Detectability of hyperbolic encounters with the gravitational wave observatories

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Timeliness of the topic & the road ahead...

- Gravitational Wave (GW) astronomy promises to observe different kinds of astrophysical sources.
- Till now we have detected binary mergers for compact objects — black holes and neutron star.
- Can we detect highly eccentric or scattering/hyperbolic orbits?
- We need the exact waveform, and modified search algorithm.
- Before that, we need to be convinced that the possible signal lies within the detector's strength — current, upgraded and upcoming. This is the question we are going to address in this talk!
- This presentation is based on arXiv:2010.00916 by S.M., Sanjit Mitra and Sourav Chatterjee, done at Inter-University Centre for Astronomy and Astrophysics (IUCAA) Pune, India.

- Hyperbolic interactions and their implications in gravitational wave (GW) astronomy are not new, rather explored extensively.
- Hyperbolic/parabolic interactions in clusters, event rates and LIGO — Kocsis et. al (2006), O'Leary et. al. (2009).
- More event rate estimations — Capozziello and Laurentis (2008), Vittori, Gopakumar and others (2012,2014).
- Primordial BHs — Bellido & Nesseris (2018).
- Energy radiation and close encounters — Berry and Gair.
- Waveform modeling — Damour (2014), and Nagar (2020).

What are we doing new!

1. Redefining the interaction model
   - Most often, when we describe hyperbolic interaction, we assume the object is coming from spatial infinity.
   - To describe encounters in a closed cluster of finite size $R_{\rm c}$, the initial distance, r_i, can be at most $R_{\rm c}$, not infinity!
   - We need a fresh outlook to this problem — considering these localized effects.
2. From observational perspective:— rate estimation with current detectors with ongoing upgrades, and upcoming third generation detectors.

To keep in mind

- Consider a simpler cluster model — uniform density, equal masses, no velocity distribution.
- While accounting for the contributions from galaxies, we assume Milky Way Equivalent Galaxy (MWEG), and the number density, $n(z)$, is a constant.
- We assume that all the objects initially move with virial velocity.
- Note black hole — BH, neutron star — NS, globular cluster — GC.

The steps at a glance!

- Introducing the model
  1. Change in the orbital parameters,
  2. The energy radiation — time and frequency domain.
- Estimating the event rates
  1. The individual probability,
  2. BH/star population inside a cluster,
  3. Redshift dependency.
- Conclusion and discussion

PART - I

The model of interaction

Hyperbolic interaction inside a closed cluster

- Conservation of momentum and energy are assumed.
- Earlier works are based on scattering problem assuming initial distance to be infinity. Requires velocity at infinity, and the impact parameter to define the system.

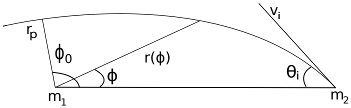


Figure: Model of interaction:
$r_i \equiv$ initial distance, $\theta_i \equiv$ initial velocity,
$vi \equiv$ virial velocity,
$m_1, m_2 \equiv$ component's mass,
$r_p \equiv$ periastron distance,
$\phi_0 \equiv$ angle at periastron,
$r(\phi)$, $\phi \equiv$ coordinates.

- But what about the finite size of the cluster? Can we assume the initial distance to be infinite within a closed cluster?
- Need to redefine the initial conditions based on local parameters — initial distance (r_i), initial angle (θ_i), and *virial* velocity (v_i).
- Respects the bulk properties of the cluster, and localized interactions.
- While $r_i \leq R_\mathrm{c}$, the cluster's radius taken as $10\mathrm{pc}$, we assume $\theta \in (\theta_\mathrm{min}, \theta_\mathrm{max})$. Decide θ_max from threshold signal to noise ratio (SNR), and θ_min from Schwarzschild radius (r_s).

