Tackling the Physicality of Space-Times from Both Ends

Ana Alonso Serrano, Jessica Santiago, **Sebastian Schuster**, Matt Visser

Ústav Teoretické Fyziky
Matematicko-Fyzikální Fakulta
Univerzita Karlova

25th July 2023

UNIVERZITA KARLOVA
Matematicko-fyzikální
fakulta

Outline

Getting Everyone on Board

My Troubles to Come

Goal: Don't leave anyone behind!

My Troubles to Come

Goal: Don't leave anyone behind!

But: The genesis of my work...

My Troubles to Come

Goal: Don't leave anyone behind!

But: The genesis of my work...

What I read:

QUANTUM ENERGY INEQUALITIES IN PREMETRIC ... PHYS. REV. D 97, 025019 (2018)

$$\textbf{Linearity}: \hat{A}(\alpha j + \beta j') = \alpha\hat{A}(j) + \beta\hat{A}(j') \quad \text{for all } \alpha, \beta \in \mathbb{C},$$
$$\textbf{Hermiticity}: \hat{A}(j)^* = \hat{A}(\bar{j}),$$
$$\textbf{Field equation}: \hat{A}(PA) = 0,$$
$$\textbf{Canonical commutation relations (CCR)}: [\hat{A}(j), \hat{A}(j')] = \mathrm{i}\sigma(j, j')\mathbb{1};$$

here, we denote the unit element of $\mathfrak{A}$ by $\mathbb{1}$ and make use of our standing conventions on j's and A's.

The algebra element $\hat{A}(j)$ can be interpreted as a smeared field $\int \hat{A}_a j^a$ (recall that j is a vector density of weight 1, so no volume element appears); later, we will discuss Hilbert space representations in which this can be taken literally, with $\hat{A}_a$ understood as an operator-valued distribution.

It is convenient to identify elements of $\mathfrak{A}$ corresponding to smeared field strengths: for any smooth compactly supported second rank contravariant tensor density t, we define

$$\hat{F}(t) \coloneqq 2\hat{A}(\operatorname{div} t), \tag{13}$$

where $(\operatorname{div} t)^a = \partial_b t^{[ab]}$ is clearly a conserved vector density; $\hat{F}(t)$ can be interpreted as a smeared field $\int \hat{F}_{ab} t^{ab}$.

The normalized positive functionals on $\mathfrak{A}$ are called *(quantum) states*. That means, Λ is a state on the field algebra $\mathfrak{A}$ if

$$\textbf{Normalization}: \Lambda(\mathbb{1}) = 1,$$
$$\textbf{Positivity}: \Lambda(a^* a) \geq 0,$$
$$\textbf{Hermiticity}: \Lambda(a^*) = \overline{\Lambda(a)}$$

for all $a \in \mathfrak{A}$. Each state Λ can be represented by a hierarchy of *n-point functions* $(\Lambda_n)_{n\geq 0}$ by setting

In the framework developed in [12], physical states in premetric electrodynamics are required to obey the *microlocal spectrum condition* (μSC), a generalization of the Hadamard condition used for QFT in curved spacetimes [30,31]:

μSC among the gauge equivalent two-point functions Λ_2 induced by the state Λ, there should be at least one that is a covector bidistribution, with wave-front set obeying

$$\mathrm{WF}(\Lambda_2) \subset \mathcal{N}^+ \times \mathcal{N}^- \subset T^*M \times T^*M \tag{14}$$

with $\mathcal{N}^\pm$ as defined in (7) or equivalently (II B), and whose antisymmetric part is fixed up to smooth terms by the *generalized CCR*[4]

$$\Lambda_2 - \Lambda_2^T = \mathrm{i}\sigma \quad (\mathrm{mod}\, C^\infty),$$

where the transposed distribution is defined by $\Lambda_2^T(f, f') = \Lambda_2(f', f)$ for general compactly supported vector densities f, f'.

The wave-front set encodes details about the singular structure of a distribution in both configuration and momentum space.[5] The theory of the wave-front set is developed, e.g., in [34]; see also [35,36] for an introduction to the subject. The condition (14) asserts that the wave-front set of Λ_2 consists of pairs $((x_1, k_1), (x_2, -k_2)) \in T^*M \times T^*M$

My Troubles to Come

Goal: Don't leave anyone behind!

But: The genesis of my work...

What I read:

QUANTUM ENERGY INEQUALITIES IN PREMETRIC ... PHYS. REV. D **97,** 025019 (2018)

$$\textbf{Linearity}: \hat{A}(\alpha j + \beta j') = \alpha\hat{A}(j) + \beta\hat{A}(j') \quad \text{for all } \alpha, \beta \in \mathbb{C},$$
$$\textbf{Hermiticity}: \hat{A}(j)^* = \hat{A}(\bar{j}),$$
$$\textbf{Field equation}: \hat{A}(PA) = 0,$$
$$\textbf{Canonical commutation relations (CCR)}: [\hat{A}(j), \hat{A}(j')] = i\sigma(j, j')\mathbb{1};$$

here, we denote the unit element of $\mathfrak{A}$ by $\mathbb{1}$ and make use of our standing conventions on j's and A's.

The algebra element $\hat{A}(j)$ can be interpreted as a smeared field $\int \hat{A}_a j^a$ (recall that j is a vector density of weight 1, so no volume element appears); later, we will discuss Hilbert space representations in which this can be taken literally, with $\hat{A}_a$ understood as an operator-valued distribution.

It is convenient to identify elements of $\mathfrak{A}$ corresponding to smeared field strengths: for any smooth compactly supported second rank contravariant tensor density t, we define

$$\hat{F}(t) := 2\hat{A}(\operatorname{div} t), \qquad (13)$$

where $(\operatorname{div} t)^a = \partial_b t^{[ab]}$ is clearly a conserved vector density; $\hat{F}(t)$ can be interpreted as a smeared field $\int \hat{F}_{ab} t^{ab}$.

The normalized positive functionals on $\mathfrak{A}$ are called *(quantum) states*. That means, Λ is a state on the field algebra $\mathfrak{A}$ if

$$\textbf{Normalization}: \Lambda(\mathbb{1}) = 1,$$
$$\textbf{Positivity}: \Lambda(a^* a) \geq 0,$$
$$\textbf{Hermiticity}: \Lambda(a^*) = \overline{\Lambda(a)}$$

for all $a \in \mathfrak{A}$. Each state Λ can be represented by a hierarchy of *n-point functions* $(\Lambda_n)_{n \geq 0}$ by setting

In the framework developed in [12], physical states in premetric electrodynamics are required to obey the *microlocal spectrum condition* (μSC), a generalization of the Hadamard condition used for QFT in curved spacetimes [30,31]:

μSC among the gauge equivalent two-point functions Λ_2 induced by the state Λ, there should be at least one that is a covector bidistribution, with wave-front set obeying

$$\mathrm{WF}(\Lambda_2) \subset \mathcal{N}^+ \times \mathcal{N}^- \subset T^*M \times T^*M \qquad (14)$$

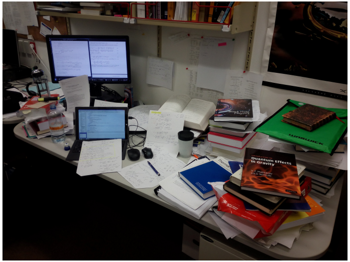
with $\mathcal{N}^\pm$ as defined in (7) or equivalently (II B), and whose antisymmetric part is fixed up to smooth terms by the *generalized CCR*[4]

$$\Lambda_2 - \Lambda_2^T = i\sigma \quad (\mathrm{mod}\ C^\infty),$$

where the transposed distribution is defined by $\Lambda_2^T(f, f') = \Lambda_2(f', f)$ for general compactly supported vector densities f, f'.

The wave-front set encodes details about the singular structure of a distribution in both configuration and momentum space.[5] The theory of the wave-front set is developed, e.g., in [34]; see also [35,36] for an introduction to the subject. The condition (14) asserts that the wave-front set of Λ_2 consists of pairs $((x_1, k_1), (x_2, -k_2)) \in T^*M \times T^*M$

How I work:



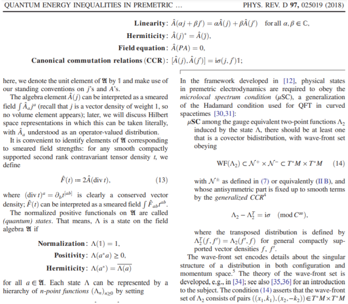
A 'Where's Waldo' for bibliophile physicists...

