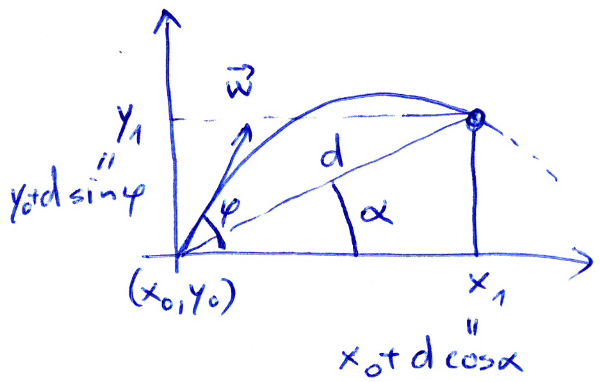

- if we insert the point (x_1, y_1) into the trajectory equation we get

$$y_1 = y_0 + \tan\varphi (x_1 - x_0) - \frac{1}{2}\frac{g}{w^2\cos^2\varphi}(x_1-x_0)^2$$

or

$$d\sin\alpha = \tan\varphi \, d\cos\alpha - \frac{1}{2}\frac{gd^2}{w^2\cos^2\varphi}\cos^2\alpha$$

which has to be solved for the angle φ

- using $\beta = \frac{gd}{w^2}$ and cancelling d, we obtain

$$\sin\alpha\cos^2\varphi + \frac{1}{2}\beta\cos^2\alpha = \cos\alpha \underbrace{\sin\varphi}_{\sqrt{1-\cos^2\varphi}}\cos\varphi$$

substitute $\sqrt{1-\cos^2\varphi}$ and square

the equation $\rightarrow$ quadratic equation for $\cos^2\varphi$:

$$\underbrace{(\sin^2\alpha + \cos^2\alpha)}_{1=a}\cos^4\varphi + \underbrace{(\beta\sin\alpha\cos^2\alpha - \cos^2\alpha)}_{b}\cos^2\varphi + \underbrace{\frac{\beta^2}{4}\cos^4\alpha}_{c} = 0$$

- discriminant $D = b^2 - 4ac = \cos^4\alpha\left[(\beta\sin\alpha - 1)^2 - \beta^2\right]$

if $D \geq 0$ we have solutions

$$\cos^2\varphi = \frac{\cos^2\alpha(1-\beta\sin\alpha) \pm \sqrt{D}}{2}$$

- see the notebook in Mathematica for an example of numerical results

f) determine the minimal speed to reach the point at the distance d and angle α

- at the minimal speed there will be just one angle, i.e. the discriminant $D = 0$

or we have the condition

$$\beta^2(\sin^2\alpha - 1) - 2\beta\sin\alpha + 1 = 0 \quad \text{with } \beta = \frac{gd}{w^2}$$

with the solution $\beta_{1,2} = \frac{\sin\alpha \pm 1}{\cos^2\alpha} = \frac{gd}{w^2}$

- physical solution is $\boxed{w \geq \sqrt{gd}\,\frac{\cos\alpha}{\sqrt{1+\sin\alpha}}}$ for $\alpha \in (0°, 90°)$