Collisional Penrose process and extraction of energy from extremal electrovacuum black holes

Filip Hejda

CEICO, Institute of Physics of the Czech Academy of Sciences, Prague, Czech Republic

in cooperation with:
J. Bičák, M. Kimura, T. Harada, O. B. Zaslavskii, J. P. S. Lemos

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History of the Penrose process

- R. Penrose, *Gravitational Collapse: the Role of General Relativity*, Rivista del Nuovo Cimento, Numero Speziale **1**, 252 (1969).
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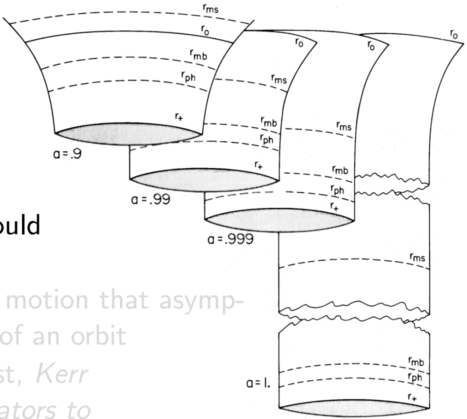

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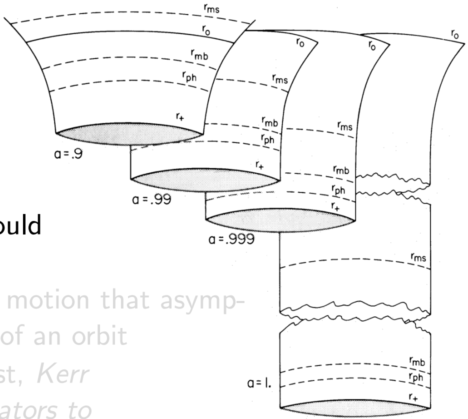

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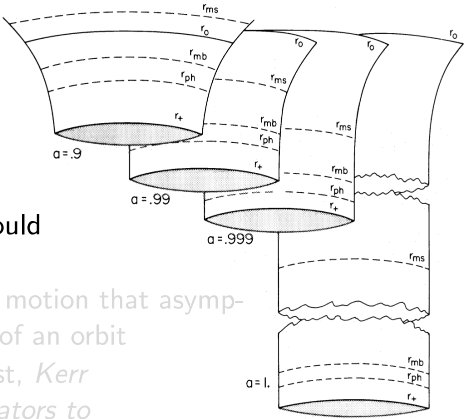

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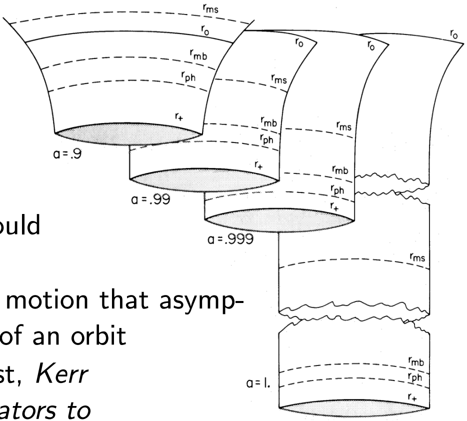

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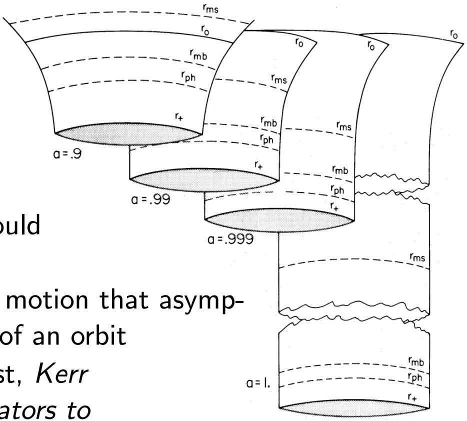