My Troubles to Come

Goal: Don't leave anyone behind!

But: The genesis of my work...

What I read:



QUANTUM ENERGY INEQUALITIES IN PREMETRIC ... PHYS. REV. D **97,** 025019 (2018)

$$\textbf{Linearity}: \hat{A}(\alpha j + \beta j') = \alpha\hat{A}(j) + \beta\hat{A}(j') \quad \text{for all } \alpha, \beta \in \mathbb{C},$$
$$\textbf{Hermiticity}: \hat{A}(j)^* = \hat{A}(\bar{j}),$$
$$\textbf{Field equation}: \hat{A}(PA) = 0,$$
$$\textbf{Canonical commutation relations (CCR)}: [\hat{A}(j), \hat{A}(j')] = i\sigma(j, j')\mathbb{1};$$

here, we denote the unit element of $\mathfrak{A}$ by $\mathbb{1}$ and make use of our standing conventions on j's and A's.

The algebra element $\hat{A}(j)$ can be interpreted as a smeared field $\int \hat{A}_a j^a$ (recall that j is a vector density of weight 1, so no volume element appears); later, we will discuss Hilbert space representations in which this can be taken literally, with $\hat{A}_a$ understood as an operator-valued distribution.

It is convenient to identify elements of $\mathfrak{A}$ corresponding to smeared field strengths: for any smooth compactly supported second rank contravariant tensor density t, we define

$$\hat{F}(t) := 2\hat{A}(\operatorname{div} t), \tag{13}$$

where $(\operatorname{div} t)^a = \partial_b t^{[ab]}$ is clearly a conserved vector density; $\hat{F}(t)$ can be interpreted as a smeared field $\int \hat{F}_{ab} t^{ab}$.

The normalized positive functionals on $\mathfrak{A}$ are called *(quantum) states*. That means, Λ is a state on the field algebra $\mathfrak{A}$ if

$$\textbf{Normalization}: \Lambda(\mathbb{1}) = 1,$$
$$\textbf{Positivity}: \Lambda(a^* a) \geq 0,$$
$$\textbf{Hermiticity}: \Lambda(a^*) = \overline{\Lambda(a)}$$

for all $a \in \mathfrak{A}$. Each state Λ can be represented by a hierarchy of *n-point functions* $(\Lambda_n)_{n \geq 0}$ by setting

In the framework developed in [12], physical states in premetric electrodynamics are required to obey the *microlocal spectrum condition* (μSC), a generalization of the Hadamard condition used for QFT in curved spacetimes [30,31]:

μSC among the gauge equivalent two-point functions Λ_2 induced by the state Λ, there should be at least one that is a covector bidistribution, with wave-front set obeying

$$\mathrm{WF}(\Lambda_2) \subset \mathcal{N}^+ \times \mathcal{N}^- \subset T^*M \times T^*M \tag{14}$$

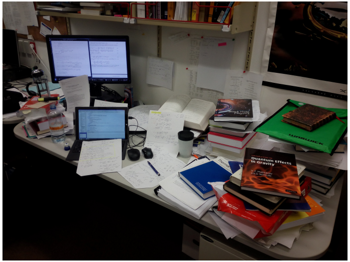
with $\mathcal{N}^\pm$ as defined in (7) or equivalently (II B), and whose antisymmetric part is fixed up to smooth terms by the *generalized CCR*[4]

$$\Lambda_2 - \Lambda_2^T = i\sigma \quad (\mathrm{mod}\ C^\infty),$$

where the transposed distribution is defined by $\Lambda_2^T(f, f') = \Lambda_2(f', f)$ for general compactly supported vector densities f, f'.

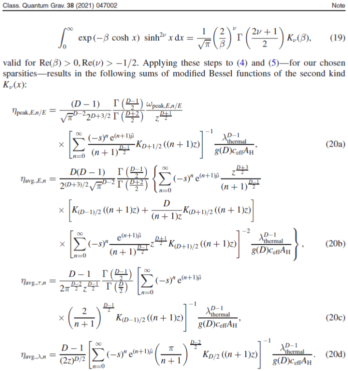
The wave-front set encodes details about the singular structure of a distribution in both configuration and momentum space.[5] The theory of the wave-front set is developed, e.g., in [34]; see also [35,36] for an introduction to the subject. The condition (14) asserts that the wave-front set of Λ_2 consists of pairs $((x_1, k_1), (x_2, -k_2)) \in T^*M \times T^*M$

How I work:



A 'Where's Waldo' for bibliophile physicists...

What I publish:



Class. Quantum Grav. **38** (2021) 047002 Note

$$\int_0^\infty \exp(-\beta \cosh x)\, \sinh^{2\nu} x\, \mathrm{d}x = \frac{1}{\sqrt{\pi}}\left(\frac{2}{\beta}\right)^\nu \Gamma\left(\frac{2\nu+1}{2}\right) K_\nu(\beta), \tag{19}$$

valid for $\mathrm{Re}(\beta) > 0, \mathrm{Re}(\nu) > -1/2$. Applying these steps to (4) and (5)—for our chosen sparsities—results in the following sums of modified Bessel functions of the second kind $K_\nu(x)$:

$$\eta_{\mathrm{peak},E,n/E} = \frac{(D-1)}{\sqrt{\pi}^{D-2} 2^{D+3/2}} \frac{\Gamma\left(\frac{D-1}{2}\right)}{\Gamma\left(\frac{D+2}{2}\right)} \frac{\omega_{\mathrm{peak},E,n/E}}{z^{\frac{D+1}{2}}} \times \left[\sum_{n=0}^\infty \frac{(-s)^n \mathrm{e}^{(n+1)\tilde{\mu}}}{(n+1)^{\frac{D-1}{2}}} K_{D+1/2}((n+1)z)\right]^{-1} \frac{\lambda_{\mathrm{thermal}}^{D-1}}{g(D) c_{\mathrm{eff}} A_{\mathrm{H}}}, \tag{20a}$$

$$\eta_{\mathrm{avg.},E,n} = \frac{D(D-1)}{2^{(D+3)/2}\sqrt{\pi}^{D-2}} \frac{\Gamma\left(\frac{D-1}{2}\right)}{\Gamma\left(\frac{D+2}{2}\right)} \left\{\sum_{n=0}^\infty (-s)^n \mathrm{e}^{(n+1)\tilde{\mu}} \frac{z^{\frac{D+3}{2}}}{(n+1)^{\frac{D-1}{2}}} \times \left[K_{(D-1)/2}((n+1)z) + \frac{D}{(n+1)z} K_{(D+1)/2}((n+1)z)\right] \times \left[\sum_{n=0}^\infty (-s)^n \frac{\mathrm{e}^{(n+1)\tilde{\mu}}}{(n+1)^{\frac{D-1}{2}}} z^{\frac{D+1}{2}} K_{(D+1)/2}((n+1)z)\right]^{-2} \frac{\lambda_{\mathrm{thermal}}^{D-1}}{g(D) c_{\mathrm{eff}} A_{\mathrm{H}}}\right\}, \tag{20b}$$

$$\eta_{\mathrm{avg.},\tau,n} = \frac{D-1}{2\pi^{\frac{D-2}{2}} z^{\frac{D-1}{2}}} \frac{\Gamma\left(\frac{D-1}{2}\right)}{\Gamma\left(\frac{D}{2}\right)} \left[\sum_{n=0}^\infty (-s)^n \mathrm{e}^{(n+1)\tilde{\mu}} \times \left(\frac{2}{n+1}\right)^{\frac{D-1}{2}} K_{(D-1)/2}((n+1)z)\right]^{-1} \frac{\lambda_{\mathrm{thermal}}^{D-1}}{g(D) c_{\mathrm{eff}} A_{\mathrm{H}}}, \tag{20c}$$

$$\eta_{\mathrm{avg.},\lambda,n} = \frac{D-1}{(2z)^{D/2}} \left[\sum_{n=0}^\infty (-s)^n \mathrm{e}^{(n+1)\tilde{\mu}} \left(\frac{\pi}{n+1}\right)^{\frac{D-2}{2}} K_{D/2}((n+1)z)\right]^{-1} \frac{\lambda_{\mathrm{thermal}}^{D-1}}{g(D) c_{\mathrm{eff}} A_{\mathrm{H}}}. \tag{20d}$$

My Troubles to Come

Goal: Don't leave anyone behind!
But: The genesis of my work…

What I read:

QUANTUM ENERGY INEQUALITIES IN PREMETRIC … PHYS. REV. D **97**, 025019 (2018)

$$\textbf{Linearity}: \hat{A}(\alpha j + \beta j') = \alpha\hat{A}(j) + \beta\hat{A}(j') \quad \text{for all } \alpha,\beta \in \mathbb{C},$$
$$\textbf{Hermiticity}: \hat{A}(j)^* = \hat{A}(\bar{\jmath}),$$
$$\textbf{Field equation}: \hat{A}(PA) = 0,$$
$$\textbf{Canonical commutation relations (CCR)}: [\hat{A}(j), \hat{A}(j')] = \mathrm{i}\sigma(j,j')\mathbb{1};$$

here, we denote the unit element of $\mathfrak{A}$ by $\mathbb{1}$ and make use of our standing conventions on j's and A's.

