell\|^2 = \langle\psi_\ell|\psi_\ell\rangle = |\eta|^2 \sum_n\sum_m \underbrace{e^{-ik_\ell n} e^{ik_\ell m}}_{1} \underbrace{\langle n|m\rangle}_{\delta_{nm}}$$

$$= |\eta|^2\sum_{n=0}^{N-1} 1 = N\cdot|\eta|^2$$

závěr: $\boxed{|\psi_\ell\rangle = \frac{1}{\sqrt N}\sum_n e^{ik_\ell n}|n\rangle}$ $\qquad \lambda_\ell = e^{-ik_\ell}$

OPERÁTORY $\hat A = \frac{R+R^\dagger}{2}$ $\qquad \hat B = \frac{R-R^\dagger}{2i}$ $\qquad [R,R^\dagger]=0$

$R = \hat A + \hat B i$ $\qquad$ vl. č. A ... Re λ_ℓ

vlč. B .. Im λ_e

$\longrightarrow$ $\lambda_e = e^{ik_e} = \cos ik_e + i \sin ik_e$

$R|\psi_e\rangle = \lambda |\psi_e\rangle$

$\langle\psi_e| R^+ = \lambda^* |\psi_e\rangle$

$\stackrel{?}{=}$ $\boxed{R^+ |\psi_e\rangle = \lambda^* |\psi_e\rangle}$

$\left(\sum_n |n+1\rangle\langle n|\right)^+$

$\sum_n |n\rangle\langle n+1|$

$= \sum_n |n-1\rangle\langle n|$

$\hat{A}|\psi_e\rangle = \left(\frac{\lambda+\lambda^*}{2}\right)|\psi_e\rangle$

$\underset{\mathrm{Re}\,\lambda}{\shortparallel}$

$\hat{B}|\psi_e\rangle = \frac{\lambda-\lambda^*}{2i}|\psi_e\rangle$