Trajectories of particles in presence of the time machine

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I Introduction

Time travel is a phenomenon which has been attracting interest in literature or in a general discussion for a long time. However, only after a formulation of the theory of relativity such considerations could be investigated on a more scientific and solid basis. Even the special relativity shows that different observers experience different times and one of them can "travel" to the future of the other by means of his relative motion (e.g., an astronaut returning from a trip to the center of our galaxy after spending 40 years in a spaceship comes to the Earth where more than 50,000 years have elapsed). The general theory of relativity describes the gravitation as a curved spacetime and that opens a possibility to consider spacetimes with even more complicated geometrical and topological structures. It allows one to study a possibility that an observer could travel even to his own past – his worldline could pass through a geometrically or topologically nontrivial area to a region where the worldline originally started. Worldlines which even cross themselves are called *closed timelike curves (CTCs)* and it is customary to say that spacetimes with CTCs contain *time machines.*

Spacetimes with time machines are causally nontrivial – in such spacetimes you can send a signal to your own past or even try to influence the past – which immediately opens a question of consistency of standard physical laws as we know them. On a formal level it is the question of the existence of solutions of physical equations of motion and the question whether the initial value problem is well possessed. On a less formal level these problems can be phrased as well-known "grandfather paradox": in spacetimes with time machines you have to face a logical riddle what happens if you travel to your own past and kill your grand-father (just for scientific reasons). It would cause that you would not be born and therefore you could not travel to the past – which is clearly inconsistent.

However, a system containing live beings is too complicated with too many unknown physical laws. For this reason in the last two decades physicists have considered various spacetimes with CTCs and studied the consistency of different physical systems in these spacetimes. Surprisingly, such studies showed that for a simple physical system pathology of spacetimes is not so severe and the equations of motion can be consistently solved.

Let us formulate this point more precisely. We consider a spacetime containing a time machine and we want to study a system with well known local physical laws (e.g.,a point particle, or electromagnetic field). We do not change these local laws, i.e., we require that they hold locally in any small spacetime domain. However, in addition to the local laws we also require so-called *principle of consistency*: there must exist a *global consistent* solution of local laws. It means that we allow the system to propagate itself to its own past, however, it must be done in a *consistent* way with the original evolution in the past.

The key question of studies of time machines is whether such globally consistent evolutions exist for given local laws and whether these global evolutions are sufficiently generic. If the local laws did not have any consistent global solution, the spacetime would be clearly pathological and we could rule it out from our consideration. Similarly, the pathology would be serious if the local physical laws had only few globally consistent solutions.

As we said, extensive studies of different systems showed that spacetimes with CTCs are not necessarily causally pathological. Let us mention results for a system of interacting particles [1] or the scalar field theory [2] where it was shown that standard local laws have globally consistent solutions even in the presence of CTCs.

Another surprising result of such studies is that the existence of time machines does not usually restrict a number of consistent solution, but in opposite, it leads to a possibility of more than one globally consistent solutions for given initial values. In spacetimes with time machines we thus usually lose the uniqueness of the evolution.

In our work, we consider a very simple toy model of an elastically interacting point particle in non-relativistic spacetime with a simple time machine. In this model we demonstrate just discussed features. We show that any initial conditions have globally consistent evolutions, that this solution of local laws is not unique, and that a number of solutions is the same even for the initial conditions which lead to an apparently inconsistent solution which mimics discussed "grandfather" paradox. Namely, we consider a situation in which we send a particle through the time machine to its past in such a way that the particle hits itself and prevents thus itself from entering the time machine.

In the next section we shortly discuss the wormhole based time machines and we specify the details of our toy model. In section III we formulate equations of motion for a point particle and discuss globally consistent solutions. In the last section before Summary we shortly discuss the behavior of balls of finite radii. We present only the main results, their derivations

will be presented in a more thorough publication [3].

II Conical spacetime with time machine

The simplest way how to construct spacetimes with CTCs is to consider a wormhole which can be then deformed to a time machine – such a procedure is described in detail , e.g., in reference [4]. The wormhole can be viewed as a shortcut between two spatial places. A simple spatial wormhole can be obtained by cutting out two spheres in a three dimensional space and gluing the surfaces of these holes together, cf. figure 1a. We thus obtain a topologically and geometrically nontrivial space – it is not a simply connected space and the geometry on the glued surface is not flat. In the full spacetime picture, a nontrivial geometry according to Einstein equations corresponds to a the presence of the stress-energy tensor – it means that the wormhole would be filled with some kind of matter. However, it is possible to deform the wormhole in such a way that some of its parts are flat, without matter.