Motivation

- There used to be two variants of the so-called BSW effect:
- Centrifugal: particles with fine-tuned angular momentum around an extremally rotating black hole, strongly limited energy extraction; T. Harada, H. Nemoto, U. Miyamoto, *Upper limits of particle emission from high-energy collision and reaction near a maximally rotating Kerr black hole*, Phys. Rev. D **86**, 024027 (2012).
- Electrostatic: particles with fine-tuned charge close to an extremally charged black hole, with no strong bounds on extracted energy; O. B. Zaslavskii, *Acceleration of particles by nonrotating charged black holes?* JETP Letters **92**, 571 (2010). O. B. Zaslavskii, *Energy extraction from extremal charged black holes due to the Banados-Silk-West effect*, Phys. Rev. D **86**, 124039 (2012).
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- Can we "bridge" the two cases? How do the bounds (dis)appear?
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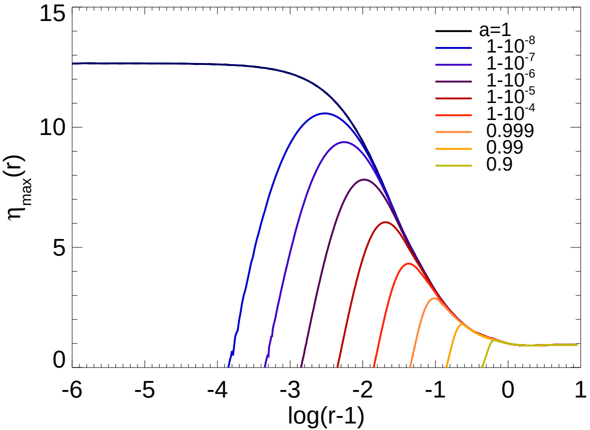
Extremal approximation (vacuum case)



- J. D. Schnittman, PRL **113**, 261102 (2014)
- Different shape (step vs. peak)
- Value for the extremal case serves as an upper bound
- Thorne limit $a < 0.998M$ gives a considerable penalty

- Additionally, near-horizon region of an extremal magnetised black hole is well approximated by extremal Kerr-Newman solution; cf. Jiří Bičák, FH, arXiv:1510.01911, PhysRevD.92.104006

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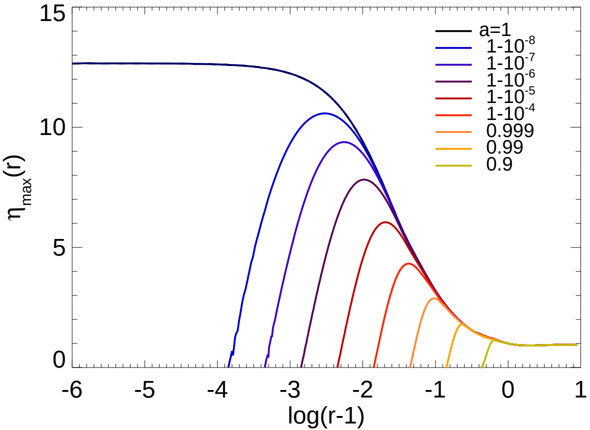

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The setup

- General axially symmetric stationary metric as a model of an isolated black hole, $N \to 0$ at the horizon(s), outer horizon at r_+, extremal r_0

$$\boldsymbol{g} = -N^2 \, \mathbf{d}t^2 + g_{\varphi\varphi} \left(\mathbf{d}\varphi - \omega \, \mathbf{d}t \right)^2 + g_{rr} \, \mathbf{d}r^2 + g_{\vartheta\vartheta} \, \mathbf{d}\vartheta^2$$

- Electromagnetic potential and generalised electrostatic potential ϕ

$$\boldsymbol{A} = A_t \, \mathbf{d}t + A_\varphi \, \mathbf{d}\varphi = -\phi \, \mathbf{d}t + A_\varphi \left(\mathbf{d}\varphi - \omega \, \mathbf{d}t \right)$$

