Point symmetries of 3D Kepler problem

Run NTMF064.Package.m first!

```
Clear["Global`*"]
```

Variables and differential equations in the form R(x,u,∂u,...) = 0

```
(* Independent variables *)
IndepVar = {t};
(* Dependent variables *)
DepVar = {x, y, z};
(* PDE *)
V[x_, y_, z_] = v[Sqrt[x^2 + y^2 + z^2]];(* general spherical potential*)
(* V[x_,y_,z_]=Z/Sqrt[x^2+y^2+z^2];(* Coulomb potential *) *)
(* V[x_,y_,z_]=Z/(x^2+y^2+z^2);(* Z/r^2 potential *) *)
PDEs = {m D[x[t], t, t] + D[V[x[t], y[t], z[t]], x[t]],
  m D[y[t], t, t] + D[V[x[t], y[t], z[t]], y[t]],
  m D[z[t], t, t] + D[V[x[t], y[t], z[t]], z[t]]}
```

$$\left\{\frac{x[t]\, v'\left[\sqrt{x[t]^2+y[t]^2+z[t]^2}\right]}{\sqrt{x[t]^2+y[t]^2+z[t]^2}}+m\, x''[t],\right.$$

$$\left.\frac{y[t]\, v'\left[\sqrt{x[t]^2+y[t]^2+z[t]^2}\right]}{\sqrt{x[t]^2+y[t]^2+z[t]^2}}+m\, y''[t],\ \frac{z[t]\, v'\left[\sqrt{x[t]^2+y[t]^2+z[t]^2}\right]}{\sqrt{x[t]^2+y[t]^2+z[t]^2}}+m\, z''[t]\right\}$$

Expression to substitute for in the infinitesimal criterion of invariance

```
subs = {D[x[t], t, t], D[y[t], t, t], D[z[t], t, t]};
sol = Solve[PDEs == 0, subs]
```

$$\left\{\left\{x''[t]\to-\frac{x[t]\, v'\left[\sqrt{x[t]^2+y[t]^2+z[t]^2}\right]}{m\sqrt{x[t]^2+y[t]^2+z[t]^2}},\right.\right.$$

$$\left.\left.y''[t]\to-\frac{y[t]\, v'\left[\sqrt{x[t]^2+y[t]^2+z[t]^2}\right]}{m\sqrt{x[t]^2+y[t]^2+z[t]^2}},\ z''[t]\to-\frac{z[t]\, v'\left[\sqrt{x[t]^2+y[t]^2+z[t]^2}\right]}{m\sqrt{x[t]^2+y[t]^2+z[t]^2}}\right\}\right\}$$

Finding point symmetries by using a more and more specific ansatz

General ansatz

```
(* Infinitesimals for all variables *)
ξ[t] = Ξ[t, x[t], y[t], z[t]];
η[x] = α[t, x[t], y[t], z[t]];
η[y] = β[t, x[t], y[t], z[t]];
η[z] = γ[t, x[t], y[t], z[t]];
(* Next expression should return zeroes
 if infinitesimals give a point symmetry of PDEs *)
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

$$x[0] = x[t]$$

$$x[1] = x'[t]$$

$$x[2] = x''[t]$$

$$y[0] = y[t]$$

$$y[1] = y'[t]$$

$$y[2] = y''[t]$$

$$z[0] = z[t]$$

$$z[1] = z'[t]$$

$$z[2] = z''[t]$$

$$\left\{-\left(\left(x[0]\,z[0]\,\gamma[t, x[0], y[0], z[0]]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/ \left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}\right) + \left(x[0]\,z[0]\,\gamma[t, x[0], y[0], z[0]]\,v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/ \left(x[0]^2+y[0]^2+z[0]^2\right) + \frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\,x[0]\,y[0]\,\beta[t, x[0], y[0], z[0]]\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right] + \sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right) + \frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\,\alpha[t, x[0], y[0], z[0]]\left(\left(y[0]^2+z[0]^2\right)v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right] + x[0]^2\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right) - \left(z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\alpha^{(0,0,0,1)}[t, x[0], y[0], z[0]]\right)\Big/ \left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right) + \left(x[1]\,z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,0,1)}[t, x[0], y[0], z[0]]\right)\Big/ \left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right) + \left(2\,x[0]\,z[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,0,1)}[t, x[0], y[0], z[0]]\right)\Big/ \left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right) + m\,z[1]^2\,\alpha^{(0,0,0,2)}[t, x[0], y[0], z[0]] - m\,x[1]\,z[1]^2\,\Xi^{(0,0,0,2)}[t, x[0], y[0], z[0]] -$$

$$\left(y[0]\, v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\alpha^{(0,0,1,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\left(x[1]\,y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,1,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\left(2\,x[0]\,y[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,1,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,y[1]\,z[1]\,\alpha^{(0,0,1,1)}[t,x[0],y[0],z[0]]-2\,m\,x[1]\,y[1]\,z[1]\,\Xi^{(0,0,1,1)}[t,x[0],y[0],z[0]]+m\,y[1]^2\,\alpha^{(0,0,2,0)}[t,x[0],y[0],z[0]]-m\,x[1]\,y[1]^2\,\Xi^{(0,0,2,0)}[t,x[0],y[0],z[0]]-\left(x[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\alpha^{(0,1,0,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\left(3\,x[0]\,x[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,1,0,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,x[1]\,z[1]\,\alpha^{(0,1,0,1)}[t,x[0],y[0],z[0]]-2\,m\,x[1]^2\,z[1]\,\Xi^{(0,1,0,1)}[t,x[0],y[0],z[0]]+2\,m\,x[1]\,y[1]\,\alpha^{(0,1,1,0)}[t,x[0],y[0],z[0]]-2\,m\,x[1]^2\,y[1]\,\Xi^{(0,1,1,0)}[t,x[0],y[0],z[0]]+m\,x[1]^2\,\alpha^{(0,2,0,0)}[t,x[0],y[0],z[0]]-m\,x[1]^3\,\Xi^{(0,2,0,0)}[t,x[0],y[0],z[0]]+\left(2\,x[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(1,0,0,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,z[1]\,\alpha^{(1,0,0,1)}[t,x[0],y[0],z[0]]-2\,m\,x[1]\,z[1]\,\Xi^{(1,0,0,1)}[t,x[0],y[0],z[0]]+2\,m\,y[1]\,\alpha^{(1,0,1,0)}[t,x[0],y[0],z[0]]-2\,m\,x[1]\,y[1]\,\Xi^{(1,0,1,0)}[t,x[0],y[0],z[0]]+2\,m\,x[1]\,\alpha^{(1,1,0,0)}[t,x[0],y[0],z[0]]-2\,m\,x[1]^2\,\Xi^{(1,1,0,0)}[t,x[0],y[0],z[0]]+m\,\alpha^{(2,0,0,0)}[t,x[0],y[0],z[0]]-m\,x[1]\,\Xi^{(2,0,0,0)}[t,x[0],y[0],z[0]],$$

$$-\left(\left(y[0]\,z[0]\,\gamma[t,x[0],y[0],z[0]]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}\right)+\left(y[0]\,z[0]\,\gamma[t,x[0],y[0],z[0]]\,v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)+\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}x[0]\,y[0]\,\alpha[t,x[0],y[0],z[0]]\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\beta[t,x[0],y[0],z[0]]\left(\left(x[0]^2+z[0]^2\right)v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+y[0]^2\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)-$$

$$\left(z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\beta^{(0,0,0,1)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\left(y[1]\,z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,0,1)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\left(2\,y[0]\,z[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,0,1)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+m\,z[1]^2\beta^{(0,0,0,2)}[t,x[0],y[0],z[0]]-m\,y[1]\,z[1]^2\Xi^{(0,0,0,2)}[t,x[0],y[0],z[0]]-\left(y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\beta^{(0,0,1,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\left(3\,y[0]\,y[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,1,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,y[1]\,z[1]\,\beta^{(0,0,1,1)}[t,x[0],y[0],z[0]]-2\,m\,y[1]^2\,z[1]\,\Xi^{(0,0,1,1)}[t,x[0],y[0],z[0]]+m\,y[1]^2\beta^{(0,0,2,0)}[t,x[0],y[0],z[0]]-m\,y[1]^3\Xi^{(0,0,2,0)}[t,x[0],y[0],z[0]]-\left(x[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\beta^{(0,1,0,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\left(2\,x[1]\,y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,1,0,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\left(x[0]\,y[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,1,0,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,x[1]\,z[1]\,\beta^{(0,1,0,1)}[t,x[0],y[0],z[0]]-2\,m\,x[1]\,y[1]\,z[1]\,\Xi^{(0,1,0,1)}[t,x[0],y[0],z[0]]+2\,m\,x[1]\,y[1]\,\beta^{(0,1,1,0)}[t,x[0],y[0],z[0]]-2\,m\,x[1]\,y[1]^2\,\Xi^{(0,1,1,0)}[t,x[0],y[0],z[0]]+m\,x[1]^2\beta^{(0,2,0,0)}[t,x[0],y[0],z[0]]-m\,x[1]^2\,y[1]\,\Xi^{(0,2,0,0)}[t,x[0],y[0],z[0]]+\left(2\,y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(1,0,0,0)}[t,x[0],y[0],z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,z[1]\,\beta^{(1,0,0,1)}[t,x[0],y[0],z[0]]-2\,m\,y[1]\,z[1]\,\Xi^{(1,0,0,1)}[t,x[0],y[0],z[0]]+2\,m\,y[1]\,\beta^{(1,0,1,0)}[t,x[0],y[0],z[0]]-2\,m\,y[1]^2\,\Xi^{(1,0,1,0)}[t,x[0],y[0],z[0]]+2\,m\,x[1]\,\beta^{(1,1,0,0)}[t,x[0],y[0],z[0]]-2\,m\,x[1]\,y[1]\,\Xi^{(1,1,0,0)}[t,x[0],y[0],z[0]]+m\,\beta^{(2,0,0,0)}[t,x[0],y[0],z[0]]-m\,y[1]\,\Xi^{(2,0,0,0)}[t,x[0],y[0],z[0]],$$

$$-\left(\left(z[0]^2\,\gamma[t,x[0],y[0],z[0]]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}\right)+\left(\gamma[t,x[0],y[0],z[0]]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$

$$\left(z[0]^2\,\gamma[t, x[0], y[0], z[0]]\,v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)+$$

$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\,x[0]\,z[0]\,\alpha[t, x[0], y[0], z[0]]$$

$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$

$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\,y[0]\,z[0]\,\beta[t, x[0], y[0], z[0]]$$

$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)-$$

$$\left(z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\gamma^{(0,0,0,1)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$

$$\left(3\,z[0]\,z[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,0,1)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+m\,z[1]^2\,\gamma^{(0,0,0,2)}[t, x[0], y[0], z[0]]-$$

$$m\,z[1]^3\,\Xi^{(0,0,0,2)}[t, x[0], y[0], z[0]]-$$

$$\left(y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\gamma^{(0,0,1,0)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$

$$\left(2\,y[1]\,z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,1,0)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$

$$\left(y[0]\,z[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,0,1,0)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,y[1]\,z[1]\,\gamma^{(0,0,1,1)}[t, x[0], y[0], z[0]]-$$

$$2\,m\,y[1]\,z[1]^2\,\Xi^{(0,0,1,1)}[t, x[0], y[0], z[0]]+m\,y[1]^2\,\gamma^{(0,0,2,0)}[t, x[0], y[0], z[0]]-$$

$$m\,y[1]^2\,z[1]\,\Xi^{(0,0,2,0)}[t, x[0], y[0], z[0]]-$$

$$\left(x[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\gamma^{(0,1,0,0)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$

$$\left(2\,x[1]\,z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,1,0,0)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$

$$\left(x[0]\,z[1]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(0,1,0,0)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,x[1]\,z[1]\,\gamma^{(0,1,0,1)}[t, x[0], y[0], z[0]]-$$

$$2\,m\,x[1]\,z[1]^2\,\Xi^{(0,1,0,1)}[t, x[0], y[0], z[0]]+$$

$$2\,m\,x[1]\,y[1]\,\gamma^{(0,1,1,0)}[t, x[0], y[0], z[0]]-$$

$$2\,m\,x[1]\,y[1]\,z[1]\,\Xi^{(0,1,1,0)}[t, x[0], y[0], z[0]]+$$

$$m\,x[1]^2\,\gamma^{(0,2,0,0)}[t, x[0], y[0], z[0]]-m\,x[1]^2\,z[1]\,\Xi^{(0,2,0,0)}[t, x[0], y[0], z[0]]+$$

$$\left(2\,z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\Xi^{(1,0,0,0)}[t, x[0], y[0], z[0]]\right)\Big/$$

$$\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+2\,m\,z[1]\,\gamma^{(1,0,0,1)}[t,x[0],y[0],z[0]]-$$
$$2\,m\,z[1]^2\,\Xi^{(1,0,0,1)}[t,x[0],y[0],z[0]]+2\,m\,y[1]\,\gamma^{(1,0,1,0)}[t,x[0],y[0],z[0]]-$$
$$2\,m\,y[1]\,z[1]\,\Xi^{(1,0,1,0)}[t,x[0],y[0],z[0]]+$$
$$2\,m\,x[1]\,\gamma^{(1,1,0,0)}[t,x[0],y[0],z[0]]-2\,m\,x[1]\,z[1]\,\Xi^{(1,1,0,0)}[t,x[0],y[0],z[0]]+$$
$$m\,\gamma^{(2,0,0,0)}[t,x[0],y[0],z[0]]-m\,z[1]\,\Xi^{(2,0,0,0)}[t,x[0],y[0],z[0]]\Big\}$$