The algebra element $\hat{A}(j)$ can be interpreted as a smeared field $\int \hat{A}_a j^a$ (recall that j is a vector density of weight 1, so no volume element appears); later, we will discuss Hilbert space representations in which this can be taken literally, with $\hat{A}_a$ understood as an operator-valued distribution.

It is convenient to identify elements of $\mathfrak{A}$ corresponding to smeared field strengths: for any smooth compactly supported second rank contravariant tensor density t, we define

$$\hat{F}(t) \coloneqq 2\hat{A}(\operatorname{div} t), \qquad (13)$$

where $(\operatorname{div} t)^a = \partial_b t^{[ab]}$ is clearly a conserved vector density; $\hat{F}(t)$ can be interpreted as a smeared field $\int \hat{F}_{ab} t^{ab}$.

The normalized positive functionals on $\mathfrak{A}$ are called *(quantum) states*. That means, Λ is a state on the field algebra $\mathfrak{A}$ if

$$\textbf{Normalization}: \Lambda(\mathbb{1}) = 1,$$
$$\textbf{Positivity}: \Lambda(a^* a) \geq 0,$$
$$\textbf{Hermiticity}: \Lambda(a^*) = \overline{\Lambda(a)}$$

for all $a \in \mathfrak{A}$. Each state Λ can be represented by a hierarchy of *n-point functions* $(\Lambda_n)_{n\geq 0}$ by setting

In the framework developed in [12], physical states in premetric electrodynamics are required to obey the *microlocal spectrum condition* (μSC), a generalization of the Hadamard condition used for QFT in curved spacetimes [30,31]:

μSC among the gauge equivalent two-point functions Λ_2 induced by the state Λ, there should be at least one that is a covector bidistribution, with wave-front set obeying

$$\mathrm{WF}(\Lambda_2) \subset \mathcal{N}^+ \times \mathcal{N}^- \subset T^*M \times T^*M \qquad (14)$$

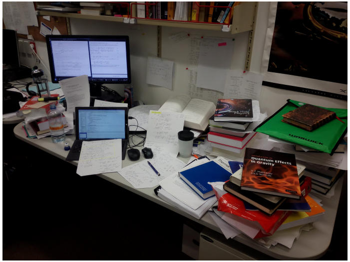
with $\mathcal{N}^\pm$ as defined in (7) or equivalently (II B), and whose antisymmetric part is fixed up to smooth terms by the *generalized CCR*[4]

$$\Lambda_2 - \Lambda_2^T = \mathrm{i}\sigma \pmod{C^\infty},$$

where the transposed distribution is defined by $\Lambda_2^T(f,f') = \Lambda_2(f',f)$ for general compactly supported vector densities f, f'.

The wave-front set encodes details about the singular structure of a distribution in both configuration and momentum space.[5] The theory of the wave-front set is developed, e.g., in [34]; see also [35,36] for an introduction to the subject. The condition (14) asserts that the wave-front set of Λ_2 consists of pairs $((x_1,k_1),(x_2,-k_2)) \in T^*M \times T^*M$

How I work:



A 'Where's Waldo' for bibliophile physicists…

What I publish:

Class. Quantum Grav. **38** (2021) 047002 Note

$$\int_0^\infty \exp(-\beta \cosh x)\, \sinh^{2\nu} x\, \mathrm{d}x = \frac{1}{\sqrt{\pi}}\left(\frac{2}{\beta}\right)^\nu \Gamma\left(\frac{2\nu+1}{2}\right) K_\nu(\beta), \qquad (19)$$

valid for $\mathrm{Re}(\beta) > 0, \mathrm{Re}(\nu) > -1/2$. Applying these steps to (4) and (5)—for our chosen sparsities—results in the following sums of modified Bessel functions of the second kind $K_\nu(x)$:

$$\eta_{\text{peak},E,n/E} = \frac{(D-1)}{\sqrt{\pi}^{D-2} 2^{D+3/2}} \frac{\Gamma\left(\frac{D-1}{2}\right)}{\Gamma\left(\frac{D+2}{2}\right)} \frac{\omega_{\text{peak},E,n/E}}{z^{\frac{D+1}{2}}} \times \left[\sum_{n=0}^\infty \frac{(-s)^n\, \mathrm{e}^{(n+1)\tilde{\mu}}}{(n+1)^{\frac{D-1}{2}}} K_{D+1/2}((n+1)z)\right]^{-1} \frac{\lambda_{\text{thermal}}^{D-1}}{g(D) c_{\text{eff}} A_{\text{H}}}, \qquad (20a)$$

$$\eta_{\text{avg.},E,n} = \frac{D(D-1)}{2^{(D+3)/2}\sqrt{\pi}^{D-2}} \frac{\Gamma\left(\frac{D-1}{2}\right)}{\Gamma\left(\frac{D+2}{2}\right)} \left\{\sum_{n=0}^\infty (-s)^n\, \mathrm{e}^{(n+1)\tilde{\mu}} \frac{z^{\frac{D+3}{2}}}{(n+1)^{\frac{D-1}{2}}} \times \left[K_{(D-1)/2}((n+1)z) + \frac{D}{(n+1)z} K_{(D+1)/2}((n+1)z)\right] \times \left[\sum_{n=0}^\infty (-s)^n \frac{\mathrm{e}^{(n+1)\tilde{\mu}}}{(n+1)^{\frac{D-1}{2}}} z^{\frac{D+1}{2}} K_{(D+1)/2}((n+1)z)\right]^{-2} \frac{\lambda_{\text{thermal}}^{D-1}}{g(D) c_{\text{eff}} A_{\text{H}}}\right\}, \qquad (20b)$$

$$\eta_{\text{avg.},\tau,n} = \frac{D-1}{2\pi^{\frac{D-2}{2}} z^{\frac{D-1}{2}}} \frac{\Gamma\left(\frac{D-1}{2}\right)}{\Gamma\left(\frac{D}{2}\right)} \left[\sum_{n=0}^\infty (-s)^n\, \mathrm{e}^{(n+1)\tilde{\mu}} \times \left(\frac{2}{n+1}\right)^{\frac{D-1}{2}} K_{(D-1)/2}((n+1)z)\right]^{-1} \frac{\lambda_{\text{thermal}}^{D-1}}{g(D) c_{\text{eff}} A_{\text{H}}}, \qquad (20c)$$

$$\eta_{\text{avg.},\lambda,n} = \frac{D-1}{(2z)^{D/2}} \left[\sum_{n=0}^\infty (-s)^n\, \mathrm{e}^{(n+1)\tilde{\mu}} \left(\frac{\pi}{n+1}\right)^{\frac{D-2}{2}} K_{D/2}((n+1)z)\right]^{-1} \frac{\lambda_{\text{thermal}}^{D-1}}{g(D) c_{\text{eff}} A_{\text{H}}}. \qquad (20d)$$

Let's see how it goes… ☺ 😇

The Backbone: Philosophy of Science

- Physics deals with (at least) three layers:
  - Our experiences (experiments)
  - Our models (mathematics/theory)
  - Our mapping of the two to each other (epistemology, ontology, psychology, ...)

The Backbone: Philosophy of Science

- Physics deals with (at least) three layers:
  - Our experiences (experiments)
  - Our models (mathematics/theory)
  - Our mapping of the two to each other (epistemology, ontology, psychology, . . .)
- These are often intimately related

The Backbone: Philosophy of Science

- Physics deals with (at least) three layers:
  - Our experiences (experiments)
  - Our models (mathematics/theory)
  - Our mapping of the two to each other (epistemology, ontology, psychology, . . .)
- These are often intimately related
- Warning! I'm a physicist (and a bit of a mathematician), not a philosopher.

