```
> restart;
> with( Physics );
```

$$[`*`, `.`, Annihilation, AntiCommutator, Antisymmetrize, Bra, Bracket, Check, Christoffel, Coefficients, Commutator, Coordinates, Creation, D\_, Dagger, Define, Dgamma, Einstein, Expand, ExteriorDerivative, FeynmanDiagrams, Fundiff, Geodesics, GrassmannParity, Gtaylor, Intc, Inverse, Ket, KillingVectors, \delta, LeviCivita, Library, LieBracket, LieDerivative, Normal, Parameters, PerformOnAnticommutativeSystem, Projector, Psigma, Ricci, Riemann, Setup, Simplify, SpaceTimeVector, SubstituteTensorIndices, SumOverRepeatedIndices, Symmetrize, TensorArray, ToFieldComponents, ToSuperfields, Trace, TransformCoordinates, Vectors, Weyl, `\hat{}`, dAlembertian, d\_, diff, g\_] \quad (1)$$

```
> Setup( dimension=2 );
```

The dimension and signature of the tensor space are set to: [2, -]

$$[dimension = 2] \quad (2)$$

```
> Coordinates( X=[z,phi],W=[u,psi] );
```

Default differentiation variables for d_, D_ and dAlembertian are: $\{\ \}$

Systems of spacetime Coordinates are: $\{W = (u, \psi), X = (z, \phi)\}$

$$\{W, X\} \quad (3)$$

```
> Setup(mathematicalnotation = true);
```

$$[mathematicalnotation = true] \quad (4)$$

```
> Setup( metric=(1+z^2/(1+z^2))*d_(z)^2+(1+z^2)*d_(phi)^2 );
```

$$\left[metric = \left\{(1,1) = 1 + \frac{z^2}{z^2+1},\ (2,2) = z^2+1\right\}\right] \quad (5)$$

```
> deq:=Geodesics();
```

$$deq := \left[\frac{d^2}{d\tau^2} z(\tau) = \frac{z(\tau)\left(z(\tau)^2+1\right)^2 \left(\frac{d}{d\tau}\phi(\tau)\right)^2 - z(\tau)\left(\frac{d}{d\tau}z(\tau)\right)^2}{2\,z(\tau)^4 + 3\,z(\tau)^2 + 1},\ \frac{d^2}{d\tau^2}\phi(\tau) = -\frac{2\,z(\tau)\left(\frac{d}{d\tau}z(\tau)\right)\left(\frac{d}{d\tau}\phi(\tau)\right)}{z(\tau)^2+1}\right] \quad (6)$$

```
> #dsolve( deq, [z(tau),phi(tau) ] );
```

```
> metric2 := TransformCoordinates( [z=sinh(u),phi=psi] , g_[mu,
  nu], [u,psi] ) ;
```

$$metric2 := \begin{bmatrix} 2\cosh(u)^2 - 1 & 0 \\ 0 & \cosh(u)^2 \end{bmatrix} \quad (7)$$

```
> restart;
  with( Physics );
```

```
Setup( dimension=2 );
Setup(mathematicalnotation = true);
Coordinates( W = [u,psi] );
Setup( metric=(2*cosh(u)^2-1)*d_(u)^2+cosh(u)^2*d_(psi)^2 );
```

[`*`, `.`, *Annihilation*, *AntiCommutator*, *Antisymmetrize*, *Bra*, *Bracket*, *Check*, *Christoffel*, *Coefficients*, *Commutator*, *Coordinates*, *Creation*, *D_*, *Dagger*, *Define*, *Dgamma*, *Einstein*, *Expand*, *ExteriorDerivative*, *FeynmanDiagrams*, *Fundiff*, *Geodesics*, *GrassmannParity*, *Gtaylor*, *Intc*, *Inverse*, *Ket*, *KillingVectors*, *δ*, *LeviCivita*, *Library*, *LieBracket*, *LieDerivative*, *Normal*, *Parameters*, *PerformOnAnticommutativeSystem*, *Projector*, *Psigma*, *Ricci*, *Riemann*, *Setup*, *Simplify*, *SpaceTimeVector*, *SubstituteTensorIndices*, *SumOverRepeatedIndices*, *Symmetrize*, *TensorArray*, *ToFieldComponents*, *ToSuperfields*, *Trace*, *TransformCoordinates*, *Vectors*, *Weyl*, `^`, *dAlembertian*, *d_*, *diff*, *g_*]

The dimension and signature of the tensor space are set to: [2, -]

[*dimension* = 2]

