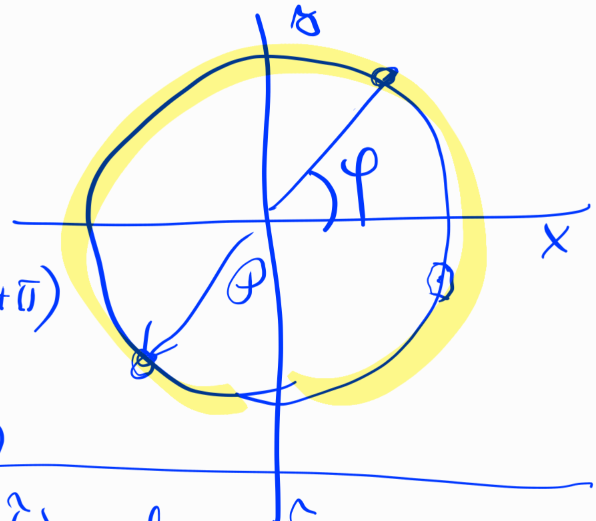
$\boxed{\text{QM-I-cv5}}$ kvantový rotor



$$\hat\phi\,\psi(\varphi) = \varphi\,\psi(\varphi)$$

$$\hat L = -i\hbar \frac{d}{d\varphi}$$

$$\hat H = \frac{\hat L^2}{2I}$$

$\hat x, \hat y$

$$\hat{\mathcal P}\psi(x) = \psi(x+\pi)$$

$$\varphi \in \langle 0, 2\pi)$$

$$\hat\phi = \int_0^{2\pi} \varphi\, |\varphi\rangle\langle\varphi|$$

↳ $d\cos\varphi$

$$\hat x = f(\hat\phi) = d\cos\hat\phi$$

$$\hat x = d\int_0^{2\pi} \cos\varphi\, |\varphi\rangle\langle\varphi|$$

$$[\hat\phi,\hat L]\psi(\varphi) = (\hat\phi\hat L - \hat L\hat\phi)\psi(\varphi) = \hat\phi(-i\hbar\psi') + i\hbar(\varphi\psi)'$$

$$= i\hbar(\psi(\varphi) + \cancel{\varphi\psi'} - \cancel{\varphi\psi'}) = i\hbar\psi(\varphi) = i\hbar\hat I\psi(\varphi)$$

tj. $[\hat\phi,\hat L] = i\hbar\hat I = i\hbar$

$$[\hat\phi,\hat{\mathcal P}]\psi(\varphi) = (\hat\phi\hat{\mathcal P} - \hat{\mathcal P}\hat\phi)\psi(\varphi)$$

$$= \hat\phi\,\psi(\varphi+\pi) - \hat{\mathcal P}(\varphi\psi(\varphi)) = \cancel{\varphi\psi(\varphi+\pi)} - (\cancel{\varphi}+\pi)\psi(\varphi+\pi)$$

φ ↳ $\varphi \to \varphi+\pi$

$$= -\pi\,\psi(\varphi+\pi) = -\pi\hat{\mathcal P}\psi(\varphi)$$

$$[\hat\phi,\hat{\mathcal P}] = -\pi\hat{\mathcal P}$$

$$[\hat\phi,\hat H] = \left[\hat\phi, \frac{\hat L^2}{2I}\right] \quad \dots \quad [\hat\phi,\hat L]$$

$$\boxed{[\hat x,\hat L] = -i\hbar\hat y} \longleftarrow [y, L]$$

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nájsť vl. v. vl. č. $\hat L$ transf. $\hat\phi$ repr. a $\hat L$ repr.

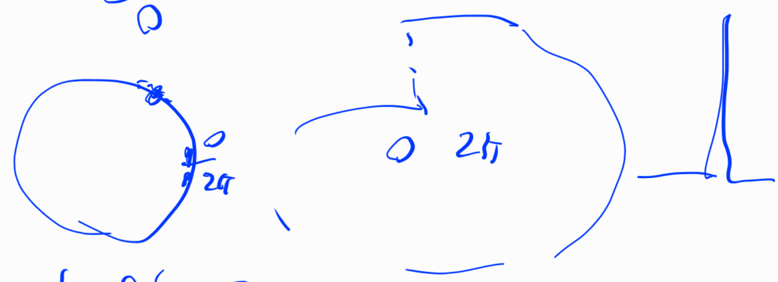
↳ $\hat L$ operátor momentu hybnosti

$\hat{L}|\psi\rangle = l|\psi\rangle$ ... ϕ-repr. $\qquad (L^2) \qquad \boxed{\psi \quad \psi'}$