The Backbone: Philosophy of Science

- Physics deals with (at least) three layers:
  - Our experiences (experiments)
  - Our models (mathematics/theory)
  - Our mapping of the two to each other (epistemology, ontology, psychology, . . .)
- These are often intimately related
- Warning! I'm a physicist (and a bit of a mathematician), not a philosopher.
- Still, philosophers' concepts can and should inform us!

The Backbone: Philosophy of Science

- Physics deals with (at least) three layers:
  - Our experiences (experiments)
  - Our models (mathematics/theory)
  - Our mapping of the two to each other (epistemology, ontology, psychology, ...)
- These are often intimately related
- Warning! I'm a physicist (and a bit of a mathematician), not a philosopher.
- Still, philosophers' concepts can and should inform us!

Tool: The Münchhausen trilemma

Ultimate options of arguments (Albert):

- Infinite regress
- Circular reasoning
- Dogma

Image: Theodor Hosemann (1840), https://commons.wikimedia.org/wiki/File:M%C3%BCnchhausen-Sumpf-Hosemann.png

The Backbone: Philosophy of Science

- Physics deals with (at least) three layers:
  - Our experiences (experiments)
  - Our models (mathematics/theory)
  - Our mapping of the two to each other (epistemology, ontology, psychology, ...)
- These are often intimately related
- **Warning!** I'm a physicist (and a bit of a mathematician), not a philosopher.
- Still, philosophers' concepts can and should inform us!

Tool: The Münchhausen polylemma

Ultimate options of arguments (Albert):

- Infinite regress
- Circular reasoning
- Dogma/Experience/Psychologism (Popper, Fries)

Image: Theodor Hosemann (1840), https://commons.wikimedia.org/wiki/File:M%C3%BCnchhausen-Sumpf-Hosemann.png

The Backbone: Philosophy of Science

- Physics deals with (at least) three layers:
  - Our experiences (experiments)
  - Our models (mathematics/theory)
  - Our mapping of the two to each other (epistemology, ontology, psychology, ...)
- These are often intimately related
- **Warning!** I'm a physicist (and a bit of a mathematician), not a philosopher.
- Still, philosophers' concepts can and should inform us!

Tool: The Münchhausen polylemma

Ultimate options of arguments (Albert):

- Infinite regress
- Circular reasoning
- Dogma/Experience/Psychologism (Popper, Fries)
- Contradiction (???)

Image: Theodor Hosemann (1840), https://commons.wikimedia.org/wiki/File:M%C3%BCnchhausen-Sumpf-Hosemann.png

Getting Everyone on Board:
General Relativity in Two Slides

Reminder: Special Relativity

Special relativity:

- Distinguish past and present by the speed of light:
  - Relativity Principle: All uniformly moving frames ('inertial frames') see the same physics
  - Constancy of c: In all inertial frames, the speed of light (in vacuum) c is the same. It's 1.

Reminder: Special Relativity

Special relativity:

- Distinguish past and present by the speed of light:
  - Relativity Principle: All uniformly moving frames ('inertial frames') see the same physics
  - Constancy of c: In all inertial frames, the speed of light (in vacuum) c is the same. It's 1.
- In Cartesian coordinates, this defines a Lorentzian metric

$$\eta = \text{diag}(-1, +1, +1, +1)$$

Reminder: Special Relativity

Special relativity:

- Distinguish past and present by the speed of light:
  - Relativity Principle: All uniformly moving frames ('inertial frames') see the same physics
  - Constancy of c: In all inertial frames, the speed of light (in vacuum) c is the same. It's 1.
- In Cartesian coordinates, this defines a Lorentzian metric

$$\eta = \text{diag}(-1, +1, +1, +1)$$

- $(\mathbb{R}^4, \eta)$ is Minkowski space

Reminder: Special Relativity

Special relativity:

- Distinguish past and present by the speed of light:
  - Relativity Principle: All uniformly moving frames ('inertial frames') see the same physics
  - Constancy of c: In all inertial frames, the speed of light (in vacuum) c is the same. It's 1.
- In Cartesian coordinates, this defines a Lorentzian metric

$$\eta = \mathrm{diag}(-1, +1, +1, +1)$$

- $(\mathbb{R}^4, \eta)$ is Minkowski space
- We call two events' X and Y separation:
  - space-like if $\eta(X - Y, X - Y) =: \eta_{ab}(X - Y)^a(X - Y)^b > 0$
  - null/light-like if $\eta(X - Y, X - Y) =: \eta_{ab}(X - Y)^a(X - Y)^b = 0$
  - time-like if $\eta(X - Y, X - Y) =: \eta_{ab}(X - Y)^a(X - Y)^b < 0$

Reminder: Special Relativity

Special relativity:

- Distinguish past and present by the speed of light:
  - Relativity Principle: All uniformly moving frames ('inertial frames') see the same physics
  - Constancy of c: In all inertial frames, the speed of light (in vacuum) c is the same. It's 1.
- In Cartesian coordinates, this defines a Lorentzian metric

$$\eta = \text{diag}(-1, +1, +1, +1)$$

- $(\mathbb{R}^4, \eta)$ is Minkowski space
- We call two events' X and Y separation:
  - space-like if $\eta(X-Y, X-Y) =: \eta_{ab}(X-Y)^a(X-Y)^b > 0$
  - null/light-like if $\eta(X-Y, X-Y) =: \eta_{ab}(X-Y)^a(X-Y)^b =$
  - time-like if $\eta(X-Y, X-Y) =: \eta_{ab}(X-Y)^a(X-Y)^b < 0$
- $\implies$ Relativity of simultaneity,
  Lorentz boosts instead of Galileo 'boosts'

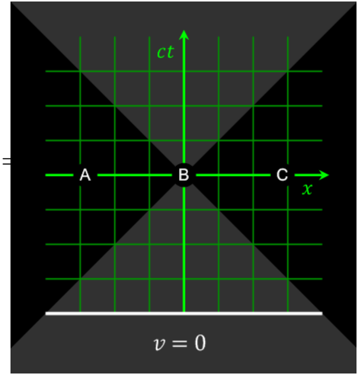


Image source: https://commons.wikimedia.org/wiki/File:Relativity_of_Simultaneity_Animation.gif

General Relativity [GR] Done Quick

- Localize the lightcone! Allow it to change direction!

General Relativity [GR] Done Quick

- Localize the lightcone! Allow it to change direction!

- The metric becomes a function of the space-time coordinates

General Relativity [GR] Done Quick

- Localize the lightcone! Allow it to change direction!
- The metric becomes a function of the space-time coordinates
- The metric has to fulfil the Einstein equation:

$$G_{ab}(g) + \Lambda g_{ab} = 8\pi T_{ab} \frac{G}{c^4}$$

General Relativity [GR] Done Quick

- Localize the lightcone! Allow it to change direction!
- The metric becomes a function of the space-time coordinates
- The metric has to fulfil the Einstein equation:

$$G_{ab}(g) + \Lambda g_{ab} = 8\pi T_{ab} \frac{G}{c^4}$$

- This only *looks* simple. It's only quasi-linear, and a coupled system for the ten components of g_{ab} with 2 physical d.o.f.

- Localize the lightcone! Allow it to change direction!
- The metric becomes a function of the space-time coordinates
- The metric has to fulfil the Einstein equation:

$$G_{ab}(g) + \Lambda g_{ab} = 8\pi T_{ab} \frac{G}{c^4}$$

- This only *looks* simple. It's only quasi-linear, and a coupled system for the ten components of g_{ab} with 2 physical d.o.f.
- Here it is as a PDE:
$\frac{1}{2}\partial_c g^{cf}[\partial_a g_{bf} + \partial_b g_{af} - \partial_f g_{ab}] - \frac{1}{2}\partial_b g^{cf}[\partial_a g_{cf}] + \frac{1}{4}g^{cg}[\partial_m g_{cg} + \partial_c g_{mg} - \partial_g g_{mc}]g^{mf}[\partial_a g_{bf} + \partial_b g_{af} - \partial_f g_{ab}] - \frac{1}{4}g^{cg}[\partial_m g_{bg} + \partial_b g_{mg} - \partial_g g_{mb}]g^{mf}[\partial_a g_{cf} + \partial_c g_{af} - \partial_f g_{ac}] - \frac{1}{2}g_{ab}g^{de}(\frac{1}{2}\partial_c g^{cf}[\partial_e g_{df} + \partial_d g_{ef} - \partial_f g_{ed}] - \frac{1}{2}\partial_d g^{cf}[\partial_e g_{cf} + \partial_c g_{ef} - \partial_f g_{ec}] + \frac{1}{4}g^{cf}[\partial_m g_{cf} + \partial_c g_{mf} - \partial_f g_{mc}]g^{mg}[\partial_e g_{dg} + \partial_d g_{eg} - \partial_g g_{ed}] - \frac{1}{4}g^{cf}[\partial_m g_{df} + \partial_d g_{mf} - \partial_f g_{md}]g^{mg}[\partial_e g_{cg} + \partial_c g_{eg} - \partial_g g_{ec}]) + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab}$

General Relativity [GR] Done Quick

- Localize the lightcone! Allow it to change direction!
- The metric becomes a function of the space-time coordinates
- The metric has to fulfil the Einstein equation:

$$G_{ab}(g) + \Lambda g_{ab} = 8\pi T_{ab} \frac{G}{c^4}$$

- This only *looks* simple. It's only quasi-linear, and a coupled system for the ten components of g_{ab} with 2 physical d.o.f.
- A moment of silence for numerical relativists. They need to discretize this. And then code the discretization...