```
variables = Flatten[Table[Table[var[j], {j, 1, 2}], {var, DepVar}]]
Column[GetConditionsForPointSymmetries[zero, variables]] /.
 {Sqrt[x[0]^2 + y[0]^2 + z[0]^2] → r,
  (x[0]^2 + y[0]^2 + z[0]^2)^(3/2) → r^3, x[0]^2 + y[0]^2 + z[0]^2 → r^2}
```

$$\{x[1],x[2],y[1],y[2],z[1],z[2]\}$$

$$m\,r^3\,\alpha^{(0,0,0,2)}[t,x[0],y[0],z[0]]$$
$$m\,r^3\,\beta^{(0,0,0,2)}[t,x[0],y[0],z[0]]$$
$$-m\,r^3\,\Xi^{(0,0,0,2)}[t,x[0],y[0],z[0]]$$
$$2\,m\,r^3\,\alpha^{(0,0,1,1)}[t,x[0],y[0],z[0]]$$
$$-2\,m\,r^3\,\Xi^{(0,0,1,1)}[t,x[0],y[0],z[0]]$$
$$m\,r^3\,\alpha^{(0,0,2,0)}[t,x[0],y[0],z[0]]$$
$$m\,r^3\,\gamma^{(0,0,2,0)}[t,x[0],y[0],z[0]]$$
$$-m\,r^3\,\Xi^{(0,0,2,0)}[t,x[0],y[0],z[0]]$$
$$2\,m\,r^3\,\beta^{(0,1,0,1)}[t,x[0],y[0],z[0]]$$
$$-2\,m\,r^3\,\Xi^{(0,1,0,1)}[t,x[0],y[0],z[0]]$$
$$2\,m\,r^3\,\gamma^{(0,1,1,0)}[t,x[0],y[0],z[0]]$$
$$-2\,m\,r^3\,\Xi^{(0,1,1,0)}[t,x[0],y[0],z[0]]$$
$$m\,r^3\,\beta^{(0,2,0,0)}[t,x[0],y[0],z[0]]$$
$$m\,r^3\,\gamma^{(0,2,0,0)}[t,x[0],y[0],z[0]]$$
$$-m\,r^3\,\Xi^{(0,2,0,0)}[t,x[0],y[0],z[0]]$$
$$2\,r^2\left(x[0]\,v'[r]\,\Xi^{(0,0,0,1)}[t,x[0],y[0],z[0]]+m\,r\,\alpha^{(1,0,0,1)}[t,x[0],y[0],z[0]]\right)$$
$$2\,r^2\left(y[0]\,v'[r]\,\Xi^{(0,0,0,1)}[t,x[0],y[0],z[0]]+m\,r\,\beta^{(1,0,0,1)}[t,x[0],y[0],z[0]]\right)$$
$$m\,r^3\left(\gamma^{(0,0,0,2)}[t,x[0],y[0],z[0]]-2\,\Xi^{(1,0,0,1)}[t,x[0],y[0],z[0]]\right)$$
$$2\,m\,r^3\left(\beta^{(0,0,1,1)}[t,x[0],y[0],z[0]]-\Xi^{(1,0,0,1)}[t,x[0],y[0],z[0]]\right)$$
$$2\,m\,r^3\left(\alpha^{(0,1,0,1)}[t,x[0],y[0],z[0]]-\Xi^{(1,0,0,1)}[t,x[0],y[0],z[0]]\right)$$
$$2\,r^2\left(x[0]\,v'[r]\,\Xi^{(0,0,1,0)}[t,x[0],y[0],z[0]]+m\,r\,\alpha^{(1,0,1,0)}[t,x[0],y[0],z[0]]\right)$$
$$2\,r^2\left(z[0]\,v'[r]\,\Xi^{(0,0,1,0)}[t,x[0],y[0],z[0]]+m\,r\,\gamma^{(1,0,1,0)}[t,x[0],y[0],z[0]]\right)$$
$$m\,r^3\left(\beta^{(0,0,2,0)}[t,x[0],y[0],z[0]]-2\,\Xi^{(1,0,1,0)}[t,x[0],y[0],z[0]]\right)$$
$$2\,m\,r^3\left(\gamma^{(0,0,1,1)}[t,x[0],y[0],z[0]]-\Xi^{(1,0,1,0)}[t,x[0],y[0],z[0]]\right)$$
$$2\,m\,r^3\left(\alpha^{(0,1,1,0)}[t,x[0],y[0],z[0]]-\Xi^{(1,0,1,0)}[t,x[0],y[0],z[0]]\right)$$
$$2\,r^2\left(y[0]\,v'[r]\,\Xi^{(0,1,0,0)}[t,x[0],y[0],z[0]]+m\,r\,\beta^{(1,1,0,0)}[t,x[0],y[0],z[0]]\right)$$
$$2\,r^2\left(z[0]\,v'[r]\,\Xi^{(0,1,0,0)}[t,x[0],y[0],z[0]]+m\,r\,\gamma^{(1,1,0,0)}[t,x[0],y[0],z[0]]\right)$$
$$m\,r^3\left(\alpha^{(0,2,0,0)}[t,x[0],y[0],z[0]]-2\,\Xi^{(1,1,0,0)}[t,x[0],y[0],z[0]]\right)$$
$$2\,m\,r^3\left(\gamma^{(0,1,0,1)}[t,x[0],y[0],z[0]]-\Xi^{(1,1,0,0)}[t,x[0],y[0],z[0]]\right)$$
$$2\,m\,r^3\left(\beta^{(0,1,1,0)}[t,x[0],y[0],z[0]]-\Xi^{(1,1,0,0)}[t,x[0],y[0],z[0]]\right)$$