The wormhole thus connects two places, which could be very distant in the surrounding space. The entries into the wormhole are called *mouths*.

In the spacetime picture, we have to specify also the moments of time when both mouths are identified. It seems natural to assume that they are identified at the same time, but it is not

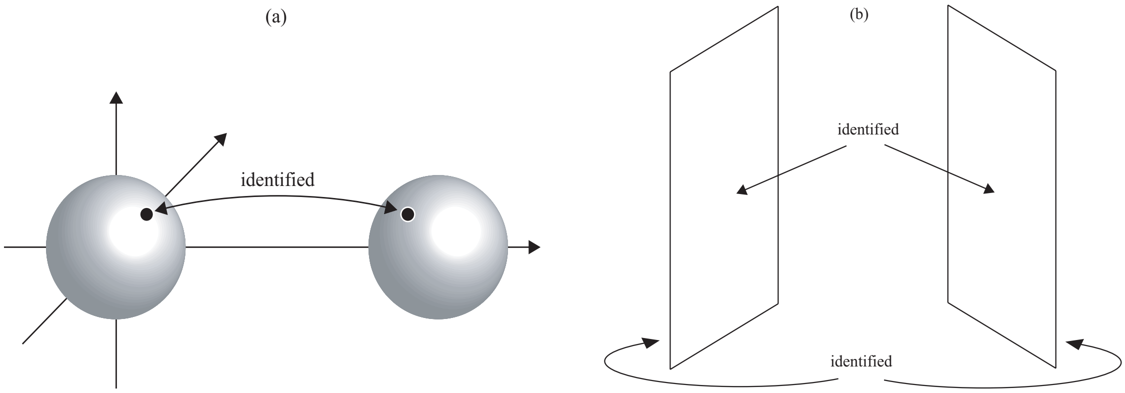

Figure 1: **Spatial representation of two simple wormholes.** (a) A wormhole formed by identification of two spherical holes in surrounding Euclidian space. Surfaces of both spheres are isometrically identified. Since the external curvature of these surfaces is non-vanishing, the geometry near the glued spherical surface is not flat. (b) A wormhole obtained by gluing two planar sections (which, could be obtained from (a) by squeezing the spheres into very thin planes). The external curvature is vanishing with the exception of the boundary of the planar sections and the geometry through the wormhole is thus flat.

necessary. In the relativistic setup, it is even not clear what "at the same moment means." Times of identification of both mouths must be specified explicitly on both sides. Of course, we have the restriction, that time must run continuously through the wormhole, i.e., locally, for the observer sitting in the wormhole, clocks on both sides must tick at the same rate without any jumps.

Let us assume as an example two mouths in the Minkowski spacetime which are at rest with respect to the same inertial frame,but they are identified with a time shift Δt equal, say, to one hour. Time on both sides of the wormhole runs in the same rate, so going through the wormhole does not affect an observer in any specific way. However, going through the wormhole, the observer arrives one hour to the future (or to the past, depending on the direction) with respect to the global inertial frame. Depending on the distance of both mouths in the surrounding space such a configuration may form the time machine: if the observer travelling through the wormhole one hour into the past is able to return through the surrounding space to his original position in less than one hour, he can form CTC, i.e., he can return to his own past and meet himself.

In our work we will consider even a simpler model of a time machine. We assume only *non-relativistic* situation, i.e., we assume that the speed of light is infinite and it determines a unique notion of simultaneity and if we use it we can define a global time – at least, before introducing a time machine. We also assume that the space is locally Euclidian.

The wormhole can be constructed in the non-relativistic spacetime in the same way as we discussed above – only in this case, thanks to the global simultaneity, we uniquely know what it means when both mouths of the wormhole are identified at the same time. If we identify them with any nonvanishing time shift Δt we immediately obtain the time machine, since the passage through the wormhole takes us to a different moment of time with respect to the global time of the surrounding spacetime. Of course, it destroys the standard causal structure of the non-relativistic spacetime (a clear distinction between future and past), but despite this we will keep using and referring to the original notion of the simultaneity and to the global time of the

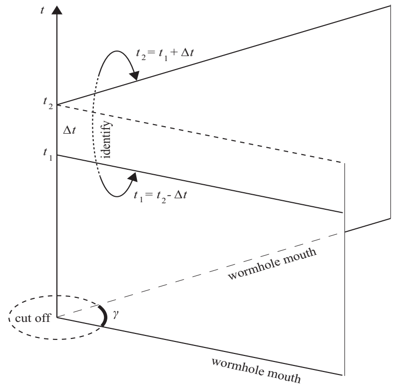

Figure 2: **A spacetime diagram of the conical time machine.** The vertical direction is temporal, horizontal planes correspond to the hypersurface of simultaneity of the original spacetime (the third spatial direction is suppressed). Two half-planes on forming the boundary of the conical spacetime represent the history of the mouths of the wormhole and they are identified with a time shift Δt.

surrounding space.