Physicality of Space-Times

Caveats & Conventions, Part I

- Signature: $-+++$
- $G = c = \hbar = 1$
- Space-time indices: $abcd\ldots$
- Spatial indices: $ijkl\ldots$
- Quasi-Cartesian coordinates where frames appear, no hatted indices needed

Physicality of Space-Times:
Why Worry?

[$Parent], Where Do Space-Times Come From?

- *Primarily*, we take a space-time (M, g) from GR

[$Parent], Where Do Space-Times Come From?

- *Primarily*, we take a space-time (M, g) from GR
- More generally, any theory with (at least) g as output

[$Parent], Where Do Space-Times Come From?

- *Primarily*, we take a space-time (M, g) from GR
- More generally, any theory with (at least) g as output
- Usually, this means field equations (PDE) involving g and *stuff* (like T_{ab})

[$Parent], Where Do Space-Times Come From?

- *Primarily*, we take a space-time (M, g) from GR
- More generally, any theory with (at least) g as output
- Usually, this means field equations (PDE) involving g and *stuff* (like T_{ab})
- Even more generally: Effective space-time geometries as in analogues

[$Parent], Where Do Space-Times Come From?

- *Primarily*, we take a space-time (M, g) from GR
- More generally, any theory with (at least) g as output
- Usually, this means field equations (PDE) involving g and *stuff* (like T_{ab})
- Even more generally: Effective space-time geometries as in analogues
- GR is what we know best; let's start there

Two Ways to Solve Einstein's Equations

Einstein's Equation: $$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}$$

Integration

- Fix T; decide on matter content
- Integrate PDE (hard) on LHS, get g
- Think about metric and its physics
- The usual approach

Differentiation/'Reverse Engineering'/ 'Metric Engineering'

- Fix g; decide what the metric should *do*
- Differentiate g (easy) in LHS to get T
- Think about what this matter is (hard)

Gödel Solution and Wormholes

- Gödel (1949): GR doesn't fulfil Mach's principle. Proof: His Universe.
- Metric:

with $t, x, y, z \in (-\infty, \infty)$:

$$ds^2 = -\frac{1}{2\omega^2}\left[-(\mathrm{d}t + e^x\,\mathrm{d}y)^2 + \mathrm{d}x^2 + \frac{1}{2}e^{2x}\,\mathrm{d}y^2 + \mathrm{d}z^2\right].$$

  - Homogeneous
  - Base manifold $\mathbb{R}^4$
  - At every point rotating about an axis

Gödel Solution and Wormholes

- Gödel (1949): GR doesn't fulfil Mach's principle. Proof: His Universe.
- Metric:

with $t, x, y, z \in (-\infty, \infty)$:

$$\mathrm{d}s^2 = -\frac{1}{2\omega^2}\left[-(\mathrm{d}t + e^x\,\mathrm{d}y)^2 + \mathrm{d}x^2 + \frac{1}{2}e^{2x}\,\mathrm{d}y^2 + \mathrm{d}z^2\right]$$

  - Homogeneous
  - Base manifold $\mathbb{R}^4$
  - At every point rotating about an axis
  - An early example of metric engineering

Since, furthermore, R is a constant, the relativistic field equations (with the x_0-lines as world lines of matter), i.e., the equations[8]

$$R_{ik} - \tfrac{1}{2}g_{ik}R = 8\pi\kappa\rho u_i u_k + \lambda g_{ik}$$

are satisfied (for a given value of ρ), if we put

$$1/a^2 = 8\pi\kappa\rho, \quad \lambda = -R/2 = -1/2a^2 = -4\pi\kappa\rho.$$

'Image' source: Gödel '49, p.448

Gödel Solution and Wormholes

- Gödel (1949): GR doesn't fulfil Mach's principle. Proof: His Universe.
- Metric:

with $t, x, y, z \in (-\infty, \infty)$:

$$ds^2 = -\frac{1}{2\omega^2}\left[-(dt + e^x\, dy)^2 + dx^2 + \frac{1}{2}e^{2x}\, dy^2 + dz^2\right].$$

  - Homogeneous
  - Base manifold $\mathbb{R}^4$
  - At every point rotating about an axis
  - An early example of metric engineering
  - Closed time-like curves (CTCs) everywhere

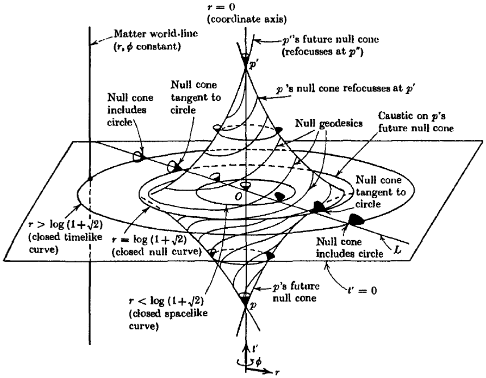


FIGURE 31. Gödel's universe with the irrelevant coordinate z suppressed. The space is rotationally symmetric about any point; the diagram represents correctly the rotational symmetry about the axis $r = 0$, and the time invariance. The light cone opens out and tips over as r increases (see line L) resulting in closed timelike curves. The diagram does not correctly represent the fact that *all* points are in fact equivalent.

Image source: Hawking & Ellis, p.169

Gödel Solution and Wormholes

- Morris & Thorne, doi:10.1119/1.15620 and Morris, Thorne & Yurtsever, doi:10.1103/PhysRevLett.61.1446: Spherically symmetric, (possibly) traversible wormholes

with $l \in (-\infty, \infty)$:

$$\mathrm{d}s^2 = -e^{2\phi(l)}\,\mathrm{d}t^2 + \mathrm{d}l^2 + r^2(l)\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\big),$$

with 2 patches, glued at throat:

$$= -e^{2\phi_\pm(r)}\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - b_\pm(r)/r} + r^2\big(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\big),$$

- *Modified* theories of gravity can easily accommodate various wormholes
- Visualized for Interstellar

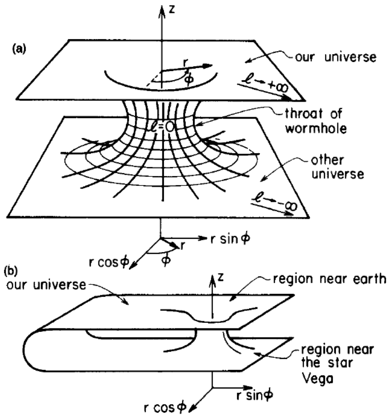


Image source: Morris & Thorne '88doi:10.1119/1.15620

The Alcubierre Warp Drive[3]

In generic Natário form:[1]

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \delta_{ij}\left(\mathrm{d}x^i - v^i(x,y,z,t)\,\mathrm{d}t\right)\left(\mathrm{d}x^j - v^j(x,y,z,t)\,\mathrm{d}t\right)$$

- ADM split, originally including global hyperbolicity
- Unit lapse, flat spatial slices
- **v** as ‘Newtonian’[2] velocity of a region of space-time
- No description of *how* this is generated/built

[1] Alcubierre ’94, arXiv:gr-qc/0110086
[2] Warning! The quotation marks do **heavy** lifting! *Cf.* Painlevé–Gullstrand coordinates!
[3] Natário ’02 arXiv:gr-qc/0009013

Adding Mass to a Warp Drive[5]

- Assume well-defined (extension) of ADM mass
- Three options:
  - Warp bubble is moving in a massive background
  - Warp bubble has mass (possibly even a horizon)
  - Warp bubble hides mass (a 'payload'/'spaceship')
- Alluded to in literature: Payloads.