$$-i\hbar \frac{d}{d\varphi}\psi(\varphi) = l\,\psi(\varphi) \to \psi(\varphi) = \eta\, e^{\frac{i}{\hbar} l\varphi}$$

$$1 = \int_0^{2\pi}|\psi|^2 d\varphi = |\eta|^2\int_0^{2\pi} 1\, d\varphi = |\eta|^2\, 2\pi \to \eta = \frac{1}{\sqrt{2\pi}}$$

L ... ψ'



spojitost ψ v okolí 0

tj. $\quad \psi(\varphi) = \psi(\varphi+2\pi) \quad 1 = e^{\frac{i}{\hbar}l\cdot 0} = e^{\frac{i}{\hbar}l\, 2\pi}$

$$l = \hbar m \qquad m\in\mathbb{Z}$$

$\to \boxed{\sigma_L = \{\hbar m,\ m\in\mathbb{Z}\}} \quad |m\rangle$ ... $\boxed{\psi_m = \frac{1}{\sqrt{2\pi}} e^{im\varphi}}$

$\psi(\varphi) \leftrightarrow |\psi\rangle - \left(c_m\right) \qquad \hat{I} = \sum_{m\in\mathbb{Z}} |m\rangle\langle m|$

$\phi \qquad \hat{L} \sim c_m \qquad \psi_m(\varphi) \equiv \langle\varphi|m\rangle$

$$\psi(\varphi) \doteq |\psi\rangle \underset{\hat{I}}{=} \sum_m \underbrace{\langle m|\psi_m\rangle}_{c_m} |m\rangle = \sum_m c_m \frac{1}{\sqrt{2\pi}} e^{im\varphi}$$

$$c_m = \int_0^{2\pi}\psi_m^*(\varphi)\,\psi(\varphi)\,d\varphi = \frac{1}{\sqrt{2\pi}}\int_0^{2\pi} e^{-im\varphi}\,\psi(\varphi)\,d\varphi$$

$\to$ Fourierova řada

105 $\quad A, A^+ \quad L, x \qquad \hat{A}^+ = \hat{x} + i\hat{y} \quad \ldots \quad \hat{L}$

$$\boxed{[\hat{L}, \hat{A}^+] = \hbar A^+}$$

$$\hat{A}^+ = d\cos\phi + i d \sin\phi = d\exp\{i\hat{\phi}\}$$

$$[-i\hbar \tfrac{d}{dx}, d e^{i\hat{\phi}}]\, \psi(\varphi) \quad \ldots$$

$$= -i\hbar d\, i\, e^{i\phi}\, \psi(\varphi)$$

$$\boxed{[\hat{L}, \hat{A}] = -\hbar \hat{A}}$$

$$\hat{L}|m\rangle = \ell|m\rangle \qquad \ell = m\hbar$$

$$|\psi\rangle = \hat{A}^+|m\rangle$$

$$\hat{L}|\psi\rangle = \underbrace{L A^+}|m\rangle = (A^+ L + \underbrace{[L, A^+]}_{\hbar \hat{A}^+})|m\rangle = (m\hbar + \hbar) A^+|m\rangle$$

$$\hat{L}|\psi\rangle = \hbar(m+1)|\psi\rangle \quad \ldots \quad |\psi\rangle \sim |m+1\rangle$$

$$A^+|m\rangle = \underline{d}\,|m+1\rangle$$

$$\langle\psi|\psi\rangle = \langle m|A A^+|m\rangle = d^2 \langle m|m\rangle = \underline{\underline{d^2}}$$

$$\underbrace{d\, e^{-i\hat{\phi}}\, d e^{i\hat{\phi}}}_{d^2 I}$$

$$A^+|m\rangle = \underset{d e^{i\varphi}}{\underline{e^{i\hat{\phi}}}} \frac{1}{\sqrt{2\pi}} e^{im\varphi} = \frac{1}{\sqrt{2\pi}} d e^{i\varphi} e^{im\varphi}$$

$$= \frac{d}{\sqrt{2\pi}} e^{i(m+1)\varphi} \quad \sim |m+1\rangle$$