```
y[0]^2 α[t, x[0], y[0], z[0]] v'[r] +
 z[0]^2 α[t, x[0], y[0], z[0]] v'[r] - x[0] y[0] β[t, x[0], y[0], z[0]] v'[r] -
 x[0] z[0] γ[t, x[0], y[0], z[0]] v'[r] + r x[0]^2 α[t, x[0], y[0], z[0]] v''[r] +
 r x[0] y[0] β[t, x[0], y[0], z[0]] v''[r] + r x[0] z[0] γ[t, x[0], y[0], z[0]] v''[r] -
 x[0]^2 z[0] v'[r] α^(0,0,0,1)[t, x[0], y[0], z[0]] -
 y[0]^2 z[0] v'[r] α^(0,0,0,1)[t, x[0], y[0], z[0]] -
 z[0]^3 v'[r] α^(0,0,0,1)[t, x[0], y[0], z[0]] -
 x[0]^2 y[0] v'[r] α^(0,0,1,0)[t, x[0], y[0], z[0]] -
 y[0]^3 v'[r] α^(0,0,1,0)[t, x[0], y[0], z[0]] -
 y[0] z[0]^2 v'[r] α^(0,0,1,0)[t, x[0], y[0], z[0]] -
 x[0]^3 v'[r] α^(0,1,0,0)[t, x[0], y[0], z[0]] -
 x[0] y[0]^2 v'[r] α^(0,1,0,0)[t, x[0], y[0], z[0]] -
 x[0] z[0]^2 v'[r] α^(0,1,0,0)[t, x[0], y[0], z[0]] +
 2 x[0]^3 v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] +
 2 x[0] y[0]^2 v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] +
 2 x[0] z[0]^2 v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] +
 m r x[0]^2 α^(2,0,0,0)[t, x[0], y[0], z[0]] +
 m r y[0]^2 α^(2,0,0,0)[t, x[0], y[0], z[0]] + m r z[0]^2 α^(2,0,0,0)[t, x[0], y[0], z[0]]
-x[0] y[0] α[t, x[0], y[0], z[0]] v'[r] +
 x[0]^2 β[t, x[0], y[0], z[0]] v'[r] + z[0]^2 β[t, x[0], y[0], z[0]] v'[r] -
 y[0] z[0] γ[t, x[0], y[0], z[0]] v'[r] + r x[0] y[0] α[t, x[0], y[0], z[0]] v''[r] +
 r y[0]^2 β[t, x[0], y[0], z[0]] v''[r] + r y[0] z[0] γ[t, x[0], y[0], z[0]] v''[r] -
 x[0]^2 z[0] v'[r] β^(0,0,0,1)[t, x[0], y[0], z[0]] -
 y[0]^2 z[0] v'[r] β^(0,0,0,1)[t, x[0], y[0], z[0]] -
 z[0]^3 v'[r] β^(0,0,0,1)[t, x[0], y[0], z[0]] -
 x[0]^2 y[0] v'[r] β^(0,0,1,0)[t, x[0], y[0], z[0]] -
 y[0]^3 v'[r] β^(0,0,1,0)[t, x[0], y[0], z[0]] -
 y[0] z[0]^2 v'[r] β^(0,0,1,0)[t, x[0], y[0], z[0]] -
 x[0]^3 v'[r] β^(0,1,0,0)[t, x[0], y[0], z[0]] -
 x[0] y[0]^2 v'[r] β^(0,1,0,0)[t, x[0], y[0], z[0]] -
 x[0] z[0]^2 v'[r] β^(0,1,0,0)[t, x[0], y[0], z[0]] +
 2 x[0]^2 y[0] v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] +
 2 y[0]^3 v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] +
 2 y[0] z[0]^2 v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] +
 m r x[0]^2 β^(2,0,0,0)[t, x[0], y[0], z[0]] +
 m r y[0]^2 β^(2,0,0,0)[t, x[0], y[0], z[0]] + m r z[0]^2 β^(2,0,0,0)[t, x[0], y[0], z[0]]
-x[0] z[0] α[t, x[0], y[0], z[0]] v'[r] - y[0] z[0] β[t, x[0], y[0], z[0]] v'[r] +
 x[0]^2 γ[t, x[0], y[0], z[0]] v'[r] + y[0]^2 γ[t, x[0], y[0], z[0]] v'[r] +
 r x[0] z[0] α[t, x[0], y[0], z[0]] v''[r] + r y[0] z[0] β[t, x[0], y[0], z[0]] v''[r] +
 r z[0]^2 γ[t, x[0], y[0], z[0]] v''[r] - x[0]^2 z[0] v'[r] γ^(0,0,0,1)[t, x[0], y[0], z[0]] -
 y[0]^2 z[0] v'[r] γ^(0,0,0,1)[t, x[0], y[0], z[0]] -
 z[0]^3 v'[r] γ^(0,0,0,1)[t, x[0], y[0], z[0]] -
 x[0]^2 y[0] v'[r] γ^(0,0,1,0)[t, x[0], y[0], z[0]] -
 y[0]^3 v'[r] γ^(0,0,1,0)[t, x[0], y[0], z[0]] -
 y[0] z[0]^2 v'[r] γ^(0,0,1,0)[t, x[0], y[0], z[0]] -
 x[0]^3 v'[r] γ^(0,1,0,0)[t, x[0], y[0], z[0]] -
 x[0] y[0]^2 v'[r] γ^(0,1,0,0)[t, x[0], y[0], z[0]] -
 x[0] z[0]^2 v'[r] γ^(0,1,0,0)[t, x[0], y[0], z[0]] +
 2 x[0]^2 z[0] v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] +
 2 y[0]^2 z[0] v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] +
 2 z[0]^3 v'[r] Ξ^(1,0,0,0)[t, x[0], y[0], z[0]] + m r x[0]^2 γ^(2,0,0,0)[t, x[0], y[0], z[0]] +
 m r y[0]^2 γ^(2,0,0,0)[t, x[0], y[0], z[0]] + m r z[0]^2 γ^(2,0,0,0)[t, x[0], y[0], z[0]]
r^2 (3 z[0] v'[r] Ξ^(0,0,0,1)[t, x[0], y[0], z[0]] +
    y[0] v'[r] Ξ^(0,0,1,0)[t, x[0], y[0], z[0]] + x[0] v'[r] Ξ^(0,1,0,0)[t, x[0], y[0], z[0]] +
    2 m r γ^(1,0,0,1)[t, x[0], y[0], z[0]] - m r Ξ^(2,0,0,0)[t, x[0], y[0], z[0]])
```

$$r^2 \left(z[0]\, v'[r]\, \Xi^{(0,0,0,1)}[t, x[0], y[0], z[0]] + 3\, y[0]\, v'[r]\, \Xi^{(0,0,1,0)}[t, x[0], y[0], z[0]] + x[0]\, v'[r]\, \Xi^{(0,1,0,0)}[t, x[0], y[0], z[0]] + 2\, m\, r\, \beta^{(1,0,1,0)}[t, x[0], y[0], z[0]] - m\, r\, \Xi^{(2,0,0,0)}[t, x[0], y[0], z[0]]\right)$$

$$r^2 \left(z[0]\, v'[r]\, \Xi^{(0,0,0,1)}[t, x[0], y[0], z[0]] + y[0]\, v'[r]\, \Xi^{(0,0,1,0)}[t, x[0], y[0], z[0]] + 3\, x[0]\, v'[r]\, \Xi^{(0,1,0,0)}[t, x[0], y[0], z[0]] + 2\, m\, r\, \alpha^{(1,1,0,0)}[t, x[0], y[0], z[0]] - m\, r\, \Xi^{(2,0,0,0)}[t, x[0], y[0], z[0]]\right)$$

Second ansatz

```
ξ[t] = δx[t] x[t] + δy[t] y[t] + δz[t] z[t] + δ0[t];
η[x] = αy[t, x[t]] y[t] + αz[t, x[t]] z[t] + α0[t, x[t]];
η[y] = βx[t, y[t]] x[t] + βz[t, y[t]] z[t] + β0[t, y[t]];
η[z] = γx[t, z[t]] x[t] + γy[t, z[t]] y[t] + γ0[t, z[t]];
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

$$x[0] = x[t]$$

$$x[1] = x'[t]$$

$$x[2] = x''[t]$$

$$y[0] = y[t]$$

$$y[1] = y'[t]$$

$$y[2] = y''[t]$$

$$z[0] = z[t]$$

$$z[1] = z'[t]$$

$$z[2] = z''[t]$$

$$\left\{-\left(\left(y[0]\, \alpha y[t, x[0]]\, v'\left[\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right]\right) \Big/ \left(\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right)\right) - \left(z[0]\, \alpha z[t, x[0]]\, v'\left[\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right]\right) \Big/ \left(\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right) + \frac{1}{\sqrt{x[0]^2 + y[0]^2 + z[0]^2}}\, 2\, x[0]\, v'\left[\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right] \left(x[1]\, \delta x[t] + y[1]\, \delta y[t] + z[1]\, \delta z[t] + \delta 0'[t] + x[0]\, \delta x'[t] + y[0]\, \delta y'[t] + z[0]\, \delta z'[t]\right) + \frac{1}{\left(x[0]^2 + y[0]^2 + z[0]^2\right)^{3/2}} x[0]\, y[0] \left(\beta 0[t, y[0]] + x[0]\, \beta x[t, y[0]] + z[0]\, \beta z[t, y[0]]\right) \left(-v'\left[\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right] + \sqrt{x[0]^2 + y[0]^2 + z[0]^2}\, v''\left[\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right]\right) + \frac{1}{\left(x[0]^2 + y[0]^2 + z[0]^2\right)^{3/2}} x[0]\, z[0] \left(\gamma 0[t, z[0]] + x[0]\, \gamma x[t, z[0]] + y[0]\, \gamma y[t, z[0]]\right) \left(-v'\left[\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right] + \sqrt{x[0]^2 + y[0]^2 + z[0]^2}\, v''\left[\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right]\right) + \frac{1}{\left(x[0]^2 + y[0]^2 + z[0]^2\right)^{3/2}} \left(\alpha 0[t, x[0]] + y[0]\, \alpha y[t, x[0]] + z[0]\, \alpha z[t, x[0]]\right) \left(\left(y[0]^2 + z[0]^2\right) v'\left[\sqrt{x[0]^2 + y[0]^2 + z[0]^2}\right] +$$

$$x[0]^2 \sqrt{x[0]^2+y[0]^2+z[0]^2}\ v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\Big) +$$

$$x[1]\ \left(\left(x[0]\ \delta x[t]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right) +\right.$$

$$\left(y[0]\ \delta y[t]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right) +$$

$$\left(z[0]\ \delta z[t]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right) -$$

$$2\,m\,x[1]\ \delta x'[t] - 2\,m\,y[1]\ \delta y'[t] - 2\,m\,z[1]\ \delta z'[t] -$$

$$\left.m\,\delta 0''[t] - m\,x[0]\ \delta x''[t] - m\,y[0]\ \delta y''[t] - m\,z[0]\ \delta z''[t]\right) -$$

$$\left(x[0]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\alpha 0^{(0,1)}[t,x[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right) +$$

$$2\,m\,y[1]\ \left(x[1]\ \alpha y^{(0,1)}[t,x[0]] + \alpha y^{(1,0)}[t,x[0]]\right) +$$

$$2\,m\,z[1]\ \left(x[1]\ \alpha z^{(0,1)}[t,x[0]] + \alpha z^{(1,0)}[t,x[0]]\right) +$$

$$m\,x[1]\ \alpha 0^{(1,1)}[t,x[0]] +$$

$$m\,x[1]\ \left(x[1]\ \alpha 0^{(0,2)}[t,x[0]] + \alpha 0^{(1,1)}[t,x[0]]\right) +$$

$$m\,\alpha 0^{(2,0)}[t,x[0]] + y[0]$$

$$\left(-\left(\left(x[0]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\alpha y^{(0,1)}[t,x[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right) +\right.$$

$$\left.m\left(x[1]^2\,\alpha y^{(0,2)}[t,x[0]] + 2\,x[1]\ \alpha y^{(1,1)}[t,x[0]] + \alpha y^{(2,0)}[t,x[0]]\right)\right) + z[0]$$

$$\left(-\left(\left(x[0]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\alpha z^{(0,1)}[t,x[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right) +\right.$$

$$\left.m\left(x[1]^2\,\alpha z^{(0,2)}[t,x[0]] + 2\,x[1]\ \alpha z^{(1,1)}[t,x[0]] + \alpha z^{(2,0)}[t,x[0]]\right)\right),$$

$$-\left(\left(x[0]\ \beta x[t,y[0]]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right) -$$

$$\left(z[0]\ \beta z[t,y[0]]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right) +$$

$$\frac{1}{\sqrt{x[0]^2+y[0]^2+z[0]^2}}$$

$$2\,y[0]\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\left(x[1]\ \delta x[t] + y[1]\ \delta y[t] + z[1]\ \delta z[t] +\right.$$

$$\left.\delta 0'[t] + x[0]\ \delta x'[t] + y[0]\ \delta y'[t] + z[0]\ \delta z'[t]\right) + \frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}$$

$$x[0]\ y[0]\ \left(\alpha 0[t,x[0]] + y[0]\ \alpha y[t,x[0]] + z[0]\ \alpha z[t,x[0]]\right)$$

$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right] + \sqrt{x[0]^2+y[0]^2+z[0]^2}\ v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\right) +$$

$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\ y[0]\ z[0]$$

$$\left(\gamma 0[t,z[0]] + x[0]\ \gamma x[t,z[0]] + y[0]\ \gamma y[t,z[0]]\right)$$

$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right] + \sqrt{x[0]^2+y[0]^2+z[0]^2}\ v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\right) +$$