[5]Santiago, SeSc, Visser '22 arXiv:2205.15950

Adding Mass to a Warp Drive[5]

- Assume well-defined (extension) of ADM mass
- Three options:
  - Warp bubble is moving in a massive background
  - Warp bubble has mass (possibly even a horizon)
  - Warp bubble hides mass (a 'payload'/'spaceship')
- Alluded to in literature: Payloads. Due to reverse-engineering irrelevant/boring/trivial.
- The other two are more interesting, but still violate the NEC[4]

[4] Santiago, SeSc, Visser '22 arXiv:2105.03079
[5] Santiago, SeSc, Visser '22 arXiv:2205.15950

Adding Mass to a Warp Drive[5]

- Assume well-defined (extension) of ADM mass
- Three options:
  - Warp bubble is moving in a massive background
  - Warp bubble has mass (possibly even a horizon)
  - Warp bubble hides mass (a ‘payload’/‘spaceship’)
- Alluded to in literature: Payloads. Due to reverse-engineering irrelevant/boring/trivial.

- The other two are more interesting, but still violate the NEC[4]

[4]Santiago, SeSc, Visser ’22 arXiv:2105.03079
[5]Santiago, SeSc, Visser ’22 arXiv:2205.15950

Adding Mass to a Warp Drive[5]

- Assume well-defined (extension) of ADM mass
- Three options:
  - Warp bubble is moving in a massive background
  - Warp bubble has mass (possibly even a horizon)
  - Warp bubble hides mass (a ‘payload’/‘spaceship’)
- Alluded to in literature: Payloads. Due to reverse-engineering irrelevant/boring/trivial.

- The other two are more interesting, but still violate the NEC[4]
- They hint at theoretical applications

---

[4] Santiago, SeSc, Visser ’22 arXiv:2105.03079

[5] Santiago, SeSc, Visser ’22 arXiv:2205.15950

Adding Mass to a Warp Drive[5]

- Assume well-defined (extension) of ADM mass
- Three options:
  - Warp bubble is moving in a massive background
  - Warp bubble has mass (possibly even a horizon)
  - Warp bubble hides mass (a ‘payload’/‘spaceship’)
- Alluded to in literature: Payloads. Due to reverse-engineering [illegible]evant/boring/trivial.

- The other two are more interesting, but [illegible]ll violate the NEC[4]

- They hint at theoretical applications

[4] Santiago, SeSc, Visser ’22 arXiv:2105.03079

[5] Santiago, SeSc, Visser ’22 arXiv:2205.15950

Tractor Beams: Modifying the Warp Drive

There is more one can do.[6]

- Slightly modify the metric to:[7]

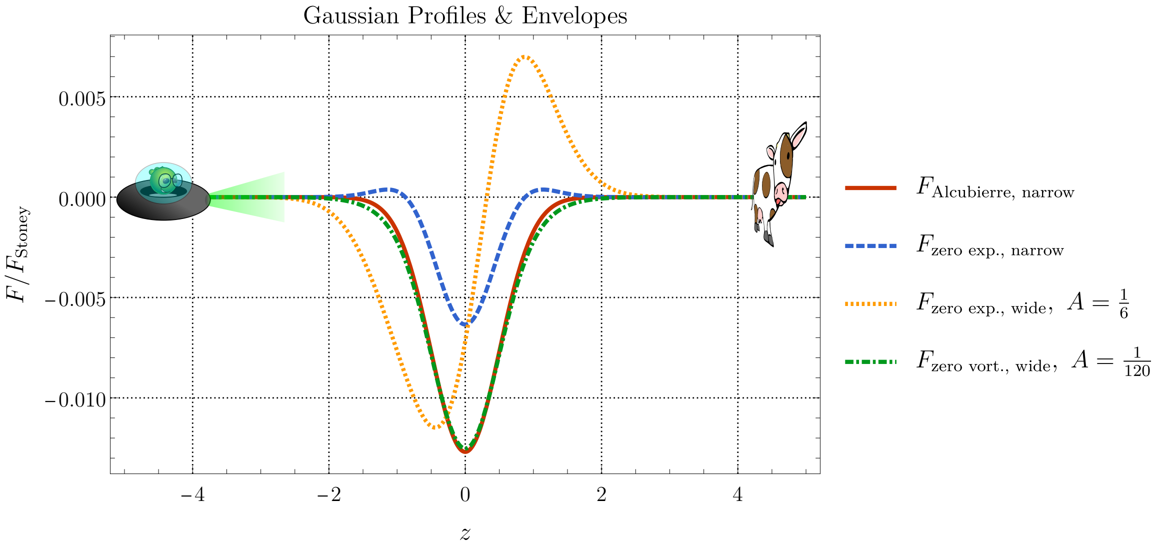
$$v_x(t,x,y,z) = k(t,z)\, x\, h(x^2+y^2),$$
$$v_y(t,x,y,z) = k(t,z)\, y\, h(x^2+y^2),$$
$$v_z(t,x,y,z) = v(t,z)\, f(x^2+y^2).$$

- Use functions k, h, v to make this into a beam along the z-axis
- Assume a ~~spherical cow in a vacuum~~ flat cow in this space-time perpendicular to beam & that beam hits it from the left
- Calculate the force on its surface from stress-energy tensor
- Explicit calculation shows (again) violations of NEC

[6] Santiago, SeSc, Visser '21 arXiv:2106.05002

[7] Warning! This does not include the original Alcubierre metric!

A Visualization of Tractor Beams



The Supposed Tool: Pointwise Energy Conditions[8]

Interpretation	**WEC**	**SEC**	**NEC**
'geometric'[a]	$\forall$ timelike V: $G_{ab}V^aV^b \geq 0$	$\forall$ timelike V: $R_{ab}V^aV^b \geq 0$	$\forall$ null k: $R_{ab}k^ak^b \geq 0$
physical	$\forall$ timelike V: $T_{ab}V^aV^b \geq 0$	$\forall$ timelike V: $(T_{ab} - \frac{1}{2}Tg_{ab})V^aV^b \geq 0$	$\forall$ null k: $T_{ab}k^ak^b \geq 0$
effective	$\rho \geq 0$ & $\forall \hat{a}$: $\rho + p_{\hat{a}} \geq 0$	$\rho + \sum_{\hat{a}} p_{\hat{a}} \geq 0$ & $\forall \hat{a}$: $\rho + p_{\hat{a}} \geq 0$	$\forall \hat{a}$: $\rho + p_{\hat{a}} \geq 0$
Interpretation	**DEC**	**†TEC†**	
'geometric'	$\forall$ timelike V, W: $G_{ab}V^aW^b \geq 0$	$\mathrm{tr}(G) \geq 0$	
physical	$\forall$ timelike V, W: $T_{ab}V^aW^b \geq 0$	$\mathrm{tr}(T) \geq 0$	
effective	$\rho \geq 0$ & $\forall \hat{a}$: $\rho \geq \lvert p_{\hat{a}} \rvert$	$\rho - \sum_{\hat{a}} p_{\hat{a}} \geq 0$	

[a] A.k.a. 'convergence conditions' (CC)

$$\mathrm{DEC} \Longrightarrow \mathrm{WEC} \Longrightarrow \mathrm{NEC} \Longleftarrow \mathrm{SEC}$$

[8] Following Curiel '14 arXiv:1405.0403 and Barceló & Visser '02 arXiv:gr-qc/0205066

The Supposed Tool: Pointwise Energy Conditions[8]

Interpretation	**WEC**	**SEC**	**NEC**
'geometric'[a]	$\forall$ timelike V: $G_{ab}V^aV^b \geq 0$	$\forall$ timelike V: $R_{ab}V^aV^b \geq 0$	$\forall$ null k: $R_{ab}k^ak^b \geq 0$
physical	$\forall$ timelike V: $T_{ab}V^aV^b \geq 0$	$\forall$ timelike V: $(T_{ab} - \frac{1}{2}Tg_{ab})V^aV^b \geq 0$	$\forall$ null k: $T_{ab}k^ak^b \geq 0$
effective	$\rho \geq 0$ & $\forall \hat{a}$: $\rho + p_{\hat{a}} \geq 0$	$\rho + \sum_{\hat{a}} p_{\hat{a}} \geq 0$ & $\forall \hat{a}$: $\rho + p_{\hat{a}} \geq 0$	$\forall \hat{a}$: $\rho + p_{\hat{a}} \geq 0$