Our second simplification is that we use the wormhole with planar mouths instead of spherical ones. Namely, we cut out from the space two planar sections which we identify as in figure 1b. Since we use *flat planar* sections their identification is geometrically trivial – spacetime in them is flat everywhere, the space is locally Euclidian, without matter. Here we ignored problematic boundaries of our planar sections. The whole curvature of the mouths is squeezed to these borders which can be understood as a kind of solid frames on which the traversable parts of the wormhole are spanned.

To avoid a discussion of the wormhole boundary we assume that the planar sections are much larger than the scales of our experiments. In this approximation we can even assume that the planar sections are infinite. To simplify the geometry even more we consider the mouths of our wormhole to be two half-planes with a common boundary line. These two half-planes form an angle γ. If we identify them (first, say, at the same moment of time) the space between them becomes a locally Euclidian space with a conical singularity localized on the axis – at the intersection of the half-planes. Indeed, if we restrict ourselves to the two dimensional picture and ignore the direction parallel to the axis, our space forms a cone with the angle γ around the vertex. Since we restrict our study only to particles moving perpendicularly to the axis, it will be sufficient to consider only this two dimensional cone.

Of course, this is over-idealized situation. We should keep in mind that the mouths of the wormhole are large but finite, so somewhere very far from the axis the conical part of the space ends and goes over to the full Euclidian space. But in our consideration we restrict ourselves only to the part of the space between the mouths of the wormhole. We thus effectively work in the conical space with angle γ around the axis.

Let us stress, that in our original construction the mouths of the wormhole are special and privileged – given by the position of the wormhole. However, after enlarging them to the semi-infinite size and restricting ourselves only to the conical space between mouths, we can no longer localise the position of the mouths by local experiments. Geometry through the mouths is locally Euclidian as everywhere else. We thus obtained a space which is axially symmetric with respect to the rotation around the axis (it has also translation symmetry along the axis). The position of the wormhole can be identified only on scales larger than the wormhole, from the surrounding globally Euclidian space – however, this region is very far and we do not consider it.

In the last three paragraphs we assumed the identification of both mouths at the same moment of global time. Since we want to study a space with a time machine, we have to identify the mouths of the wormhole with a time shift Δt. This can be visualized in the three-dimensional spacetime diagram in figure 2 where we draw two important spatial directions in horizontal directions. The vertical direction corresponds to time. Semi-planar mouths of the wormhole at one moment are thus depicted as horizontal semi-lines, their time evolution as vertical half-planes. The identification of such two half-planes is not on the same vertical level, but with the vertical shift Δt. We assume that going through the wormhole in anticlockwise direction takes us time $\Delta t > 0$ to the past, in clockwise direction to the future.

After such an identification the spacetime is still locally Euclidian, even through the wormhole, (of course, except the axis), but endowed with a strange causal structure. Hypersurfaces of simultaneity (originally horizontal planes) propagate through the wormhole and form "helical" surfaces winding around the vertex. This explicitly demonstrates that the spacetime contains CTCs.

In the just described conical spacetime with the time machine we investigate a motion of a particle which can interact with a similar particle by elastic collisions described by the standard non-relativistic local laws. Namely, we assume the validity of the local conservation of energy and momentum. To study elastic collisions we consider finite solid spherical balls of radius R (this is also the main reason for the restriction to the non-relativistic case). However, in this paper we concentrate mainly on the simpler limit $R \to 0$ for which the discussion simplifies considerably since it turns out to be scale invariant. The case of finite balls will be shortly discussed at the end and thoroughly presented elsewhere [3].

III Point particle

As we described above, we consider a point particle moving in two dimensional conical space with a positive time shift Δt when going in the clockwise direction around the vertex of the cone. We also assume that the angle γ around the vertex of the cone is smaller than π since only for such angles we obtain interesting situations of self-collisions of the particle. Indeed, on the cone with $\gamma < \pi$ a straight line intersects itself at least once. Since a free particle is moving along the straight line, after passing through the time machine its trajectory must intersect the trajectory along which the same particle approached the time machine. If the trajectory crosses itself in different times we will speak about *self-intersection.* If the particle intersects its trajectory exactly at the same times, it hits itself and we speak about *self-collision.* In the following we will specify the conditions for the self-collision and find consistent solutions of a particle motion with one self-collision.