- Equations of motion for a test particle with mass m, charge $q = m\tilde{q}$, energy $E = m\varepsilon$ and angular momentum $L = ml$

$$u^t = \frac{(\varepsilon - V_+) + (\varepsilon - V_-)}{2N^2} \qquad u^r = \sigma \sqrt{\frac{1}{N^2 g_{rr}} (\varepsilon - V_+)(\varepsilon - V_-)}$$

- $\varepsilon \geqslant V_+$ implies $(u^r)^2 > 0$ and $u^t > 0$; $V_+ \equiv V$ is effective potential

$$V_\pm = \omega l + \tilde{q}\phi \pm N \sqrt{1 + \frac{(l - \tilde{q}A_\varphi)^2}{g_{\varphi\varphi}}}$$

- Valid for equatorial and also axial (with $l = 0$, $A_\varphi = 0$) motion

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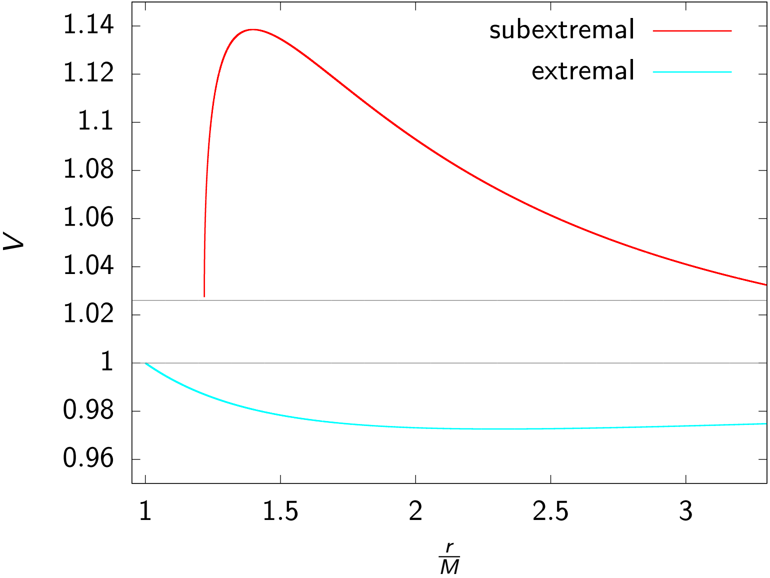

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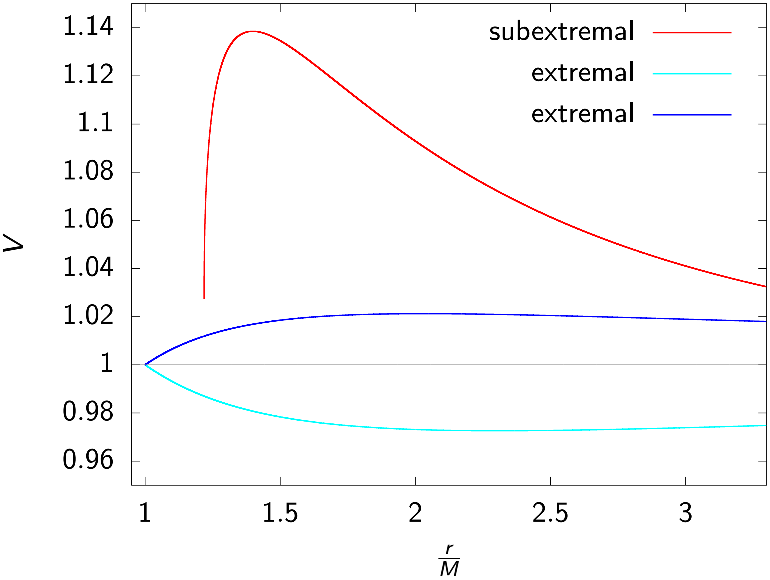