Interpretation	**DEC**	**†TEC†**
'geometric'	$\forall$ timelike V, W: $G_{ab}V^aW^b \geq 0$	$\mathrm{tr}(G) \geq 0$
physical	$\forall$ timelike V, W: $T_{ab}V^aW^b \geq 0$	$\mathrm{tr}(T) \geq 0$
effective	$\rho \geq 0$ & $\forall \hat{a}$: $\rho \geq \lvert p_{\hat{a}} \rvert$	$\rho - \sum_{\hat{a}} p_{\hat{a}} \geq 0$

[a] A.k.a. 'convergence conditions' (CC)

$$\text{DEC} \implies \text{WEC} \implies \text{NEC} \impliedby \text{SEC}$$

As the name suggests—the NEC is the weakest.😇

[8] Following Curiel '14 arXiv:1405.0403 and Barceló & Visser '02 arXiv:gr-qc/0205066

Their Uses & Their Issues[10]

They find much use
(mostly in mathematical relativity):

- Stand-in for unknown equations of state
- Positive mass theorems
- Singularity theorems (cosmological and black holes)
- Cosmic no-hair theorem ($\Lambda > 0$ approaches de Sitter)
- *'Ruling out' exotic space-times*

There is an increasing list of physically viable violations of various kinds:

[10] Martín-Moruno & Visser '17 arXiv:1702.05915

Their Uses & Their Issues[10]

They find much use
(mostly in mathematical relativity):

- Stand-in for unknown equations of state
- Positive mass theorems
- Singularity theorems (cosmological and black holes)
- Cosmic no-hair theorem ($\Lambda > 0$ approaches de Sitter)
- *'Ruling out' exotic space-times*

There is an increasing list of physically viable violations of various kinds:

TEC
- EoS of neutron star matter $\longrightarrow \dagger(\leq 1961)$[9]

[9]Zel'dovich '62 JETP **14**(5), 1143–1147
[10]Martín-Moruno & Visser '17 arXiv:1702.05915

Their Uses & Their Issues[10]

They find much use
(mostly in mathematical relativity):

- Stand-in for unknown equations of state
- Positive mass theorems
- Singularity theorems (cosmological and black holes)
- Cosmic no-hair theorem ($\Lambda > 0$ approaches de Sitter)
- *'Ruling out' exotic space-times*

There is an increasing list of physically viable violations of various kinds:

TEC
- EoS of neutron star matter $\longrightarrow$ †(≤ 1961)[9]

NEC
- Non-minimally coupled, classical scalar fields
- Casimir effect

[9] Zel'dovich '62 JETP **14**(5), 1143–1147
[10] Martín-Moruno & Visser '17 arXiv:1702.05915

Their Uses & Their Issues[10]

They find much use
(mostly in mathematical relativity):

- Stand-in for unknown equations of state
- Positive mass theorems
- Singularity theorems (cosmological and black holes)
- Cosmic no-hair theorem ($\Lambda > 0$ approaches de Sitter)
- *'Ruling out' exotic space-times*

There is an increasing list of physically viable violations of various kinds:

TEC
- EoS of neutron star matter $\longrightarrow \dagger(\leq 1961)$[9]

NEC
- Non-minimally coupled, classical scalar fields
- Casimir effect

WEC
- $\Lambda < 0$

[9] Zel'dovich '62 JETP **14**(5), 1143–1147
[10] Martín-Moruno & Visser '17 arXiv:1702.05915

Their Uses & Their Issues[10]

They find much use
(mostly in mathematical relativity):

- Stand-in for unknown equations of state
- Positive mass theorems
- Singularity theorems (cosmological and black holes)
- Cosmic no-hair theorem ($\Lambda > 0$ approaches de Sitter)
- *'Ruling out' exotic space-times*

There is an increasing list of physically viable violations of various kinds:

TEC
- EoS of neutron star matter $\longrightarrow$ †(≤ 1961)[9]

NEC
- Non-minimally coupled, classical scalar fields
- Casimir effect

WEC
- $\Lambda < 0$

SEC
- $\Lambda > 0$
- Massive, minimally-coupled, non-tachyonic scalar fields (*e.g.*, inflatons)
- *Present* accelerated cosmological expansion

[9] Zel'dovich '62 JETP **14**(5), 1143–1147

[10] Martín-Moruno & Visser '17 arXiv:1702.05915

Their Uses & Their Issues[10]

They find much use
(mostly in mathematical relativity):

- Stand-in for unknown equations of state
- Positive mass theorems
- Singularity theorems (cosmological and black holes)
- Cosmic no-hair theorem ($\Lambda > 0$ approaches de Sitter)
- *'Ruling out' exotic space-times*

There is an increasing list of physically viable violations of various kinds:

TEC	• EoS of neutron star matter $\longrightarrow$ †(≤ 1961)[9]
NEC	• Non-minimally coupled, classical scalar fields
	• Casimir effect
WEC	• $\Lambda < 0$
SEC	• $\Lambda > 0$
	• Massive, minimally-coupled, non-tachyonic scalar fields (*e.g.*, inflatons)
	• *Present* accelerated cosmological expansion
DEC	• [. . .]

[9] Zel'dovich '62 JETP **14**(5), 1143–1147

[10] Martín-Moruno & Visser '17 arXiv:1702.05915

Physicality of Space-Times:

Competing Notions—General Relativity

Physical Solutions: The Good, the Bad, the Ugly

- Theorists (usually) evaluate physics' objects based on 'physicality'
- This is by no means a clear concept:

Physical Solutions: The Good, the Bad, the Ugly

- Theorists (usually) evaluate physics' objects based on 'physicality'
- This is by no means a clear concept:
  - Different, 'obviously physical' notions of inextensibility may compete (Manchak 2021, doi:10.1007/978-3-030-64187-0_17)

Physical Solutions: The Good, the Bad, the Ugly

- Theorists (usually) evaluate physics' objects based on 'physicality'
- This is by no means a clear concept:
    - Different, 'obviously physical' notions of inextensibility may compete (Manchak 2021, doi:10.1007/978-3-030-64187-0_17)
    - Some notions are not falsifiable (*e.g.*, excluding a single point)

Physical Solutions: The Good, the Bad, the Ugly

- Theorists (usually) evaluate physics' objects based on 'physicality'
- This is by no means a clear concept:
  - Different, 'obviously physical' notions of inextensibility may compete (Manchak 2021, doi:10.1007/978-3-030-64187-0_17)
  - Some notions are not falsifiable (*e.g.*, excluding a single point)
  - GR may have different claims on physicality than other theories of gravity

Physical Solutions: The Good, the Bad, the Ugly

- Theorists (usually) evaluate physics' objects based on 'physicality'
- This is by no means a clear concept:
  - Different, 'obviously physical' notions of inextensibility may compete (Manchak 2021, doi:10.1007/978-3-030-64187-0_17)
  - Some notions are not falsifiable (*e.g.*, excluding a single point)
  - GR may have different claims on physicality than other theories of gravity
  - Quantum theory will not agree with classical theory

Physical Solutions: The Good, the Bad, the Ugly

- Theorists (usually) evaluate physics' objects based on 'physicality'
- This is by no means a clear concept:
  - Different, 'obviously physical' notions of inextensibility may compete (Manchak 2021, doi:10.1007/978-3-030-64187-0_17)
  - Some notions are not falsifiable (*e.g.*, excluding a single point)
  - GR may have different claims on physicality than other theories of gravity
  - Quantum theory will not agree with classical theory
  - Analogue metrics differ from astrophysical metrics

Physical Solutions: The Good, the Bad, the Ugly

- Theorists (usually) evaluate physics' objects based on 'physicality'
- This is by no means a clear concept:
  - Different, 'obviously physical' notions of inextensibility may compete (Manchak 2021, doi:10.1007/978-3-030-64187-0_17)
  - Some notions are not falsifiable (*e.g.*, excluding a single point)
  - GR may have different claims on physicality than other theories of gravity
  - Quantum theory will not agree with classical theory
  - Analogue metrics differ from astrophysical metrics
  - Toy/local models need not fulfil all 'physicalities' ($\rightarrow$ utility of homogeneous magnetic fields!)