$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\ \left(\beta 0[t,y[0]] + x[0]\ \beta x[t,y[0]] + z[0]\ \beta z[t,y[0]]\right)$$

$$\left(\left(x[0]^2+z[0]^2\right)\ v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right] +\right.$$

$$\left.y[0]^2 \sqrt{x[0]^2+y[0]^2+z[0]^2}\ v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\ \right]\right) +$$

$$
\begin{aligned}
&y[1]\left(\left(x[0]\,\delta x[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\right.\\
&\quad\left(y[0]\,\delta y[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\\
&\quad\left(z[0]\,\delta z[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)-\\
&\quad 2\,m\,x[1]\,\delta x'[t]-2\,m\,y[1]\,\delta y'[t]-2\,m\,z[1]\,\delta z'[t]-\\
&\quad\left.m\,\delta 0''[t]-m\,x[0]\,\delta x''[t]-m\,y[0]\,\delta y''[t]-m\,z[0]\,\delta z''[t]\right)-\\
&\left(y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\beta 0^{(0,1)}[t,y[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\\
&2\,m\,x[1]\left(y[1]\,\beta x^{(0,1)}[t,y[0]]+\beta x^{(1,0)}[t,y[0]]\right)+\\
&2\,m\,z[1]\left(y[1]\,\beta z^{(0,1)}[t,y[0]]+\beta z^{(1,0)}[t,y[0]]\right)+\\
&m\,y[1]\,\beta 0^{(1,1)}[t,y[0]]+\\
&m\,y[1]\left(y[1]\,\beta 0^{(0,2)}[t,y[0]]+\beta 0^{(1,1)}[t,y[0]]\right)+\\
&m\,\beta 0^{(2,0)}[t,y[0]]+x[0]\\
&\quad\left(-\left(\left(y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\beta x^{(0,1)}[t,y[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right)+\right.\\
&\quad\left.m\left(y[1]^2\,\beta x^{(0,2)}[t,y[0]]+2\,y[1]\,\beta x^{(1,1)}[t,y[0]]+\beta x^{(2,0)}[t,y[0]]\right)\right)+z[0]\\
&\quad\left(-\left(\left(y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\beta z^{(0,1)}[t,y[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right)+\right.\\
&\quad\left.m\left(y[1]^2\,\beta z^{(0,2)}[t,y[0]]+2\,y[1]\,\beta z^{(1,1)}[t,y[0]]+\beta z^{(2,0)}[t,y[0]]\right)\right),\\
&-\left(\left(x[0]\,\gamma x[t,z[0]]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right)-\\
&\left(y[0]\,\gamma y[t,z[0]]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\\
&\frac{1}{\sqrt{x[0]^2+y[0]^2+z[0]^2}}\\
&\quad 2\,z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\left(x[1]\,\delta x[t]+y[1]\,\delta y[t]+z[1]\,\delta z[t]+\right.\\
&\quad\left.\delta 0'[t]+x[0]\,\delta x'[t]+y[0]\,\delta y'[t]+z[0]\,\delta z'[t]\right)+\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\\
&\quad x[0]\,z[0]\left(\alpha 0[t,x[0]]+y[0]\,\alpha y[t,x[0]]+z[0]\,\alpha z[t,x[0]]\right)\\
&\quad\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+\\
&\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\,y[0]\,z[0]\\
&\quad\left(\beta 0[t,y[0]]+x[0]\,\beta x[t,y[0]]+z[0]\,\beta z[t,y[0]]\right)\\
&\quad\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+\\
&\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\left(\gamma 0[t,z[0]]+x[0]\,\gamma x[t,z[0]]+y[0]\,\gamma y[t,z[0]]\right)\\
&\quad\left(\left(x[0]^2+y[0]^2\right)v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.\\
&\quad\left.z[0]^2\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+\\
&z[1]\left(\left(x[0]\,\delta x[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+\right.
\end{aligned}
$$

$$\left(y[0]\,\delta y[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(z[0]\,\delta z[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)-$$
$$2\,m\,x[1]\,\delta x'[t]-2\,m\,y[1]\,\delta y'[t]-2\,m\,z[1]\,\delta z'[t]-$$
$$m\,\delta 0''[t]-m\,x[0]\,\delta x''[t]-m\,y[0]\,\delta y''[t]-m\,z[0]\,\delta z''[t]\Big)-$$
$$\left(z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\gamma 0^{(0,1)}[t,z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$2\,m\,x[1]\left(z[1]\,\gamma x^{(0,1)}[t,z[0]]+\gamma x^{(1,0)}[t,z[0]]\right)+$$
$$2\,m\,y[1]\left(z[1]\,\gamma y^{(0,1)}[t,z[0]]+\gamma y^{(1,0)}[t,z[0]]\right)+$$
$$m\,z[1]\,\gamma 0^{(1,1)}[t,z[0]]+$$
$$m\,z[1]\left(z[1]\,\gamma 0^{(0,2)}[t,z[0]]+\gamma 0^{(1,1)}[t,z[0]]\right)+$$
$$m\,\gamma 0^{(2,0)}[t,z[0]]+x[0]$$
$$\left(-\left(\left(z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\gamma x^{(0,1)}[t,z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right)+\right.$$
$$\left.m\left(z[1]^2\,\gamma x^{(0,2)}[t,z[0]]+2\,z[1]\,\gamma x^{(1,1)}[t,z[0]]+\gamma x^{(2,0)}[t,z[0]]\right)\right)+y[0]$$
$$\left(-\left(\left(z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\gamma y^{(0,1)}[t,z[0]]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right)+\right.$$
$$\left.m\left(z[1]^2\,\gamma y^{(0,2)}[t,z[0]]+2\,z[1]\,\gamma y^{(1,1)}[t,z[0]]+\gamma y^{(2,0)}[t,z[0]]\right)\right)\Big\}$$

```
variables = Flatten[Table[Table[var[j], {j, 1, 2}], {var, DepVar}]]
Column[GetConditionsForPointSymmetries[zero, variables]] /.
 {Sqrt[x[0]^2 + y[0]^2 + z[0]^2] → r,
  (x[0]^2 + y[0]^2 + z[0]^2)^(3/2) → r^3, x[0]^2 + y[0]^2 + z[0]^2 → r^2}
```

$$\{x[1],\,x[2],\,y[1],\,y[2],\,z[1],\,z[2]\}$$

$$-2\,m\,r^3\left(\delta y'[t]-\alpha y^{(0,1)}[t,x[0]]\right)$$
$$-2\,m\,r^3\left(\delta z'[t]-\alpha z^{(0,1)}[t,x[0]]\right)$$
$$-2\,m\,r^3\left(\delta x'[t]-\beta x^{(0,1)}[t,y[0]]\right)$$
$$-2\,m\,r^3\left(\delta z'[t]-\beta z^{(0,1)}[t,y[0]]\right)$$
$$-2\,m\,r^3\left(\delta x'[t]-\gamma x^{(0,1)}[t,z[0]]\right)$$
$$-2\,m\,r^3\left(\delta y'[t]-\gamma y^{(0,1)}[t,z[0]]\right)$$
$$m\,r^3\left(-2\,\delta x'[t]+\alpha 0^{(0,2)}[t,x[0]]+y[0]\,\alpha y^{(0,2)}[t,x[0]]+z[0]\,\alpha z^{(0,2)}[t,x[0]]\right)$$
$$m\,r^3\left(-2\,\delta y'[t]+\beta 0^{(0,2)}[t,y[0]]+x[0]\,\beta x^{(0,2)}[t,y[0]]+z[0]\,\beta z^{(0,2)}[t,y[0]]\right)$$
$$m\,r^3\left(-2\,\delta z'[t]+\gamma 0^{(0,2)}[t,z[0]]+x[0]\,\gamma x^{(0,2)}[t,z[0]]+y[0]\,\gamma y^{(0,2)}[t,z[0]]\right)$$
$$2\,r^2\left(x[0]\,\delta y[t]\,v'[r]+m\,r\,\alpha y^{(1,0)}[t,x[0]]\right)$$
$$2\,r^2\left(x[0]\,\delta z[t]\,v'[r]+m\,r\,\alpha z^{(1,0)}[t,x[0]]\right)$$
$$2\,r^2\left(y[0]\,\delta x[t]\,v'[r]+m\,r\,\beta x^{(1,0)}[t,y[0]]\right)$$
$$2\,r^2\left(y[0]\,\delta z[t]\,v'[r]+m\,r\,\beta z^{(1,0)}[t,y[0]]\right)$$
$$2\,r^2\left(z[0]\,\delta x[t]\,v'[r]+m\,r\,\gamma x^{(1,0)}[t,z[0]]\right)$$
$$2\,r^2\left(z[0]\,\delta y[t]\,v'[r]+m\,r\,\gamma y^{(1,0)}[t,z[0]]\right)$$
$$r^2\left(3\,x[0]\,\delta x[t]\,v'[r]+y[0]\,\delta y[t]\,v'[r]+z[0]\,\delta z[t]\,v'[r]-\right.$$
$$m\,r\,\delta 0''[t]-m\,r\,x[0]\,\delta x''[t]-m\,r\,y[0]\,\delta y''[t]-m\,r\,z[0]\,\delta z''[t]+$$
$$\left.2\,m\,r\,\alpha 0^{(1,1)}[t,x[0]]+2\,m\,r\,y[0]\,\alpha y^{(1,1)}[t,x[0]]+2\,m\,r\,z[0]\,\alpha z^{(1,1)}[t,x[0]]\right)$$