Critical particles and the approach phase of the process

- Due to the causality restriction ($u^t > 0$), we can say that particles with $\varepsilon > V|_{r_+}$ fall into the black hole, whereas particles with $\varepsilon < V|_{r_+}$ can not get close to horizon
- Fine-tuned particles with $\varepsilon = V|_{r_+}$ are on the verge between those cases, so they are called "critical particles"
- Clearly, the effective potential must decrease in order for the motion of critical particles towards r_+ to be allowed
- In the subextremal case the derivative $\partial V/\partial r|_{r_+}$ is always infinite and positive, so no critical particle can approach r_+
- For extremal black holes, the derivative is finite. The condition $\partial V/\partial r|_{r=r_0} = 0$ corresponds to a branch of a hyperbola in the parameter space of $l, \tilde{q}$ (energy is determined by the fine-tuning through relation $\varepsilon_{\mathrm{cr}} = l\omega_{\mathrm{H}} + \tilde{q}\phi_{\mathrm{H}}$)
- See FH, J. Bičák, arXiv:1612.04959, PhysRevD.95.084055 for details about the approach phase and the hyperbola orientation

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Conservation of momentum

- Let us consider a $2 \to 2$ scattering process involving a critical particle 1 and a usual particle 2, both moving towards extremal black hole
- E_{CM} attainable in such processes diverges $\sim (r_{\mathrm{C}} - r_0)^{-1}$
- Can we produce highly energetic particles and extract energy?
- Conservation of charge, energy and angular momentum
- For radial momentum, we can do a resummation. For usual particles:

$$N^2 p^t - \sigma N \sqrt{g_{rr}} p^r \sim (r_{\mathrm{C}} - r_0)^2$$
$$N^2 p^t + \sigma N \sqrt{g_{rr}} p^r \doteq 2 (E - L\omega_{\mathrm{H}} - q\phi_{\mathrm{H}}) + \dots$$

- In contrast, for (nearly) critical particles

$$N^2 p^t \pm N \sqrt{g_{rr}} p^r \sim (r_{\mathrm{C}} - r_0)$$

- Thus, we can split orders by summing the time and radial component

$$N^2 \left(p^t_{(1)} + p^t_{(2)}\right) + N\sqrt{g_{rr}} \left(p^r_{(1)} + p^r_{(2)}\right) = N^2 \left(p^t_{(3)} + p^t_{(4)}\right) + N\sqrt{g_{rr}} \left(p^r_{(3)} + p^r_{(4)}\right)$$

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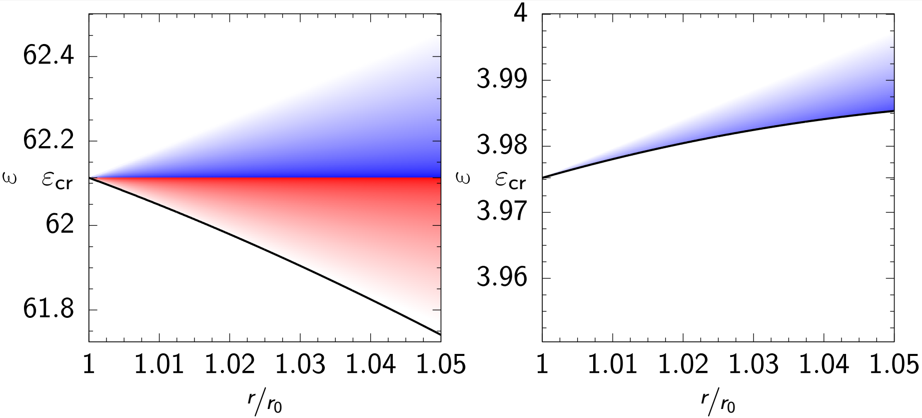

- Four ways to produce particle 3: $+/-$ combined with IN/OUT
- $+/-$ is the sign of C_3 in the formal expansion

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- Figure from: FH, J. Bičák, O. B. Zaslavskii, PhysRevD.100.064041