[*mathematicalnotation* = *true*]

Default differentiation variables for d_, D_ and dAlembertian are: $\{W = (u, \psi)\}$

Systems of spacetime Coordinates are: $\{W = (u, \psi)\}$

$\{W\}$

$$\left[metric = \{(1, 1) = 2\cosh(u)^2 - 1,\ (2, 2) = \cosh(u)^2\}\right] \tag{8}$$

```
> Simplify( Geodesics() );
```

$$\left[\frac{d^2}{d\tau^2}u(\tau) = -\frac{2\cosh(u(\tau))\left(\left(\frac{d}{d\tau}u(\tau)\right)^2 - \frac{\left(\frac{d}{d\tau}\psi(\tau)\right)^2}{2}\right)\sinh(u(\tau))}{1 + 2\sinh(u(\tau))^2},\ \frac{d^2}{d\tau^2}\psi(\tau) = -\frac{2\sinh(u(\tau))\left(\frac{d}{d\tau}u(\tau)\right)\left(\frac{d}{d\tau}\psi(\tau)\right)}{\cosh(u(\tau))}\right] \tag{9}$$

```
> dsolve(%,[u(tau), psi(tau)]);
```

$$\left[\left\{u(\tau) = RootOf\left(-\sqrt{2}\left(\right.\right.\right.\right. \tag{10}$$

$$\int^{\_Z} \frac{\left(\left(e^{\_f}\right)^4+1\right)\left(\left(e^{\_f}\right)^2+1\right)}{\sqrt{\frac{\left(\left(e^{\_f}\right)^4+1\right)\left(\left(e^{\_f}\right)^4+4\,\_C2\left(e^{\_f}\right)^2+2\left(e^{\_f}\right)^2+1\right)}{\left(e^{\_f}\right)^2\_C3}}\left(e^{\_f}\right)^2\_C3}\,d\_f$$

$$+2\,\_C4+2\,\tau\Bigg), u(\tau)=RootOf\Bigg(\sqrt{2}\Bigg($$

$$\int^{\_Z} \frac{\left(\left(e^{\_f}\right)^4+1\right)\left(\left(e^{\_f}\right)^2+1\right)}{\sqrt{\frac{\left(\left(e^{\_f}\right)^4+1\right)\left(\left(e^{\_f}\right)^4+4\,\_C2\left(e^{\_f}\right)^2+2\left(e^{\_f}\right)^2+1\right)}{\left(e^{\_f}\right)^2\_C3}}\left(e^{\_f}\right)^2\_C3}\,d\_f$$

$$+2\,\_C4+2\,\tau\Bigg)\Bigg)\Bigg\}, \Bigg\{\psi(\tau)=$$

$$\int \frac{1}{\left(e^{u(\tau)}-1\right)\left(e^{u(\tau)}+1\right)\left(\left(e^{u(\tau)}\right)^2+1\right)}\Bigg(\sqrt{2}\Bigg(\left(e^{u(\tau)}-1\right)\left(e^{u(\tau)}+1\right)\Big(\left(e^{u(\tau)}\right)^2$$

$$+1\Big)\left(\left(e^{u(\tau)}\right)^4\left(\frac{d}{d\tau}u(\tau)\right)^2+\left(e^{u(\tau)}\right)^4\left(\frac{d^2}{d\tau^2}u(\tau)\right)-\left(\frac{d}{d\tau}u(\tau)\right)^2+\frac{d^2}{d\tau^2}u(\tau)\right)\Bigg)$$

$$^{1/2}\Bigg)\,d\tau+\_C1, \psi(\tau)=\int$$

$$-\frac{1}{\left(e^{u(\tau)}-1\right)\left(e^{u(\tau)}+1\right)\left(\left(e^{u(\tau)}\right)^2+1\right)}\Bigg(\sqrt{2}\Bigg(\left(e^{u(\tau)}-1\right)\left(e^{u(\tau)}+1\right)\Big(\left(e^{u(\tau)}\right)^2$$

$$+1\Big)\left(\left(e^{u(\tau)}\right)^4\left(\frac{d}{d\tau}u(\tau)\right)^2+\left(e^{u(\tau)}\right)^4\left(\frac{d^2}{d\tau^2}u(\tau)\right)-\left(\frac{d}{d\tau}u(\tau)\right)^2+\frac{d^2}{d\tau^2}u(\tau)\right)\Bigg)$$

$$\left.\left.\left.{}^{1/2}\right)\mathrm{d}\tau + \_C1\right\}\right]$$