$$
\begin{aligned}
&r^2 \left(x[0]\,\delta x[t]\,v'[r] + 3\,y[0]\,\delta y[t]\,v'[r] + z[0]\,\delta z[t]\,v'[r] - \right.\\
&\quad m\,r\,\delta 0''[t] - m\,r\,x[0]\,\delta x''[t] - m\,r\,y[0]\,\delta y''[t] - m\,r\,z[0]\,\delta z''[t] + \\
&\quad \left. 2\,m\,r\,\beta 0^{(1,1)}[t, y[0]] + 2\,m\,r\,x[0]\,\beta x^{(1,1)}[t, y[0]] + 2\,m\,r\,z[0]\,\beta z^{(1,1)}[t, y[0]]\right)\\
&r^2 \left(x[0]\,\delta x[t]\,v'[r] + y[0]\,\delta y[t]\,v'[r] + 3\,z[0]\,\delta z[t]\,v'[r] - \right.\\
&\quad m\,r\,\delta 0''[t] - m\,r\,x[0]\,\delta x''[t] - m\,r\,y[0]\,\delta y''[t] - m\,r\,z[0]\,\delta z''[t] + \\
&\quad \left. 2\,m\,r\,\gamma 0^{(1,1)}[t, z[0]] + 2\,m\,r\,x[0]\,\gamma x^{(1,1)}[t, z[0]] + 2\,m\,r\,y[0]\,\gamma y^{(1,1)}[t, z[0]]\right)\\
&y[0]^2\,\alpha 0[t, x[0]]\,v'[r] + z[0]^2\,\alpha 0[t, x[0]]\,v'[r] - x[0]^2\,y[0]\,\alpha y[t, x[0]]\,v'[r] - \\
&\ x[0]^2\,z[0]\,\alpha z[t, x[0]]\,v'[r] - x[0]\,y[0]\,\beta 0[t, y[0]]\,v'[r] - x[0]^2\,y[0]\,\beta x[t, y[0]]\,v'[r] - \\
&\ x[0]\,y[0]\,z[0]\,\beta z[t, y[0]]\,v'[r] - x[0]\,z[0]\,\gamma 0[t, z[0]]\,v'[r] - \\
&\ x[0]^2\,z[0]\,\gamma x[t, z[0]]\,v'[r] - x[0]\,y[0]\,z[0]\,\gamma y[t, z[0]]\,v'[r] + 2\,x[0]^3\,v'[r]\,\delta 0'[t] + \\
&\ 2\,x[0]\,y[0]^2\,v'[r]\,\delta 0'[t] + 2\,x[0]\,z[0]^2\,v'[r]\,\delta 0'[t] + 2\,x[0]^4\,v'[r]\,\delta x'[t] + \\
&\ 2\,x[0]^2\,y[0]^2\,v'[r]\,\delta x'[t] + 2\,x[0]^2\,z[0]^2\,v'[r]\,\delta x'[t] + 2\,x[0]^3\,y[0]\,v'[r]\,\delta y'[t] + \\
&\ 2\,x[0]\,y[0]^3\,v'[r]\,\delta y'[t] + 2\,x[0]\,y[0]\,z[0]^2\,v'[r]\,\delta y'[t] + 2\,x[0]^3\,z[0]\,v'[r]\,\delta z'[t] + \\
&\ 2\,x[0]\,y[0]^2\,z[0]\,v'[r]\,\delta z'[t] + 2\,x[0]\,z[0]^3\,v'[r]\,\delta z'[t] + r\,x[0]^2\,\alpha 0[t, x[0]]\,v''[r] + \\
&\ r\,x[0]^2\,y[0]\,\alpha y[t, x[0]]\,v''[r] + r\,x[0]^2\,z[0]\,\alpha z[t, x[0]]\,v''[r] + \\
&\ r\,x[0]\,y[0]\,\beta 0[t, y[0]]\,v''[r] + r\,x[0]^2\,y[0]\,\beta x[t, y[0]]\,v''[r] + \\
&\ r\,x[0]\,y[0]\,z[0]\,\beta z[t, y[0]]\,v''[r] + r\,x[0]\,z[0]\,\gamma 0[t, z[0]]\,v''[r] + \\
&\ r\,x[0]^2\,z[0]\,\gamma x[t, z[0]]\,v''[r] + r\,x[0]\,y[0]\,z[0]\,\gamma y[t, z[0]]\,v''[r] - \\
&\ x[0]^3\,v'[r]\,\alpha 0^{(0,1)}[t, x[0]] - x[0]\,y[0]^2\,v'[r]\,\alpha 0^{(0,1)}[t, x[0]] - \\
&\ x[0]\,z[0]^2\,v'[r]\,\alpha 0^{(0,1)}[t, x[0]] - x[0]^3\,y[0]\,v'[r]\,\alpha y^{(0,1)}[t, x[0]] - \\
&\ x[0]\,y[0]^3\,v'[r]\,\alpha y^{(0,1)}[t, x[0]] - x[0]\,y[0]\,z[0]^2\,v'[r]\,\alpha y^{(0,1)}[t, x[0]] - \\
&\ x[0]^3\,z[0]\,v'[r]\,\alpha z^{(0,1)}[t, x[0]] - x[0]\,y[0]^2\,z[0]\,v'[r]\,\alpha z^{(0,1)}[t, x[0]] - \\
&\ x[0]\,z[0]^3\,v'[r]\,\alpha z^{(0,1)}[t, x[0]] + m\,r\,x[0]^2\,\alpha 0^{(2,0)}[t, x[0]] + m\,r\,y[0]^2\,\alpha 0^{(2,0)}[t, x[0]] + \\
&\ m\,r\,z[0]^2\,\alpha 0^{(2,0)}[t, x[0]] + m\,r\,x[0]^2\,y[0]\,\alpha y^{(2,0)}[t, x[0]] + m\,r\,y[0]^3\,\alpha y^{(2,0)}[t, x[0]] + \\
&\ m\,r\,y[0]\,z[0]^2\,\alpha y^{(2,0)}[t, x[0]] + m\,r\,x[0]^2\,z[0]\,\alpha z^{(2,0)}[t, x[0]] + \\
&\ m\,r\,y[0]^2\,z[0]\,\alpha z^{(2,0)}[t, x[0]] + m\,r\,z[0]^3\,\alpha z^{(2,0)}[t, x[0]]\\
&-x[0]\,y[0]\,\alpha 0[t, x[0]]\,v'[r] - x[0]\,y[0]^2\,\alpha y[t, x[0]]\,v'[r] - \\
&\ x[0]\,y[0]\,z[0]\,\alpha z[t, x[0]]\,v'[r] + x[0]^2\,\beta 0[t, y[0]]\,v'[r] + z[0]^2\,\beta 0[t, y[0]]\,v'[r] - \\
&\ x[0]\,y[0]^2\,\beta x[t, y[0]]\,v'[r] - y[0]^2\,z[0]\,\beta z[t, y[0]]\,v'[r] - \\
&\ y[0]\,z[0]\,\gamma 0[t, z[0]]\,v'[r] - x[0]\,y[0]\,z[0]\,\gamma x[t, z[0]]\,v'[r] - \\
&\ y[0]^2\,z[0]\,\gamma y[t, z[0]]\,v'[r] + 2\,x[0]^2\,y[0]\,v'[r]\,\delta 0'[t] + 2\,y[0]^3\,v'[r]\,\delta 0'[t] + \\
&\ 2\,y[0]\,z[0]^2\,v'[r]\,\delta 0'[t] + 2\,x[0]^3\,y[0]\,v'[r]\,\delta x'[t] + 2\,x[0]\,y[0]^3\,v'[r]\,\delta x'[t] + \\
&\ 2\,x[0]\,y[0]\,z[0]^2\,v'[r]\,\delta x'[t] + 2\,x[0]^2\,y[0]^2\,v'[r]\,\delta y'[t] + \\
&\ 2\,y[0]^4\,v'[r]\,\delta y'[t] + 2\,y[0]^2\,z[0]^2\,v'[r]\,\delta y'[t] + 2\,x[0]^2\,y[0]\,z[0]\,v'[r]\,\delta z'[t] + \\
&\ 2\,y[0]^3\,z[0]\,v'[r]\,\delta z'[t] + 2\,y[0]\,z[0]^3\,v'[r]\,\delta z'[t] + r\,x[0]\,y[0]\,\alpha 0[t, x[0]]\,v''[r] + \\
&\ r\,x[0]\,y[0]^2\,\alpha y[t, x[0]]\,v''[r] + r\,x[0]\,y[0]\,z[0]\,\alpha z[t, x[0]]\,v''[r] + \\
&\ r\,y[0]^2\,\beta 0[t, y[0]]\,v''[r] + r\,x[0]\,y[0]^2\,\beta x[t, y[0]]\,v''[r] + \\
&\ r\,y[0]^2\,z[0]\,\beta z[t, y[0]]\,v''[r] + r\,y[0]\,z[0]\,\gamma 0[t, z[0]]\,v''[r] + \\
&\ r\,x[0]\,y[0]\,z[0]\,\gamma x[t, z[0]]\,v''[r] + r\,y[0]^2\,z[0]\,\gamma y[t, z[0]]\,v''[r] - \\
&\ x[0]^2\,y[0]\,v'[r]\,\beta 0^{(0,1)}[t, y[0]] - y[0]^3\,v'[r]\,\beta 0^{(0,1)}[t, y[0]] - \\
&\ y[0]\,z[0]^2\,v'[r]\,\beta 0^{(0,1)}[t, y[0]] - x[0]^3\,y[0]\,v'[r]\,\beta x^{(0,1)}[t, y[0]] - \\
&\ x[0]\,y[0]^3\,v'[r]\,\beta x^{(0,1)}[t, y[0]] - x[0]\,y[0]\,z[0]^2\,v'[r]\,\beta x^{(0,1)}[t, y[0]] - \\
&\ x[0]^2\,y[0]\,z[0]\,v'[r]\,\beta z^{(0,1)}[t, y[0]] - y[0]^3\,z[0]\,v'[r]\,\beta z^{(0,1)}[t, y[0]] - \\
&\ y[0]\,z[0]^3\,v'[r]\,\beta z^{(0,1)}[t, y[0]] + m\,r\,x[0]^2\,\beta 0^{(2,0)}[t, y[0]] + \\
&\ m\,r\,y[0]^2\,\beta 0^{(2,0)}[t, y[0]] + m\,r\,z[0]^2\,\beta 0^{(2,0)}[t, y[0]] + m\,r\,x[0]^3\,\beta x^{(2,0)}[t, y[0]] + \\
&\ m\,r\,x[0]\,y[0]^2\,\beta x^{(2,0)}[t, y[0]] + m\,r\,x[0]\,z[0]^2\,\beta x^{(2,0)}[t, y[0]] + \\
&\ m\,r\,x[0]^2\,z[0]\,\beta z^{(2,0)}[t, y[0]] + m\,r\,y[0]^2\,z[0]\,\beta z^{(2,0)}[t, y[0]] + m\,r\,z[0]^3\,\beta z^{(2,0)}[t, y[0]]
\end{aligned}
$$

```
-x[0] z[0] α0[t, x[0]] v'[r] - x[0] y[0] z[0] αy[t, x[0]] v'[r] -
 x[0] z[0]^2 αz[t, x[0]] v'[r] - y[0] z[0] β0[t, y[0]] v'[r] -
 x[0] y[0] z[0] βx[t, y[0]] v'[r] - y[0] z[0]^2 βz[t, y[0]] v'[r] +
 x[0]^2 γ0[t, z[0]] v'[r] + y[0]^2 γ0[t, z[0]] v'[r] - x[0] z[0]^2 γx[t, z[0]] v'[r] -
 y[0] z[0]^2 γy[t, z[0]] v'[r] + 2 x[0]^2 z[0] v'[r] δ0'[t] + 2 y[0]^2 z[0] v'[r] δ0'[t] +
 2 z[0]^3 v'[r] δ0'[t] + 2 x[0]^3 z[0] v'[r] δx'[t] + 2 x[0] y[0]^2 z[0] v'[r] δx'[t] +
 2 x[0] z[0]^3 v'[r] δx'[t] + 2 x[0]^2 y[0] z[0] v'[r] δy'[t] +
 2 y[0]^3 z[0] v'[r] δy'[t] + 2 y[0] z[0]^3 v'[r] δy'[t] + 2 x[0]^2 z[0]^2 v'[r] δz'[t] +
 2 y[0]^2 z[0]^2 v'[r] δz'[t] + 2 z[0]^4 v'[r] δz'[t] + r x[0] z[0] α0[t, x[0]] v''[r] +
 r x[0] y[0] z[0] αy[t, x[0]] v''[r] + r x[0] z[0]^2 αz[t, x[0]] v''[r] +
 r y[0] z[0] β0[t, y[0]] v''[r] + r x[0] y[0] z[0] βx[t, y[0]] v''[r] +
 r y[0] z[0]^2 βz[t, y[0]] v''[r] + r z[0]^2 γ0[t, z[0]] v''[r] +
 r x[0] z[0]^2 γx[t, z[0]] v''[r] + r y[0] z[0]^2 γy[t, z[0]] v''[r] -
 x[0]^2 z[0] v'[r] γ0^(0,1)[t, z[0]] - y[0]^2 z[0] v'[r] γ0^(0,1)[t, z[0]] -
 z[0]^3 v'[r] γ0^(0,1)[t, z[0]] - x[0]^3 z[0] v'[r] γx^(0,1)[t, z[0]] -
 x[0] y[0]^2 z[0] v'[r] γx^(0,1)[t, z[0]] - x[0] z[0]^3 v'[r] γx^(0,1)[t, z[0]] -
 x[0]^2 y[0] z[0] v'[r] γy^(0,1)[t, z[0]] - y[0]^3 z[0] v'[r] γy^(0,1)[t, z[0]] -
 y[0] z[0]^3 v'[r] γy^(0,1)[t, z[0]] + m r x[0]^2 γ0^(2,0)[t, z[0]] +
 m r y[0]^2 γ0^(2,0)[t, z[0]] + m r z[0]^2 γ0^(2,0)[t, z[0]] + m r x[0]^3 γx^(2,0)[t, z[0]] +
 m r x[0] y[0]^2 γx^(2,0)[t, z[0]] + m r x[0] z[0]^2 γx^(2,0)[t, z[0]] +
 m r x[0]^2 y[0] γy^(2,0)[t, z[0]] + m r y[0]^3 γy^(2,0)[t, z[0]] + m r y[0] z[0]^2 γy^(2,0)[t, z[0]]
```