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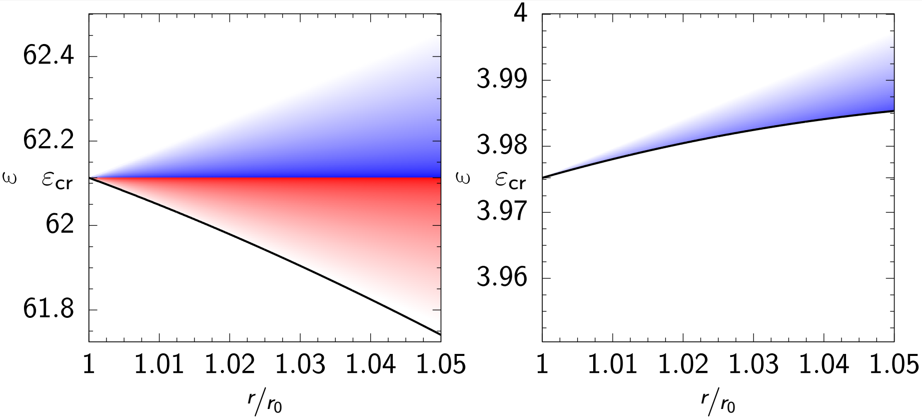

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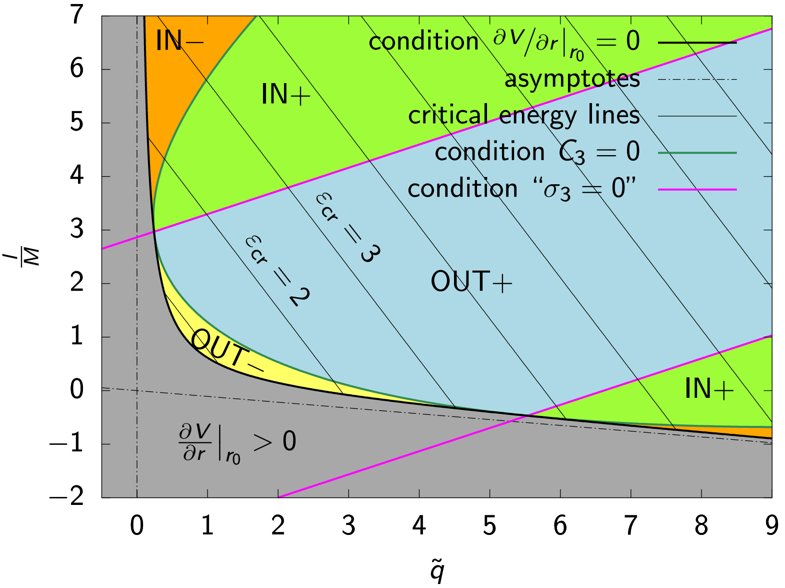
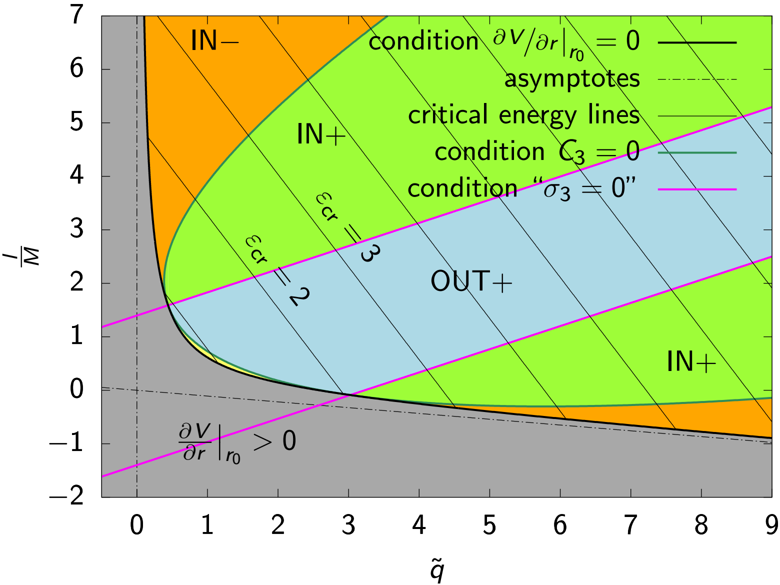
- Figure from: FH, J. Bičák, O. B. Zaslavskii, PhysRevD.100.064041