A trajectory of the particle is determined by two initial parameters: the *impact parameter* ρ which gives the distance of the initial trajectory from the vertex of the cone, and the *magnitude of initial velocity* $u > 0$, see figure 3a. We adopt the convention that the impact parameter ρ is positive if the particle circles the cone in the counterclockwise direction and it is negative if it

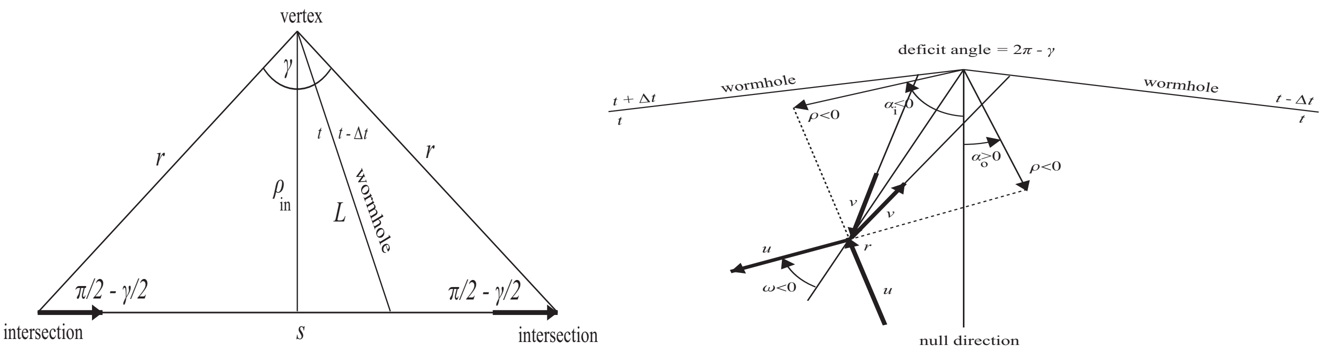

Figure 3: **Geometry of self-colliding trajectory.** (a) A typical self-colliding trajectory in the conical space with a time machine. A point particle is approaching the wormhole from infinity with a velocity u, it collides with the version of itself which already passed through the time machine, and with a velocity v moves toward the wormhole. After passing it and self-colliding with itself, it moves with the velocity u back to infinity. The conical space is illustrated as an angle boundary of which represent mouths of the wormhole and should be identified. (b) The trajectory between the self-collision is a straight line, which can be clearly demonstrated if we cut the conical space not along the wormhole but along the radial line going through the self-collision. The wormhole is than depicted as another radial line. Since the particle passes the wormhole freely, its trajectory must be straight across the wormhole line. It starts and ends at the self-collision which is represented as two points on the boundary semi-lines at the distance r from the vertex. The same geometry applies also for a segment of the collision-free trajectory between its self-intersection. We immediately see that length between the self-intersection is $2r\sin\frac{\gamma}{2}$.

circles the cone in the clockwise direction.

The parameters u and ρ do not determine the initial trajectory uniquely since these parameters do not distinguish between trajectories which are just rotated around the vertex of the cone. We should specify also an angle $\alpha_{\rm i}$ of the incoming trajectory with respect to some chosen "null" direction. However, since the conical space is symmetric under rotations, the angle $\alpha_{\rm i}$ does not affect a character of the motion and we can ignore it.

A generic physical solution to initial data is represented by a collision-free trajectory. A particle which moves along such a trajectory intersects itself without a self-collision. The collision-free trajectory can take the particle to the past, or to the future, according to the direction in which it passes the wormhole. If the collision-free trajectory is determined by a negative impact parameter $\rho < 0$ the time machine takes the particle to the future, if the collision-free trajectory is defined by positive impact parameter $\rho > 0$ it takes the particle back in time by $-\Delta t$.

The length s of the straight trajectory between its self-intersection is given by the conical geometry as can be seen in figure 3,

$$s = 2\rho\,\tan\frac{\gamma}{2}\ . \tag{1}$$

The time needed to circle the cone is thus $s/u = \frac{2\rho\tan\gamma/2}{u}$.

Now, there are two ways in which the particle can travel back in time without self-collision. (a) Either the movement around the cone takes longer time than the time thus gained, namely $\Delta t < s/u$, in which case the older version of the particle (i.e., the one that already passed the wormhole) gets late with respect to the younger version of the particle which moves through the point of intersection as the first. (b) Or the orbit around the cone takes shorter time than time thus gained, $\Delta t > s/u$, in which case the older version of the particle moves through the point of intersection as the first.