- The eccentricity (e), angle at periastron (ϕ_0), and periastron distance (r_p):

$$e^2 = 1 + \frac{L^2 vi^2}{G^2 M^2}\left\{1 - \left(\frac{2GM}{v_i^2}\right)\frac{1}{r_i}\right\},$$
$$\tan\phi_0 = \frac{L v_i \cos\theta_i}{L^2/r_i - GM} = -\frac{L v_i}{GM}\left\{\frac{\cos\theta_i}{1 - L^2/(GMr_i)}\right\},$$
$$r_p = \frac{L^2\cos\phi_0}{L^2/r_i - GM(1-\cos\phi_0)} = \frac{L^2}{GM(1+e)}, \tag{1}$$

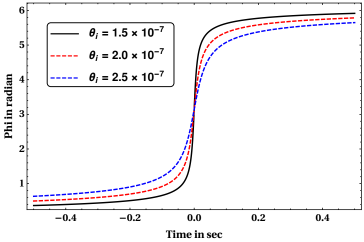
  $L \equiv$ conserved momentum $= r_i v_i \sin\theta_i$, $M \equiv$ total mass $= m_1 + m_2$.
- With $r_i \to \infty$, and $\theta_i \to 0$, such that the angular momentum $L = r_i v_i \sin\theta_i$ remains finite, we get back the results in literature (Vittori et. al. (2012)).
- Note that we use virial velocity, $v_i = \sqrt{\frac{G\bar{M}}{3R_c}}$, where average mass of the cluster $\bar{M} = m * n_{\rm star}$, $m = 10 M_\odot$ is the average mass of a compact object, and $n_{\rm star} = 8 * 10^5$ is the total number of objects inside the cluster.

The orbital evolution in time

- The orbital trajectory can be characterized by

$$r(\phi) = \frac{L^2/(GM)}{1 + [L^2/(GMr_p) - 1]\cos(\phi - \phi_0)}, \tag{2}$$

which follows the initial condition, at $\phi = \phi_0$, $r = r_p$.



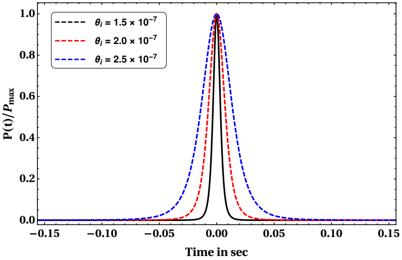
- Figure represents the ϕ *vs* t plot, $r_i = 1$pc, and $m_1 = m_2 = 10M_\odot$, $v_i \cong 10.69$km/sec.
- Note that the relation between ϕ and t follows from, $L = [r(\phi)]^2\dot{\phi}$, where 'dot' is a differentiation with respect to time.
- The $t = 0$ represents the closest distance, $r = r_p$.
- Lesser the θ_i, stronger the interaction!

For the power radiation, we use the following formulas in time and frequency domains:

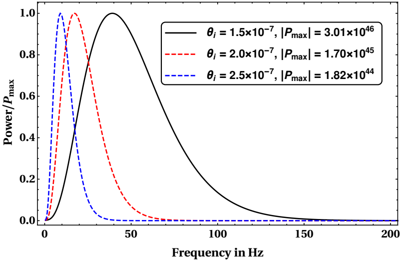
$$\text{time domain}, \; P(t) = -\frac{G}{45c^5} < \dddot{D}^{ij}\dddot{D}_{ij} >,$$

$$\text{frequency domain}, \; \text{Fourier transform of } P(t),$$

where, $D_{ij} = \mu(3x_i x_j - \delta_{ij} r^2)$ is the quadrupole moment, and μ is the reduced mass.



(a) Normalized power radiation in time domain with $r_i = 1\text{pc}$, $m_1 = m_2 = 10M_\odot$.



(b) Normalized power radiation in frequency domain with $r_i = 1\text{pc}$, $m_1 = m_2 = 10M_\odot$.

Till now...

- We revised our model of interaction.
- The orbital dynamics explicitly depends on the initial conditions.
- The power peaks at the periapsis, the closest distance that can be approached.
- Obtained the power spectrum in frequency domain — the results are consistent with the parabolic limit, shown by Berry & Gair (2010) at large periapsis, within the numerical error 1%.

PART - II

The event rate calculation

We need to answer

- What is the probability of an individual encounter? This will require the informations from different detectors, such as noise profile!

- What is the total probability by considering the entire cluster? To answer this, we need to know how many compact objects (BH/NS) are there in a closed cluster?

- Number of GCs in a galaxy?

- Number of galaxies in a given redshift?