Third ansatz

```
αy[t, x[t]] = αy0[t] + δy'[t] x[t];
αz[t, x[t]] = αz0[t] + δz'[t] x[t];
βx[t, y[t]] = βx0[t] + δx'[t] y[t];
βz[t, y[t]] = βz0[t] + δz'[t] y[t];
γx[t, z[t]] = γx0[t] + δx'[t] z[t];
γy[t, z[t]] = γy0[t] + δy'[t] z[t];
α0[t, x[t]] = α00[t] + α01[t] x[t] + δx'[t] x[t]^2;
β0[t, y[t]] = β00[t] + β01[t] y[t] + δy'[t] y[t]^2;
γ0[t, z[t]] = γ00[t] + γ01[t] z[t] + δz'[t] z[t]^2;
ξ[t] = δx[t] x[t] + δy[t] y[t] + δz[t] z[t] + δ0[t];
η[x] = αy[t, x[t]] y[t] + αz[t, x[t]] z[t] + α0[t, x[t]];
η[y] = βx[t, y[t]] x[t] + βz[t, y[t]] z[t] + β0[t, y[t]];
η[z] = γx[t, z[t]] x[t] + γy[t, z[t]] y[t] + γ0[t, z[t]];
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

```
x[0] = x[t]

x[1] = x'[t]

x[2] = x''[t]

y[0] = y[t]

y[1] = y'[t]

y[2] = y''[t]

z[0] = z[t]

z[1] = z'[t]

z[2] = z''[t]
```

```
{-((x[0] α01[t] v'[√(x[0]^2 + y[0]^2 + z[0]^2)]) / (√(x[0]^2 + y[0]^2 + z[0]^2))) -
```

$$\left(y[0]\,\alpha y0[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)-$$
$$\left(z[0]\,\alpha z0[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(3\,x[0]\,x[1]\,\delta x[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(x[1]\,y[0]\,\delta y[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(2\,x[0]\,y[1]\,\delta y[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(x[1]\,z[0]\,\delta z[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(2\,x[0]\,z[1]\,\delta z[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$2\,m\,x[1]\,\alpha 01'[t]+2\,m\,y[1]\,\alpha y0'[t]+2\,m\,z[1]\,\alpha z0'[t]+$$
$$\left(2\,x[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\delta 0'[t]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}x[0]\,y[0]\,\left(\beta 00[t]+y[0]\,\beta 01[t]+x[0]\,\beta x0[t]+\right.$$
$$\left.z[0]\,\beta z0[t]+x[0]\,y[0]\,\delta x'[t]+y[0]^2\,\delta y'[t]+y[0]\,z[0]\,\delta z'[t]\right)$$
$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}x[0]\,z[0]\,\left(\gamma 00[t]+z[0]\,\gamma 01[t]+x[0]\,\gamma x0[t]+\right.$$
$$\left.y[0]\,\gamma y0[t]+x[0]\,z[0]\,\delta x'[t]+y[0]\,z[0]\,\delta y'[t]+z[0]^2\,\delta z'[t]\right)$$
$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\left(\alpha 00[t]+x[0]\,\alpha 01[t]+y[0]\,\alpha y0[t]+\right.$$
$$\left.z[0]\,\alpha z0[t]+x[0]^2\,\delta x'[t]+x[0]\,y[0]\,\delta y'[t]+x[0]\,z[0]\,\delta z'[t]\right)$$
$$\left(\left(y[0]^2+z[0]^2\right)v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.$$
$$\left.x[0]^2\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$m\,\alpha 00''[t]+m\,x[0]\,\alpha 01''[t]+m\,y[0]\,\alpha y0''[t]+m\,z[0]\,\alpha z0''[t]-m\,x[1]\,\delta 0''[t]+$$
$$3\,m\,x[0]\,x[1]\,\delta x''[t]+m\,x[1]\,y[0]\,\delta y''[t]+2\,m\,x[0]\,y[1]\,\delta y''[t]+$$
$$m\,x[1]\,z[0]\,\delta z''[t]+2\,m\,x[0]\,z[1]\,\delta z''[t]+m\,x[0]^2\,\delta x^{(3)}[t]+$$
$$m\,x[0]\,y[0]\,\delta y^{(3)}[t]+m\,x[0]\,z[0]\,\delta z^{(3)}[t],$$
$$-\left(\left(y[0]\,\beta 01[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right)-$$
$$\left(x[0]\,\beta x0[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)-$$
$$\left(z[0]\,\beta z0[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(2\,x[1]\,y[0]\,\delta x[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(x[0]\,y[1]\,\delta x[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(3\,y[0]\,y[1]\,\delta y[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$

$$\left(y[1]\,z[0]\,\delta z[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(2\,y[0]\,z[1]\,\delta z[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$2\,m\,y[1]\,\beta 01'[t]+2\,m\,x[1]\,\beta x0'[t]+2\,m\,z[1]\,\beta z0'[t]+$$
$$\left(2\,y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\delta 0'[t]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}x[0]\,y[0]\,\left(\alpha 00[t]+x[0]\,\alpha 01[t]+y[0]\,\alpha y0[t]+\right.$$
$$\left.z[0]\,\alpha z0[t]+x[0]^2\,\delta x'[t]+x[0]\,y[0]\,\delta y'[t]+x[0]\,z[0]\,\delta z'[t]\right)$$
$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}y[0]\,z[0]\,\left(\gamma 00[t]+z[0]\,\gamma 01[t]+x[0]\,\gamma x0[t]+\right.$$
$$\left.y[0]\,\gamma y0[t]+x[0]\,z[0]\,\delta x'[t]+y[0]\,z[0]\,\delta y'[t]+z[0]^2\,\delta z'[t]\right)$$
$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\left(\beta 00[t]+y[0]\,\beta 01[t]+x[0]\,\beta x0[t]+\right.$$
$$\left.z[0]\,\beta z0[t]+x[0]\,y[0]\,\delta x'[t]+y[0]^2\,\delta y'[t]+y[0]\,z[0]\,\delta z'[t]\right)$$
$$\left(\left(x[0]^2+z[0]^2\right)v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.$$
$$\left.y[0]^2\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$m\,\beta 00''[t]+m\,y[0]\,\beta 01''[t]+m\,x[0]\,\beta x0''[t]+m\,z[0]\,\beta z0''[t]-m\,y[1]\,\delta 0''[t]+$$
$$2\,m\,x[1]\,y[0]\,\delta x''[t]+m\,x[0]\,y[1]\,\delta x''[t]+$$
$$3\,m\,y[0]\,y[1]\,\delta y''[t]+m\,y[1]\,z[0]\,\delta z''[t]+2\,m\,y[0]\,z[1]\,\delta z''[t]+$$
$$m\,x[0]\,y[0]\,\delta x^{(3)}[t]+m\,y[0]^2\,\delta y^{(3)}[t]+m\,y[0]\,z[0]\,\delta z^{(3)}[t],$$
$$-\left(\left(z[0]\,\gamma 01[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right)-$$
$$\left(x[0]\,\gamma x0[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)-$$
$$\left(y[0]\,\gamma y0[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(2\,x[1]\,z[0]\,\delta x[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(x[0]\,z[1]\,\delta x[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(2\,y[1]\,z[0]\,\delta y[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(y[0]\,z[1]\,\delta y[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(3\,z[0]\,z[1]\,\delta z[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$2\,m\,z[1]\,\gamma 01'[t]+2\,m\,x[1]\,\gamma x0'[t]+2\,m\,y[1]\,\gamma y0'[t]+$$
$$\left(2\,z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\delta 0'[t]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}x[0]\,z[0]\,\left(\alpha 00[t]+x[0]\,\alpha 01[t]+y[0]\,\alpha y0[t]+\right.$$
$$\left.z[0]\,\alpha z0[t]+x[0]^2\,\delta x'[t]+x[0]\,y[0]\,\delta y'[t]+x[0]\,z[0]\,\delta z'[t]\right)$$

$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}y[0]\,z[0]\,\left(\beta 00[t]+y[0]\,\beta 01[t]+x[0]\,\beta x0[t]+\right.$$
$$\left.z[0]\,\beta z0[t]+x[0]\,y[0]\,\delta x'[t]+y[0]^2\,\delta y'[t]+y[0]\,z[0]\,\delta z'[t]\right)$$
$$\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\left(\gamma 00[t]+z[0]\,\gamma 01[t]+x[0]\,\gamma x0[t]+\right.$$
$$\left.y[0]\,\gamma y0[t]+x[0]\,z[0]\,\delta x'[t]+y[0]\,z[0]\,\delta y'[t]+z[0]^2\,\delta z'[t]\right)$$
$$\left(\left(x[0]^2+y[0]^2\right)v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.$$
$$\left.z[0]^2\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)+$$
$$m\,\gamma 00''[t]+m\,z[0]\,\gamma 01''[t]+m\,x[0]\,\gamma x0''[t]+m\,y[0]\,\gamma y0''[t]-m\,z[1]\,\delta 0''[t]+$$
$$2\,m\,x[1]\,z[0]\,\delta x''[t]+m\,x[0]\,z[1]\,\delta x''[t]+$$
$$2\,m\,y[1]\,z[0]\,\delta y''[t]+m\,y[0]\,z[1]\,\delta y''[t]+3\,m\,z[0]\,z[1]\,\delta z''[t]+$$
$$m\,x[0]\,z[0]\,\delta x^{(3)}[t]+m\,y[0]\,z[0]\,\delta y^{(3)}[t]+m\,z[0]^2\,\delta z^{(3)}[t]\Big\}$$

```
variables = Flatten[Table[Table[var[j], {j, 0, 2}], {var, DepVar}]]
Column[GetConditionsForPointSymmetries[zero (x[0]^2 + y[0]^2 + z[0]^2)^3/2, variables]] /.
 {Sqrt[x[0]^2 + y[0]^2 + z[0]^2] → r,
  (x[0]^2 + y[0]^2 + z[0]^2)^3/2 → r^3, x[0]^2 + y[0]^2 + z[0]^2 → r^2}
```