For a special choice of parameters ρ and u the particle self-interacts. In this case while the older version of the particle passes through the time machine and leaves it, the younger version of the same particle moves from a distant region toward the time machine exactly in such a way that both versions of the particle collide. Such an initial condition is potentially *dangerous* since the self-collision could prevent the particle from passing the wormhole which would be inconsistent. However, for the same dangerous initial condition we could still find a consistent solution with a self-collision which would be fine-tuned in such a way that the self-interacting particle consistently moves through the time machine. To convince ourselves that it is really true we first investigate a general particle motion with one self-collision.

Let us assume that the self-collision takes place on a radial ray at the distance r from the vertex of the cone, see figure 3a. For a symmetry reason (justified also by a detailed study of collision of finite balls [3]) the self-collision will be symmetric with respect to the radial line. We will define the oriented angle $\omega \in [-\pi/2, \pi/2]$ between the radial ray and the outgoing trajectory – it is thus a half of the angle between ingoing and outgoing trajectories. We will see that the initial parameters u and ρ are uniquely related to the quantities ω and r and can be interchanged mutually. We also denote v the magnitude of the particle velocity between the self-collision and ρ_{in} the impact parameter of the trajectory between the self-collision.

For the collision-free trajectories we can define analogous quantities r and ω which refer to the point of self-intersection instead of self-collision. In this case $\rho_{\text{in}} = \rho$ and the angle ω is given just by the conical geometry, namely

$$\omega = \omega_{\text{crit}} \equiv \frac{\pi - \gamma}{2} \ . \tag{2}$$

From the definition of ρ, r, and ω we immediately find that for both self-interacting and collision-free trajectories the radial distance r is related to ρ by

$$\rho = r \, \sin\omega \ , \qquad \rho_{\text{in}} = r \, \sin\omega_{\text{crit}} = r \, \cos\frac{\gamma}{2} \ . \tag{3}$$

For a trajectory between the self-collision the "inner" velocity v is given by the condition that the particle has to travel the distance s in time Δt, i.e.,

$$v = \frac{s}{\Delta t} = \frac{2r\sin\frac{\gamma}{2}}{\Delta t} = \frac{2\rho}{\Delta t}\frac{\sin\frac{\gamma}{2}}{\sin\omega} \ , \tag{4}$$

where we expressed the length s using the radial distance r, see figure 3, which is given in terms of ρ and ω by equation (3).

The remaining relation between parameters ρ, u, and r, ω follows from the equations for the self-collision, namely from the momentum and energy conservation during the self-collision. A detailed study [3] of the self-collision of a finite ball of radius R shows that the particle can self-collide in two ways and the type of the self-collision is determined by the angle ω. For $\omega < \omega_{\text{crit}}$ the trajectory of the particle after the self-collision is directed to the wormhole closer to the vertex than if it followed the collision-free trajectory. Thus the older version of the ball touches the younger one by its rear part. Let us denote such a case as the self-collision of the *type I*. If

$\omega > \omega_{\rm crit}$ then the trajectory of the particle is directed to the wormhole farther from the vertex than if it moved along the collision-free trajectory and the older version touches the younger version by its frontal part. We denote this case as the self-collision of the *type II*.

In the limit $R \to 0$ of the point particle the self-collision of the first type changes continuously into the self-collision of the second type and both types are described by the same equation. It turns to be equivalent to the conservation of the radial momentum during the self-collision. Comparing radial momentum of the particle between the self-collision given by $v \sin \frac{\gamma}{2}$ and radial momentum of the incoming trajectory $u \cos \omega$ we find

$$u \cos \omega = v \sin \frac{\gamma}{2} \,. \tag{5}$$

Substituting expression (4) we obtain ω in terms of ρ and u

$$\sin(2\omega) = \frac{4\rho \sin^2 \frac{\gamma}{2}}{u \Delta t} \,. \tag{6}$$

Finally, inserting the expression for ω into (3) we get r in terms of ρ and u.