Detectors at a glance

Detectors: current, upgrade, and upcoming

- Advanced LIGO, ongoing,
- LIGO A+, new upgrades,
- LIGO Voyager, new upgrades,
- Einstein telescope, third generation detector,
- Cosmic explorer, third generation detector.
- The nose profiles can be found in — https://dcc.ligo.org/LIGO-T1800084/public, https://dcc.ligo.org/LIGO-T1800042-v4/public, and https://cosmicexplorer.org/.
- For the signal to noise ration (SNR) computation, we follow the paper by Flanagan and Hughes 1998.

Individual probability

Individual event

- Given a fixed r_i, the solid angle Ω as a function of $\theta_{\max}(r_i)$ and $\theta_{\min}(r_i)$, which results in a detectable signal ($\theta_i \ll 1$):

$$\Omega = 2\pi \int_{\theta_{\min}(r_i)}^{\theta_{\max}(r_i)} \sin\theta d\theta = \pi \left[\theta_{\max}(r_i)^2 - \theta_{\min}(r_i)^2\right],$$

and assuming that the objects inside the cluster are uniformly distributed, the probability of selecting a fraction of particles fall within the above solid angle is

$$P_{\rm theta} = \frac{1}{4}\left[\theta_{\max}(r_i)^2 - \theta_{\min}(r_i)^2\right]. \quad (3)$$

- Note that $\theta_{\max}$ is obtained from the SNR constrained, and $\theta_{\min}$ from the condition $r_p \geq 2r_{\rm s}$, $r_{\rm s}$ is the Schwarzschild radius.
- For an individual object inside the cluster, the event rate or number of events per unit time becomes:

$$P_{\rm indv} = \int_{R_{\min}}^{R_{\rm c}} \left(\frac{P_{\rm theta}}{t_{\rm col}}\right) 4\pi r_i^2 n_{\rm s} dr_i, \quad (4)$$

where, $t_{\rm col}$ is the average time of a collision, and can be written as $t_{\rm col} \leq r_i/v_i$, $n_{\rm s}$ is the uniform volume density of the stars, i.e., $n_{\rm s} = 3n_{\rm star}/(4\pi R_{\rm c}^3)$, and $R_{\min}$ is the lower radial cut-off (described in the next slide).

Individual event continued...

- After the integration, we arrive at

$$P_{\rm indv} = \frac{3 v_i n_{\rm star} \left(b^2_{\rm max} - b^2_{\rm min}\right)}{4 R_{\rm c}^3} \ln(R_{\rm c}/R_{\rm min}). \tag{5}$$

- Considering $n_{\rm co}$ number of compact objects are distributed within radius $R_{\rm co}$ (which may be less than the radius of the whole cluster $R_{\rm c}$ for a dense core), $R_{\rm min}$ can be obtained from the following expression:

$$\frac{4\pi}{3} R^3_{\rm min} n_{\rm co} = \frac{4\pi}{3} R^3_{\rm co} \,, \tag{6}$$

  which naively summarize the fact that total volume of the cluster is composed of $n_{\rm star}$ number of spheres with radius $R_{\rm min}$.
- For the entire cluster, the probability of detectable hyperbolic interaction events becomes $P_{\rm clus} = n_{\rm co} P_{\rm indv}$, where the small scale structures of the cluster are ignored. We discuss more about $n_{\rm co}$ next.

What about the cluster!

To keep in mind

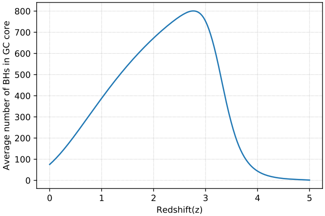
- They are typically of $\sim$ 10pc radius, with up to a few million stars, including white dwarfs, neutron stars and black holes. Their cores of are dense, $\sim 10^5/\mathrm{pc}^3$ stars (Ivanova 2007)!

- On an average, few 100 BHs exist in GC cores up to a redshift of a few. A recent study found $\sim 50 - 100$ BHs in the core $\sim$ 0.5pc region of the Milky Way GCs at the present epoch (Weatherford 2020), while the number was $\sim$ 1000 at redshift $\sim$ 2.

- Present simulations show that at present in the central GC cores, there are $\sim 200 - 300$ NSs, which seems to be not evolving significantly with time (Kremer 2019).

Different model to target different scenarios!