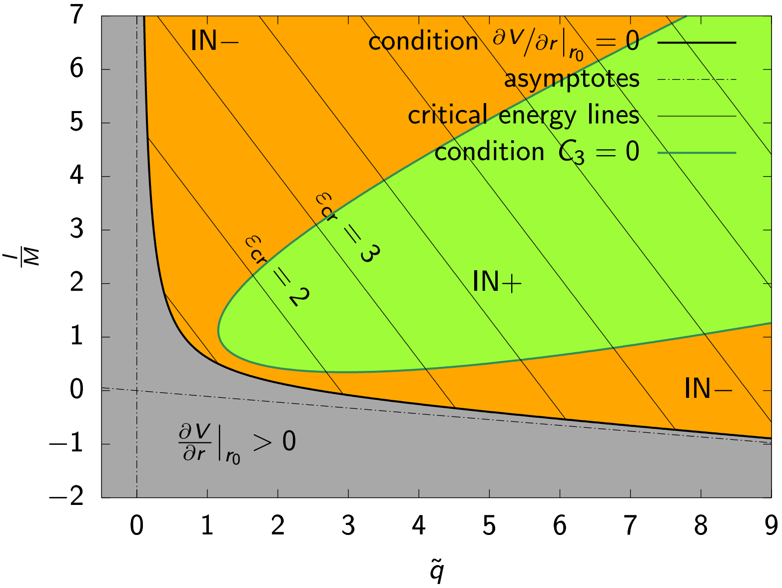
Extremal Kerr-Newman with $\frac{a}{M} = \frac{1}{2}$, $\frac{Q}{M} = \frac{\sqrt{3}}{2}$; process with $\mathfrak{A}_1 = 2.5 m_3$



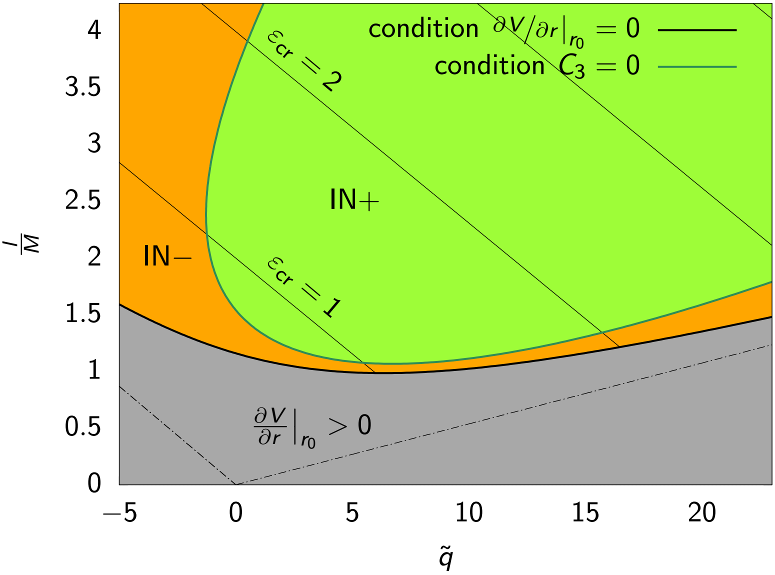
Extremal Kerr-Newman with $\frac{a}{M} = \frac{1}{2}$, $\frac{Q}{M} = \frac{\sqrt{3}}{2}$; process with $\mathfrak{A}_1 = 1.5 m_3$



Extremal Kerr-Newman with $\frac{a}{M} = \frac{1}{2}$, $\frac{Q}{M} = \frac{\sqrt{3}}{2}$; process with $\mathfrak{A}_1 = 0.5 m_3$