Before a discussion of these relation let us return to the dangerous initial conditions mentioned above. Clearly, the dangerous initial parameters would be those for which the self-intersection becomes the self-collision. It means that $\omega = \omega_{\rm crit}$ and $v = u$. For the given initial velocity u we denote the dangerous value of the impact parameters as $\rho_{\rm px}$. Substituting $\omega = \omega_{\rm crit}$ and $v = u$ into (4) we find

$$\rho_{\rm px} = \frac{u \, \Delta t}{2 \tan \frac{\gamma}{2}} \,. \tag{7}$$

Let us reformulate equations (6) and (3) in such a way that the radial distance and the impact parameter are expressed as functions of the angle ω and of the velocity u

$$\rho(\omega) = \frac{u \Delta t \sin \omega \cos \omega}{2 \sin^2 \frac{\gamma}{2}} \,, \qquad r(\omega) = \frac{u \Delta t \cos \omega}{2 \sin^2 \frac{\gamma}{2}} \,. \tag{8}$$

Dividing these equations by $u\Delta t$ we obtain relations for dimensionless quantities (distances measured in units of $u\Delta t$) which do not depend on the velocity u anymore. It means that point particle configurations with different initial velocities u are related just by a simple rescaling. Therefore, we will discuss only the relation between ρ, r, and ω. This dependence is depicted in figure 4. On the left we can see the function $\rho = \rho(\omega)$, on the right the parametric curve $[\rho(\omega), r(\omega)]$ in the plane r-ρ, with $\omega \in [-\pi/2, \pi/2]$. From these graphs we can easily read the desired dependence $\omega(\rho)$ and $r(\rho)$.

The first equation of (8) also determines the interval of the impact parameter ρ for which the self-interaction is possible. Clearly, $\rho \in [\rho_{\rm min}, \rho_{\rm max}]$ with $\rho_{\rm min} \equiv -\frac{\Delta t u}{4 \sin^2 \gamma/2}$ and $\rho_{\rm max} \equiv \frac{\Delta t u}{4 \sin^2 \gamma/2}$. Similarly for the radial distance of the self-collision r we find $r \in [0, \frac{\Delta t u}{2 \sin^2 \gamma/2}]$. Trajectories outside this region cross themselves too far from the vertex with the time shift too short to self-interact.

Inspecting figure 4a we thus see that for $\rho \notin [\rho_{\rm min}, \rho_{\rm max}]$ we have exactly one solution for ω, r which corresponds to the collision-free trajectory. However, for $\rho \in (\rho_{\rm min}, \rho_{\rm max})$, $\rho \neq \rho_{\rm px}$ we find three solutions for ω, r – one corresponding again to the collision-free trajectory, and

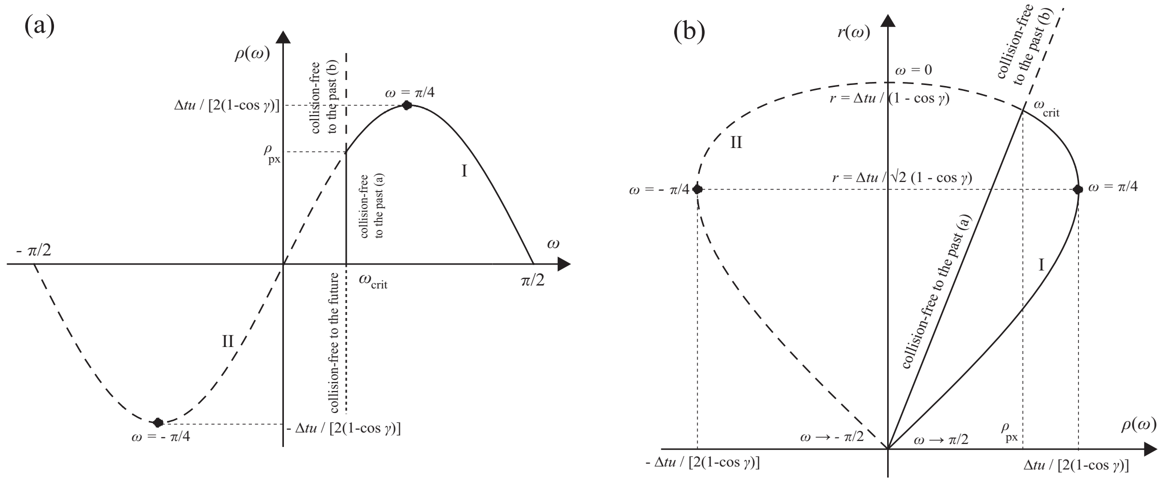

Figure 4: **Relation between ρ, r, and ω for a point particle.** (a) The dependence $\rho = \rho(\omega)$ and (b) the parametric curve $[\rho(\omega), r(\omega)]$. Relations for a self-colliding particle are given by eqs. (8). Relations for collision-free trajectories are given by eqs. (2) and (3). The angle ω runs in the interval $[-\pi/2, \pi/2]$.

two corresponding to trajectories with a self-collision. In the limiting cases $\rho = \rho_{\min}, \rho_{\max}$ two self-colliding solutions coincide.