- Model I: We assume 10^3 BHs with an average mass of $10M_\odot$ which is fixed over z and a redshift cutoff of $z_{\rm max} = 3.5$ (Kocsis 2006). This is the simplest model, but useful, as the rates can be scaled as $n_{\rm co}^2 \log(n_{\rm co})$, other parameters and detector sensitivities remaining the same.
- Model II: This model considers only NSs, we take a total of 10 retention fraction for all formation channels, that is, 400 NS in the core up to $z_{\rm max} = 3.5$, which does not evolve significantly with lookback time (Kremer 2019).



- Model III: This is a highly realistic scenario.
- The core is composed of BHs only.
- Assume 0.1% of the stars became BH in a short period of time starting from a lookback time of 12Gyr to 10Gyr and 90% of them escaped from the GCs by the current epoch.

OK! now the galaxies

The integration at different redshift

- Considering the Milky Way Equivalent Galaxy (MWEG), and the number density, $n(z)$, is a constant, the total event rate becomes:

$$P_{\rm tot} = \int_{z_{\rm min}}^{z_{\rm max}} P_{\rm clus}\mathcal{N}(z) n_{\rm gc} dz = \int_{z_{\rm min}}^{z_{\rm max}} \tau_{\rm tot} dz, \quad (7)$$

where $\mathcal{N}(z)$ number of galaxies between z to $z + dz$, and $n_{\rm gc}$ is the number of globular clusters per galaxy.

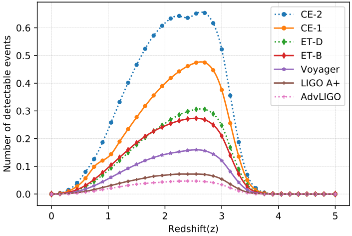


Figure: The event rate is shown corresponds to Model-III for different detectors for the following parameters, $n(z) = 0.0116$ Mpc^{-3}, $n_{\rm gc} = 200$, $R_{\rm co} = 0.5$ pc, and $m_1 = m_2 = 10M_\odot$. In order to reproduce the above results, we assume that the initial distance to be 0.5 pc.

Table: Event rates per year for different detectors

Detector	Expected event rate (per year) Model I $n_{\rm co}=1000$ $m=10M_\odot$	Model II $n_{\rm co}=400$ $m=2M_\odot$	Model III $n_{\rm co}$: Fig. 2 $m=10M_\odot$
Adv. LIGO	0.34	0.002	0.09
LIGO A+	0.51	0.002	0.14
Voyager	1.04	0.005	0.30
ET-B	1.80	0.009	0.52
ET-D	1.81	0.009	0.55
CE-1	2.97	0.014	0.87
CE-2	4.47	0.020	1.29

- For a single detector SNR ≥ 5 (network $\mathrm{SNR} > 7$ for two detectors).
- The rates can increase significantly due to the density profile peaking at the centre.
- Note that the event rates are listed for a year, though *each of these facilities are expected to operate for at least* 10 *years at different sensitivity levels and* $4-5$ *detectors could be operating at A+ like sensitivity in the next few years*, so the integrated detection rate is much higher.

Discussions

Key aspects of the work

- The present study provides a fresh outlook to the hyperbolic/scattering interactions by introducing the localized effects.
- To ensure that our calculations are conservative, we use extremely conservative estimates of these relevant properties, including how the number of BHs varies with cluster's time, and consider a highly conservative stellar cluster model.
- Model of interaction critically hinges on the initial conditions — distance and angle.
- Remarkably, the competition between very low probability of impact and very large number of objects (galaxies!) lead to a reasonable number of detectable events.
- The results clearly show that potential of these events to be detected by future generation detectors and even for detectors with ongoing upgrades.

Future perspective

- In addition to the compact binary coalescences and the associated stochastic background, we propose that hyperbolic encounters are also possible sources for the next generation gravitational wave detectors.
- Given that globular clusters are among the most favorable to host these events, possible detections of hyperbolic encounters may shed light on the BH population (not in binaries) and density profile inside these clusters.
- It is believed that the fraction of BHs in compact binaries is at most a few percent, while most BHs remain single! Thus, putting constraints, any constraints, is a significant stride towards a better understanding of BH physics.
- Like the rate estimation, waveform calculation would also depends on the initial conditions and localized effects! We need accurate waveform.
- The relativistic corrections can be included in our future works.
- Estimating rates with more realistic cluster model may change the rates quite a bit!