Extremal Kerr-Newman with $\frac{a}{M} = \frac{\sqrt{35}}{6}$, $\frac{Q}{M} = \frac{1}{6}$; process with $3\mathfrak{A}_1 = 2m_3$



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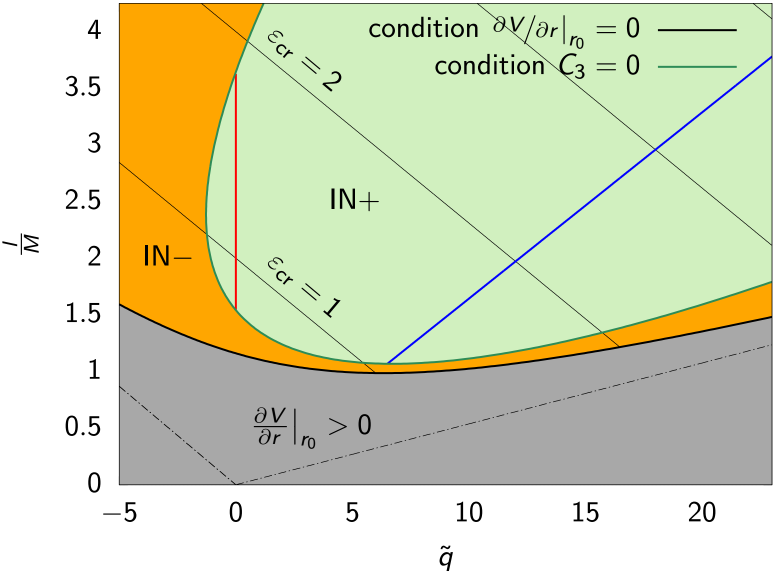

Caveats

- No unconditional bounds on energy extraction from extremal electrovacuum black holes via charged particle collisions
- However, microscopic particles have enormous values of specific charge $\tilde{q}$ (roughly $5 \cdot 10^{17}$ for protons and 10^{21} for electrons)
- Problems arise due to the proportionality of the critical energy to the charge $\varepsilon_{\mathrm{cr}} = l\omega_{\mathrm{H}} + \tilde{q}\phi_{\mathrm{H}}$
- This is particularly troublesome for the axial motion, for which $l = 0$
- In addition, we need $|q_3| > |q_1| > 0$ for $E_3 > E_1$ in the axial case
- This requires processes involving atomic nuclei, which are unrealistic and reduce efficiency; toy model
- We can consider black holes with $|Q| \ll M$ (i.e. $|\phi_{\mathrm{H}}| \ll 1$)
- However, it turns out that the process is possible only for $Q/M > 10^{-6}$, and consequetnly for $\varepsilon_1 > 5 \cdot 10^{11}$

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Conclusion

- On the other hand, the problem with the proportionality $\varepsilon_{\mathrm{cr}} = l\omega_{\mathrm{H}} + \tilde{q}\phi_{\mathrm{H}}$ can be remedied in the equatorial case
- We can consider either uncharged initial particles, or particle 1 with a value of angular momentum that leads to cancellation of the terms
- The latter variant works only for $-\operatorname{sgn} q_1 = \operatorname{sgn} q_3 = \operatorname{sgn} Q$ and $|Q| \ll M$
- The key point: the extracted energy goes like

$$E_3 \approx 10^{27}\,\mathrm{eV}\,\frac{Q}{M}\,\frac{q_3}{q_{\mathrm{elem}}}$$

- Thus, in order to throw a proton with energy at the GZK limit ($5 \cdot 10^{19}\,\mathrm{eV}$), we need just seemingly insignificant $Q/M \doteq 5 \cdot 10^{-8}$
- Our setup neglects a lot of things, including back-reaction…
- Nevertheless, for uncharged particles, unconditional bounds on E_3 arise even in this simplified case, as seen in the previous figure

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