The special case of the dangerous initial conditions $\rho = \rho_{\mathrm{px}}$ seems to have only two solutions with collision-free trajectory degenerating to the paradoxical case. The exact behavior of the particle in this case cannot be solved on the level of a point particle model but can be found by investigating finite balls as we will do in the next section.

IV Balls of a finite radius

In this section we would like to discuss shortly the motion of solid balls of a finite radius. We review only the main results, technical details can be found in [3].

The discussion of this case goes along the same line as for a point particle – we can introduce initial parameters ρ and u and parameters r and ω related to the self-collision. However, the finite radius of balls modifies the description of the self-collision. The radius R introduces a new scale into the theory and a dependence on the initial velocity is not so simple. Moreover, as we discussed above, there are two types of self-collision, however, for $R > 0$ the equations describing them are different.

The solutions of these equations split into two classes – we have physically realistic solutions which resemble those discussed in the case of point particle. Additionally, we also have unphysical solutions for which the particle during the self-collision would have exchange a negative momentum. So, if we do not consider "sticky" balls we have to rule these solutions out. The relation among ρ, r and ω with fixed u for finite balls is showen in figure 5 in a similar manner as we did for a point particle in figure 4. Since the equation for collisions of type I and II are different we have two curves representing dependencies $\rho(\omega)$ and $r(\omega)$ (lines I and II in the figure). However, only a part of these curves correspond to physical solutions (thick lines). Spurious solutions are depicted by thin lines. Similarly as for the case of point particle we also included lines for collision-free

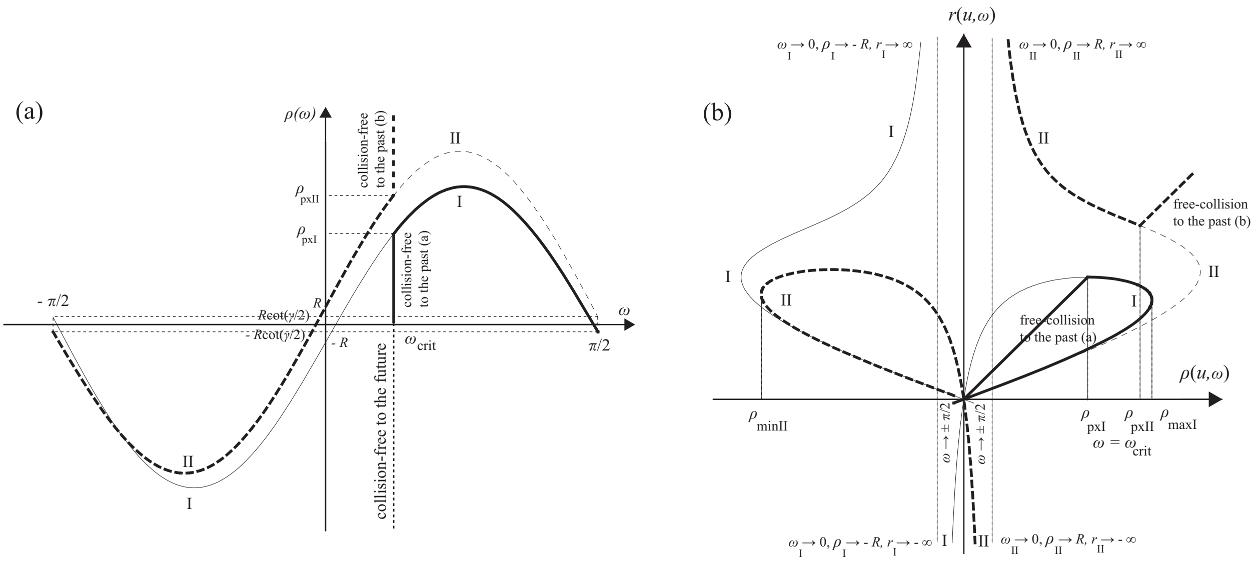

Figure 5: **Relation between ρ, r, and ω for a particle of a finite radius R.** (a) The dependence $\rho = \rho(\omega)$ and (b) the parametric curve $[\rho(\omega), r(\omega)]$ with u fixed and $\omega \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. Curves corresponding to the physical solutions are represented by thick solid (type I) and dashed (type II) lines while curves corresponding to the spurious solutions are represented by thin lines.

trajectories given by $\omega = \omega_{\text{crit}}$ and with ρ not corresponding to paradoxical values.