```
{x[0], x[1], x[2], y[0], y[1], y[2], z[0], z[1], z[2]}
```

$$2\,m\,r\,\alpha y0'[t]$$
$$2\,m\,r\,\alpha z0'[t]$$
$$2\,m\,r\,\beta x0'[t]$$
$$2\,m\,r\,\beta z0'[t]$$
$$2\,m\,r\,\gamma x0'[t]$$
$$2\,m\,r\,\gamma y0'[t]$$
$$-\alpha 00[t]\,(v'[r]-r\,v''[r])$$
$$-\beta 00[t]\,(v'[r]-r\,v''[r])$$
$$-\left(\alpha y0[t]+\beta x0[t]\right)\,(v'[r]-r\,v''[r])$$
$$-\gamma 00[t]\,(v'[r]-r\,v''[r])$$
$$-\left(\alpha z0[t]+\gamma x0[t]\right)\,(v'[r]-r\,v''[r])$$
$$-\left(\beta z0[t]+\gamma y0[t]\right)\,(v'[r]-r\,v''[r])$$
$$r\left(\alpha 00[t]\,v''[r]+m\,\alpha 00''[t]\right)$$
$$\alpha 00[t]\,v'[r]+m\,r\,\alpha 00''[t]$$
$$-\alpha 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\alpha 01[t]\,v''[r]+m\,r\,\alpha 01''[t]$$
$$-\beta 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\beta 01[t]\,v''[r]+m\,r\,\alpha 01''[t]$$
$$-\gamma 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\gamma 01[t]\,v''[r]+m\,r\,\alpha 01''[t]$$
$$m\,r\,\alpha y0''[t]$$
$$-\alpha y0[t]\,v'[r]-\beta x0[t]\,v'[r]+r\,\alpha y0[t]\,v''[r]+r\,\beta x0[t]\,v''[r]+m\,r\,\alpha y0''[t]$$
$$m\,r\,\alpha z0''[t]$$
$$-\alpha z0[t]\,v'[r]-\gamma x0[t]\,v'[r]+r\,\alpha z0[t]\,v''[r]+r\,\gamma x0[t]\,v''[r]+m\,r\,\alpha z0''[t]$$
$$r\left(\beta 00[t]\,v''[r]+m\,\beta 00''[t]\right)$$
$$\beta 00[t]\,v'[r]+m\,r\,\beta 00''[t]$$
$$-\alpha 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\alpha 01[t]\,v''[r]+m\,r\,\beta 01''[t]$$
$$-\beta 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\beta 01[t]\,v''[r]+m\,r\,\beta 01''[t]$$
$$-\gamma 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\gamma 01[t]\,v''[r]+m\,r\,\beta 01''[t]$$
$$m\,r\,\beta x0''[t]$$
$$-\alpha y0[t]\,v'[r]-\beta x0[t]\,v'[r]+r\,\alpha y0[t]\,v''[r]+r\,\beta x0[t]\,v''[r]+m\,r\,\beta x0''[t]$$
$$m\,r\,\beta z0''[t]$$
$$-\beta z0[t]\,v'[r]-\gamma y0[t]\,v'[r]+r\,\beta z0[t]\,v''[r]+r\,\gamma y0[t]\,v''[r]+m\,r\,\beta z0''[t]$$
$$r\left(\gamma 00[t]\,v''[r]+m\,\gamma 00''[t]\right)$$
$$\gamma 00[t]\,v'[r]+m\,r\,\gamma 00''[t]$$
$$-\alpha 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\alpha 01[t]\,v''[r]+m\,r\,\gamma 01''[t]$$
$$-\beta 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\beta 01[t]\,v''[r]+m\,r\,\gamma 01''[t]$$
$$-\gamma 01[t]\,v'[r]+2\,v'[r]\,\delta 0'[t]+r\,\gamma 01[t]\,v''[r]+m\,r\,\gamma 01''[t]$$
$$m\,r\,\gamma x0''[t]$$
$$-\alpha z0[t]\,v'[r]-\gamma x0[t]\,v'[r]+r\,\alpha z0[t]\,v''[r]+r\,\gamma x0[t]\,v''[r]+m\,r\,\gamma x0''[t]$$
$$m\,r\,\gamma y0''[t]$$
$$-\beta z0[t]\,v'[r]-\gamma y0[t]\,v'[r]+r\,\beta z0[t]\,v''[r]+r\,\gamma y0[t]\,v''[r]+m\,r\,\gamma y0''[t]$$
$$m\,r\left(2\,\alpha 01'[t]-\delta 0''[t]\right)$$
$$m\,r\left(2\,\beta 01'[t]-\delta 0''[t]\right)$$
$$m\,r\left(2\,\gamma 01'[t]-\delta 0''[t]\right)$$
$$\delta x[t]\,v'[r]+m\,r\,\delta x''[t]$$
$$2\left(\delta x[t]\,v'[r]+m\,r\,\delta x''[t]\right)$$
$$3\left(\delta x[t]\,v'[r]+m\,r\,\delta x''[t]\right)$$
$$\delta y[t]\,v'[r]+m\,r\,\delta y''[t]$$
$$2\left(\delta y[t]\,v'[r]+m\,r\,\delta y''[t]\right)$$
$$3\left(\delta y[t]\,v'[r]+m\,r\,\delta y''[t]\right)$$
$$\delta z[t]\,v'[r]+m\,r\,\delta z''[t]$$
$$2\left(\delta z[t]\,v'[r]+m\,r\,\delta z''[t]\right)$$
$$3\left(\delta z[t]\,v'[r]+m\,r\,\delta z''[t]\right)$$
$$r\left(\delta x'[t]\,v''[r]+m\,\delta x^{(3)}[t]\right)$$
$$r\left(\delta y'[t]\,v''[r]+m\,\delta y^{(3)}[t]\right)$$
$$r\left(\delta z'[t]\,v''[r]+m\,\delta z^{(3)}[t]\right)$$

Fourth ansatz

```
(* only for V(r) ≠ a r^2 + b *)
αy[t, x[t]] = c[1];
αz[t, x[t]] = -c[3];
βx[t, y[t]] = -c[1];
βz[t, y[t]] = c[2];
γx[t, z[t]] = c[3];
γy[t, z[t]] = -c[2];
α0[t, x[t]] = α01[t] x[t];
β0[t, y[t]] = β01[t] y[t];
γ0[t, z[t]] = γ01[t] z[t];
ξ[t] = δ0[t];
η[x] = αy[t, x[t]] y[t] + αz[t, x[t]] z[t] + α0[t, x[t]];
η[y] = βx[t, y[t]] x[t] + βz[t, y[t]] z[t] + β0[t, y[t]];
η[z] = γx[t, z[t]] x[t] + γy[t, z[t]] y[t] + γ0[t, z[t]];
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

```
x[0] = x[t]

x[1] = x'[t]

x[2] = x''[t]

y[0] = y[t]

y[1] = y'[t]

y[2] = y''[t]

z[0] = z[t]

z[1] = z'[t]

z[2] = z''[t]
```

$$\left\{-\left(\left(x[0]\,z[0]^2\,\gamma 01[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}\right)+2\,m\,x[1]\right.$$
$$\alpha 01'[t]+\left(2\,x[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\delta 0'[t]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(x[0]\,z[0]^2\,\gamma 01[t]\,v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)+$$
$$\left(x[0]^3\,\alpha 01[t]\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.\right.$$
$$\left.\left.\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/$$
$$\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}+\left(x[0]\,y[0]^2\,\beta 01[t]\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.\right.$$
$$\left.\left.\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/$$
$$\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}+m\,x[0]\,\alpha 01''[t]-m\,x[1]\,\delta 0''[t],$$
$$-\left(\left(y[0]\,z[0]^2\,\gamma 01[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}\right)+$$
$$2\,m\,y[1]\,\beta 01'[t]+$$
$$\left(2\,y[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\delta 0'[t]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(y[0]\,z[0]^2\,\gamma 01[t]\,v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)+$$
$$\left(x[0]^2\,y[0]\,\alpha 01[t]\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.\right.$$
$$\left.\left.\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/$$
$$\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}+\left(y[0]^3\,\beta 01[t]\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.\right.$$
$$\left.\left.\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/$$
$$\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}+m\,y[0]\,\beta 01''[t]-m\,y[1]\,\delta 0''[t],$$
$$-\left(\left(z[0]^3\,\gamma 01[t]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}\right)+$$
$$2\,m\,z[1]\,\gamma 01'[t]+$$
$$\left(2\,z[0]\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\delta 0'[t]\right)\Big/\left(\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)+$$
$$\left(z[0]^3\,\gamma 01[t]\,v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\Big/\left(x[0]^2+y[0]^2+z[0]^2\right)+$$
$$\left(x[0]^2\,z[0]\,\alpha 01[t]\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.\right.$$
$$\left.\left.\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/$$
$$\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}+\left(y[0]^2\,z[0]\,\beta 01[t]\left(-v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\right.\right.$$
$$\left.\left.\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/$$
$$\left.\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}+m\,z[0]\,\gamma 01''[t]-m\,z[1]\,\delta 0''[t]\right\}$$

```
variables = Flatten[Table[Table[var[j], {j, 0, 2}], {var, DepVar}]]
Column[GetConditionsForPointSymmetries[zero (x[0]^2 + y[0]^2 + z[0]^2)^(3/2), variables]] /.
 {Sqrt[x[0]^2 + y[0]^2 + z[0]^2] → r,
  (x[0]^2 + y[0]^2 + z[0]^2)^(3/2) → r^3, x[0]^2 + y[0]^2 + z[0]^2 → r^2}
```

```
{x[0], x[1], x[2], y[0], y[1], y[2], z[0], z[1], z[2]}
```

```
-α01[t] v'[r] + 2 v'[r] δ0'[t] + r α01[t] v''[r] + m r α01''[t]
-β01[t] v'[r] + 2 v'[r] δ0'[t] + r β01[t] v''[r] + m r α01''[t]
-γ01[t] v'[r] + 2 v'[r] δ0'[t] + r γ01[t] v''[r] + m r α01''[t]
-α01[t] v'[r] + 2 v'[r] δ0'[t] + r α01[t] v''[r] + m r β01''[t]
-β01[t] v'[r] + 2 v'[r] δ0'[t] + r β01[t] v''[r] + m r β01''[t]
-γ01[t] v'[r] + 2 v'[r] δ0'[t] + r γ01[t] v''[r] + m r β01''[t]
-α01[t] v'[r] + 2 v'[r] δ0'[t] + r α01[t] v''[r] + m r γ01''[t]
-β01[t] v'[r] + 2 v'[r] δ0'[t] + r β01[t] v''[r] + m r γ01''[t]
-γ01[t] v'[r] + 2 v'[r] δ0'[t] + r γ01[t] v''[r] + m r γ01''[t]
m r (2 α01'[t] - δ0''[t])
m r (2 β01'[t] - δ0''[t])
m r (2 γ01'[t] - δ0''[t])
```

Fifth ansatz

```
(* only for V(r) ≠ a r^2 + b *)
αy[t, x[t]] = c[1];
αz[t, x[t]] = -c[3];
βx[t, y[t]] = -c[1];
βz[t, y[t]] = c[2];
γx[t, z[t]] = c[3];
γy[t, z[t]] = -c[2];
α0[t, x[t]] = (δ02 t + α00) x[t];
β0[t, y[t]] = (δ02 t + β00) y[t];
γ0[t, z[t]] = (δ02 t + γ00) z[t];
ξ[t] = δ02 t^2 + δ01 t + δ00;
η[x] = αy[t, x[t]] y[t] + αz[t, x[t]] z[t] + α0[t, x[t]];
η[y] = βx[t, y[t]] x[t] + βz[t, y[t]] z[t] + β0[t, y[t]];
η[z] = γx[t, z[t]] x[t] + γy[t, z[t]] y[t] + γ0[t, z[t]];
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

```
x[0] = x[t]

x[1] = x'[t]

x[2] = x''[t]

y[0] = y[t]

y[1] = y'[t]

y[2] = y''[t]

z[0] = z[t]

z[1] = z'[t]

z[2] = z''[t]
```

$$\left\{\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}\right.$$
$$x[0]\left(\left(-\alpha 00\,x[0]^2+3\,t\,\delta 02\,x[0]^2-\beta 00\,y[0]^2+3\,t\,\delta 02\,y[0]^2-\gamma 00\,z[0]^2+\right.\right.$$
$$\left.\left.3\,t\,\delta 02\,z[0]^2+2\,\delta 01\left(x[0]^2+y[0]^2+z[0]^2\right)\right)v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+$$
$$\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\left(\alpha 00\,x[0]^2+\beta 00\,y[0]^2+\gamma 00\,z[0]^2+\right.$$
$$\left.\left.t\,\delta 02\left(x[0]^2+y[0]^2+z[0]^2\right)\right)v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right),$$
$$\frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}y[0]\left(\left(-\alpha 00\,x[0]^2+3\,t\,\delta 02\,x[0]^2-\beta 00\,y[0]^2+\right.\right.$$
$$\left.\left.3\,t\,\delta 02\,y[0]^2-\gamma 00\,z[0]^2+3\,t\,\delta 02\,z[0]^2+2\,\delta 01\left(x[0]^2+y[0]^2+z[0]^2\right)\right)\right.$$
$$v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}$$
$$\left(\alpha 00\,x[0]^2+\beta 00\,y[0]^2+\gamma 00\,z[0]^2+t\,\delta 02\left(x[0]^2+y[0]^2+z[0]^2\right)\right)$$
$$\left.v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right),\ \frac{1}{\left(x[0]^2+y[0]^2+z[0]^2\right)^{3/2}}$$
$$z[0]\left(\left(-\alpha 00\,x[0]^2+3\,t\,\delta 02\,x[0]^2-\beta 00\,y[0]^2+3\,t\,\delta 02\,y[0]^2-\gamma 00\,z[0]^2+\right.\right.$$
$$\left.\left.3\,t\,\delta 02\,z[0]^2+2\,\delta 01\left(x[0]^2+y[0]^2+z[0]^2\right)\right)v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+$$
$$\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\left(\alpha 00\,x[0]^2+\beta 00\,y[0]^2+\gamma 00\,z[0]^2+\right.$$
$$\left.\left.t\,\delta 02\left(x[0]^2+y[0]^2+z[0]^2\right)\right)v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\}$$

```
variables = Flatten[Table[Table[var[j], {j, 0, 2}], {var, DepVar}]];
AppendTo[variables, t]
Column[GetConditionsForPointSymmetries[zero (x[0]^2 + y[0]^2 + z[0]^2)^(3/2), variables]] /.
 {Sqrt[x[0]^2 + y[0]^2 + z[0]^2] -> r,
  (x[0]^2 + y[0]^2 + z[0]^2)^(3/2) -> r^3, x[0]^2 + y[0]^2 + z[0]^2 -> r^2}
```

```
{x[0], x[1], x[2], y[0], y[1], y[2], z[0], z[1], z[2], t}
```

$$\delta 02\left(3\,v'[r]+r\,v''[r]\right)$$
$$-\alpha 00\,v'[r]+2\,\delta 01\,v'[r]+r\,\alpha 00\,v''[r]$$
$$-\beta 00\,v'[r]+2\,\delta 01\,v'[r]+r\,\beta 00\,v''[r]$$
$$-\gamma 00\,v'[r]+2\,\delta 01\,v'[r]+r\,\gamma 00\,v''[r]$$

The last ansatz

```
nc = 4;
αy[t, x[t]] = c[1];
αz[t, x[t]] = -c[3];
βx[t, y[t]] = -c[1];
βz[t, y[t]] = c[2];
γx[t, z[t]] = c[3];
γy[t, z[t]] = -c[2];
α0[t, x[t]] = 0;
β0[t, y[t]] = 0;
γ0[t, z[t]] = 0;
ξ[t] = c[4];
η[x] = αy[t, x[t]] y[t] + αz[t, x[t]] z[t] + α0[t, x[t]];
η[y] = βx[t, y[t]] x[t] + βz[t, y[t]] z[t] + β0[t, y[t]];
η[z] = γx[t, z[t]] x[t] + γy[t, z[t]] y[t] + γ0[t, z[t]];
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

```
x[0] = x[t]

x[1] = x'[t]

x[2] = x''[t]

y[0] = y[t]

y[1] = y'[t]

y[2] = y''[t]

z[0] = z[t]

z[1] = z'[t]

z[2] = z''[t]
```

```
{0, 0, 0}
```

```
(* Coulomb ansatz *)
nc = 5;
αy[t, x[t]] = c[1];
αz[t, x[t]] = -c[3];
βx[t, y[t]] = -c[1];
βz[t, y[t]] = c[2];
γx[t, z[t]] = c[3];
γy[t, z[t]] = -c[2];
α0[t, x[t]] = 2 / 3 c[5] x[t];
β0[t, y[t]] = 2 / 3 c[5] y[t];
γ0[t, z[t]] = 2 / 3 c[5] z[t];
ξ[t] = c[4] + c[5] t;
η[x] = αy[t, x[t]] y[t] + αz[t, x[t]] z[t] + α0[t, x[t]];
η[y] = βx[t, y[t]] x[t] + βz[t, y[t]] z[t] + β0[t, y[t]];
η[z] = γx[t, z[t]] x[t] + γy[t, z[t]] y[t] + γ0[t, z[t]];
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

$$x[0] = x[t]$$

$$x[1] = x'[t]$$

$$x[2] = x''[t]$$

$$y[0] = y[t]$$

$$y[1] = y'[t]$$

$$y[2] = y''[t]$$

$$z[0] = z[t]$$

$$z[1] = z'[t]$$

$$z[2] = z''[t]$$

$$\left\{\frac{2}{3} c[5]\, x[0] \left(\frac{2\, v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]}{\sqrt{x[0]^2+y[0]^2+z[0]^2}} + v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right),\right.$$

$$\frac{2}{3} c[5]\, y[0] \left(\frac{2\, v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]}{\sqrt{x[0]^2+y[0]^2+z[0]^2}} + v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right),$$

$$\left.\frac{2}{3} c[5]\, z[0] \left(\frac{2\, v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]}{\sqrt{x[0]^2+y[0]^2+z[0]^2}} + v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right\}$$

```
(* Z/r^2 ansatz *)
nc = 6;
αy[t, x[t]] = c[1];
αz[t, x[t]] = -c[3];
βx[t, y[t]] = -c[1];
βz[t, y[t]] = c[2];
γx[t, z[t]] = c[3];
γy[t, z[t]] = -c[2];
α0[t, x[t]] = 1/2 c[5] x[t] + c[6] t x[t];
β0[t, y[t]] = 1/2 c[5] y[t] + c[6] t y[t];
γ0[t, z[t]] = 1/2 c[5] z[t] + c[6] t z[t];
ξ[t] = c[4] + c[5] t + c[6] t^2;
η[x] = αy[t, x[t]] y[t] + αz[t, x[t]] z[t] + α0[t, x[t]];
η[y] = βx[t, y[t]] x[t] + βz[t, y[t]] z[t] + β0[t, y[t]];
η[z] = γx[t, z[t]] x[t] + γy[t, z[t]] y[t] + γ0[t, z[t]];
zero = CheckPointSymmetryOfDE[PDEs, subs, IndepVar, DepVar, ξ, η]
```

```
x[0] = x[t]
x[1] = x'[t]
x[2] = x''[t]
y[0] = y[t]
y[1] = y'[t]
y[2] = y''[t]
z[0] = z[t]
z[1] = z'[t]
z[2] = z''[t]
```

$$\left\{\left(\left(c[5]+2\,t\,c[6]\right)x[0]\left(3\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/\left(2\sqrt{x[0]^2+y[0]^2+z[0]^2}\right),\right.$$
$$\left(\left(c[5]+2\,t\,c[6]\right)y[0]\left(3\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/\left(2\sqrt{x[0]^2+y[0]^2+z[0]^2}\right),$$
$$\left.\left(\left(c[5]+2\,t\,c[6]\right)z[0]\left(3\,v'\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]+\sqrt{x[0]^2+y[0]^2+z[0]^2}\;v''\left[\sqrt{x[0]^2+y[0]^2+z[0]^2}\,\right]\right)\right)\Big/\left(2\sqrt{x[0]^2+y[0]^2+z[0]^2}\right)\right\}$$

Infinitesimal generators, point transformations and commutator table from the last ansatz

```
ShowPointSymmetriesAndCommutationRelations[X, f, ϵ, IndepVar, DepVar, ξ, η, c, nc, {}]
```

Infinitesimal operators:

$$X[1]f[t,x,y,z] = -x\,f^{(0,0,1,0)}[t,x,y,z] + y\,f^{(0,1,0,0)}[t,x,y,z]$$

$$X[2]f[t,x,y,z] = -y\,f^{(0,0,0,1)}[t,x,y,z] + z\,f^{(0,0,1,0)}[t,x,y,z]$$

$$X[3]f[t,x,y,z] = x\,f^{(0,0,0,1)}[t,x,y,z] - z\,f^{(0,1,0,0)}[t,x,y,z]$$

$$X[4]f[t,x,y,z] = f^{(1,0,0,0)}[t,x,y,z]$$

Corresponding global transformations:

$$X[1]\text{ gives }\{t[\epsilon]\to t,\ x[\epsilon]\to x\,\text{Cos}[\epsilon]+y\,\text{Sin}[\epsilon],\ y[\epsilon]\to y\,\text{Cos}[\epsilon]-x\,\text{Sin}[\epsilon],\ z[\epsilon]\to z\}$$

$$X[2]\text{ gives }\{t[\epsilon]\to t,\ x[\epsilon]\to x,\ y[\epsilon]\to y\,\text{Cos}[\epsilon]+z\,\text{Sin}[\epsilon],\ z[\epsilon]\to z\,\text{Cos}[\epsilon]-y\,\text{Sin}[\epsilon]\}$$

$$X[3]\text{ gives }\{t[\epsilon]\to t,\ x[\epsilon]\to x\,\text{Cos}[\epsilon]-z\,\text{Sin}[\epsilon],\ z[\epsilon]\to z\,\text{Cos}[\epsilon]+x\,\text{Sin}[\epsilon],\ y[\epsilon]\to y\}$$

$$X[4]\text{ gives }\{t[\epsilon]\to t+\epsilon,\ x[\epsilon]\to x,\ y[\epsilon]\to y,\ z[\epsilon]\to z\}$$

Commutator table:

	1	2	3	4
1	0	X[3]	-X[2]	0
2	-X[3]	0	X[1]	0
3	X[2]	-X[1]	0	0
4	0	0	0	0