However, for a finite radius R, instead of only one paradoxical value ρ_{px} as in the case of a point particle we have an entire interval $[\rho_{\text{px}_{\text{I}}}, \rho_{\text{px}_{\text{II}}}]$ of dangerous values of ρ (for fixed u) for which a collision-free trajectory is inconsistent, see figure 5. Boundary values $\rho_{\text{px}_{\text{I}}}$ and $\rho_{\text{px}_{\text{II}}}$ of the interval of dangerous values of ρ represent solutions for which both versions of the ball just touch each other without exchanging any momentum. Thus they can be considered both collision-free and self-interactiong trajectories.

Now, let us discuss a number of consistent solutions for different initial values ρ and u. For fixed u it can be easily read from figure 5. For a large positive or negative ρ we have again only one collision-free trajectory. For ρ sufficiently small but out of the range of dangerous values, $\rho \notin [\rho_{\text{px}_{\text{I}}}, \rho_{\text{px}_{\text{II}}}]$, we have one collision-free trajectory and two physical self-colliding trajectories. Surprisingly, for dangerous values $\rho \in [\rho_{\text{px}_{\text{I}}}, \rho_{\text{px}_{\text{II}}}]$ we also have three solutions, all of them with a self-collision. One of them is of type II, two other solutions are of type I. An inconsistent collision-free trajectory thus changes to a new self-colliding solution.

The geometry of all these three consistent self-colliding solutions corresponding to a dangerous initial data is explicitly shown in figure 6.

Summary

We have discussed the motion of a point particle and of a solid ball in the non-relativistic conical space with a time machine. The point particle model documents a non-uniqueness of the evolutions in the presence of time machines. However, this model is not rich enough to investigate structure of trajectories corresponding to dangerous initial values. Such a discussion can be done

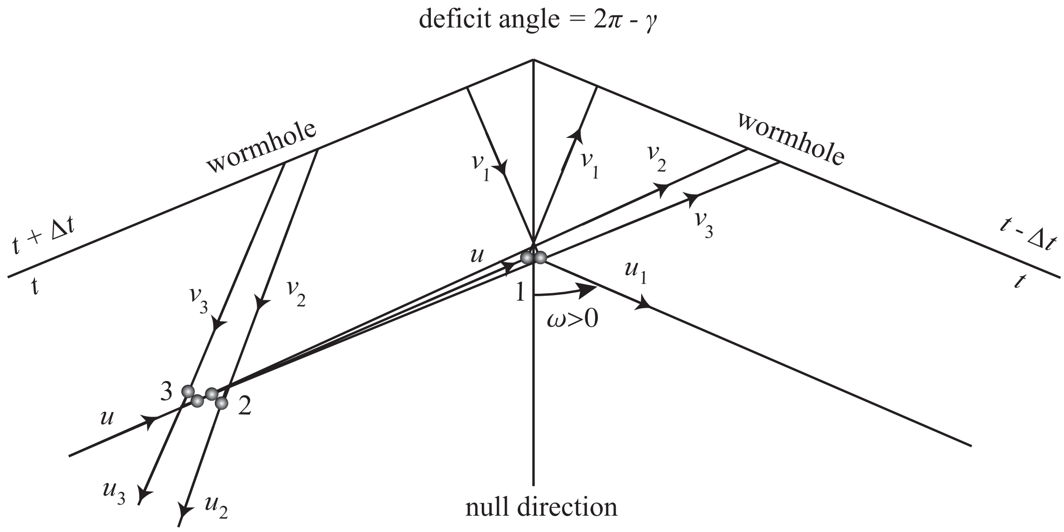

Figure 6: **Solutions for "paradoxical" initial data.** Figure shows three consistent solutions 1, 2, and 3 for "paradoxical" or "dangerous" initial data. We can identify three solutions with a self-collision, two of them are similar to the case of generic initial data. However, for the paradoxical initial data there is no collision-free solution. Instead, it is compensated by the third self-colliding solution.

in the case of finite balls for which we found that a number of solutions for given initial data is the same for dangerous initial data as for generic ones. An inconsistent collision-free trajectory is replaced by a new self-colliding solution.

Finally, let us mention that we considered only trajectories with one self-collision or no collision at all. In principle, it can and it does happen that the particle self-collides or self-intersects more times. Such a possibility enlarges a number of solutions for given initial data and complicates a discussion of all possible particle motions. For a point particle such the discussion is done in [3].

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