- **The classic:** Presence of singularities (in the sense of inextendible geodesics)

VS

Causality Conditions + Energy Conditions + Curvature Conditions

Some Examples in General Relativity

- **The classic:** Presence of singularities (in the sense of inextendible geodesics)

  VS

  Causality Conditions + Energy Conditions + Curvature Conditions

- Global hyperbolicity VS cosmology

Some Examples in General Relativity

- **The classic:** Presence of singularities (in the sense of inextendible geodesics)

  VS

  Causality Conditions + Energy Conditions + Curvature Conditions

- Global hyperbolicity VS cosmology

- **Less well-known:** Inextendibility VS chronological/causal/. . . [11]

[11] Manchak (2021), doi:10.1007/978-3-030-64187-0_17

Some Examples in General Relativity

- **The classic:** Presence of singularities (in the sense of inextendible geodesics)

  VS

  Causality Conditions + Energy Conditions + Curvature Conditions

- Global hyperbolicity VS cosmology

- **Less well-known:** Inextendibility VS chronological/causal/. . . [11]

- **Bitter:** Hole-free/inextendibility VS CTCs. . . [12]

[11] Manchak (2021), doi:10.1007/978-3-030-64187-0_17

[12] Manchak (2009), doi:10.1086/605806

Some Examples in General Relativity

- **The classic:** Presence of singularities (in the sense of inextendible geodesics)

  VS

  Causality Conditions + Energy Conditions + Curvature Conditions

- Global hyperbolicity VS cosmology

- **Less well-known:** Inextendibility VS chronological/causal/...[11]

- **Bitter:** Hole-free/inextendibility VS CTCs...[12]

...

[11] Manchak (2021), doi:10.1007/978-3-030-64187-0_17

[12] Manchak (2009), doi:10.1086/605806

Physicality of Space-Times:
Inapplicable Notions—Analogues

Quick Example: Fluid Analogues

- Perturbations ϕ_1 on a potential flow $\mathbf{v} = -\nabla\phi_0$ have to fulfil

$$\Box\phi_1 := \frac{1}{\sqrt{-g_{\text{eff}}}}\partial_\mu\left(\sqrt{-g_{\text{eff}}}\, g_{\text{eff}}^{\mu\nu}\,\partial_\nu\phi_1\right) = 0.$$

  with

$$\mathrm{d}s^2 = -\frac{\rho}{c_\text{s}}\left[\left(c_\text{s}^2 - \mathbf{v}^2\right)\mathrm{d}t^2 - 2v_i\,\mathrm{d}x^i\,\mathrm{d}t + \mathbb{1}\,\mathrm{d}\mathbf{x}^2\right] =: \quad g^{\text{eff}}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

Quick Example: Fluid Analogues

- Perturbations ϕ_1 on a potential flow $\mathbf{v} = -\nabla\phi_0$ have to fulfil

$$\Box\phi_1 := \frac{1}{\sqrt{-g_{\text{eff}}}}\partial_\mu\left(\sqrt{-g_{\text{eff}}}\, g_{\text{eff}}^{\mu\nu}\,\partial_\nu\phi_1\right) = 0.$$

with

$$\mathrm{d}s^2 = -\frac{\rho}{c_\mathrm{s}}\left[\left(c_\mathrm{s}^2 - \mathbf{v}^2\right)\mathrm{d}t^2 - 2v_i\,\mathrm{d}x^i\,\mathrm{d}t + \mathbb{1}\,\mathrm{d}\mathbf{x}^2\right] =: \quad g_{\mu\nu}^{\text{eff}}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

- The irrotational vortex, a.k.a. draining bath tub, gives a background flow

$$\mathbf{v} = -\nabla\phi_0 = \frac{A\hat{r} + B\hat{\theta}}{r}$$

Quick Example: Fluid Analogues

- Perturbations ϕ_1 on a potential flow $\mathbf{v} = -\nabla\phi_0$ have to fulfil

$$\Box\phi_1 := \frac{1}{\sqrt{-g_{\text{eff}}}}\partial_\mu\left(\sqrt{-g_{\text{eff}}}\, g^{\mu\nu}_{\text{eff}}\,\partial_\nu\phi_1\right) = 0.$$

with

$$\mathrm{d}s^2 = -\frac{\rho}{c_s}\left[\left(c_s^2 - \mathbf{v}^2\right)\mathrm{d}t^2 - 2v_i\,\mathrm{d}x^i\,\mathrm{d}t + \mathbb{1}\,\mathrm{d}\mathbf{x}^2\right] =: \quad g^{\text{eff}}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

- The irrotational vortex, a.k.a. draining bath tub, gives a background flow

$$\mathbf{v} = -\nabla\phi_0 = \frac{A\hat{r} + B\hat{\theta}}{r}$$

- This looks something like

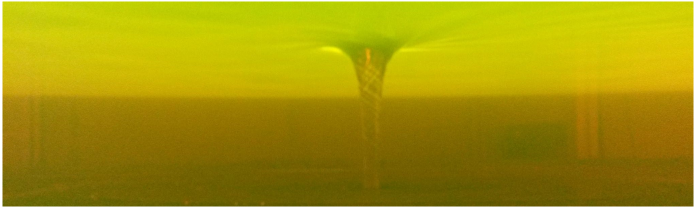


Image credit: Jessica Santiago (2017)

Counterfactuals[13]

- For the draining bathtub:

$$\mathbf{v} = -\nabla\phi_0 = \frac{A\hat{r} + B\hat{\theta}}{r}$$

- Suppose, the resulting metric

$$\mathrm{d}s^2 = -\frac{\rho}{c_\mathrm{s}}\left[\left(c_\mathrm{s}^2 - \frac{A^2 + B^2}{r^2}\right)\mathrm{d}t^2 - 2\frac{A}{r}\,\mathrm{d}r\,\mathrm{d}t - 2B\,\mathrm{d}\theta\,\mathrm{d}t + \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + \mathrm{d}z^2\right]$$

  did arise from GR
- Then we *could* link T_{ab} and G_{ab}

[13] SeSc 2023 arXiv:2305.08725

Counterfactuals[13]

- For the draining bathtub:

$$\mathbf{v} = -\nabla\phi_0 = \frac{A\hat{r} + B\hat{\theta}}{r}$$

- Suppose, the resulting metric

$$\mathrm{d}s^2 = -\frac{\rho}{c_\mathrm{s}}\left[\left(c_\mathrm{s}^2 - \frac{A^2+B^2}{r^2}\right)\mathrm{d}t^2 - 2\frac{A}{r}\,\mathrm{d}r\,\mathrm{d}t - 2B\,\mathrm{d}\theta\,\mathrm{d}t + \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + \mathrm{d}z^2\right]$$

  did arise from GR
- Then we *could* link T_{ab} and G_{ab}
- Then

$$G^{\mathrm{eff}}_{\mu\nu}V^\mu V^\nu = -\frac{A^2+B^2}{r^4\rho c_\mathrm{s}}$$

[13] SeSc 2023 arXiv:2305.08725

Counterfactuals[13]

- For the draining bathtub:

$$\mathbf{v} = -\nabla\phi_0 = \frac{A\hat{r} + B\hat{\theta}}{r}$$

- Suppose, the resulting metric

$$\mathrm{d}s^2 = -\frac{\rho}{c_\mathrm{s}}\left[\left(c_\mathrm{s}^2 - \frac{A^2+B^2}{r^2}\right)\mathrm{d}t^2 - 2\frac{A}{r}\,\mathrm{d}r\,\mathrm{d}t - 2B\,\mathrm{d}\theta\,\mathrm{d}t + \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + \mathrm{d}z^2\right]$$

did arise from GR

- Then we *could* link T_{ab} and G_{ab}
- Then

$$G^{\mathrm{eff}}_{\mu\nu}V^\mu V^\nu = -\frac{A^2+B^2}{r^4\rho c_\mathrm{s}}$$

- This violates the WEC.
- This is not a surprise; the metric isn't GR.

[13] SeSc 2023 arXiv:2305.08725

Counterfactuals[13]

- For the draining bathtub:

$$\mathbf{v} = -\nabla\phi_0 = \frac{A\hat{r} + B\hat{\theta}}{r}$$

- Suppose, the resulting metric

$$\mathrm{d}s^2 = -\frac{\rho}{c_\mathrm{s}}\left[\left(c_\mathrm{s}^2 - \frac{A^2+B^2}{r^2}\right)\mathrm{d}t^2 - 2\frac{A}{r}\,\mathrm{d}r\,\mathrm{d}t - 2B\,\mathrm{d}\theta\,\mathrm{d}t + \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + \mathrm{d}z^2\right]$$

  did arise from GR
- Then we *could* link T_{ab} and G_{ab}
- Then

$$G^{\mathrm{eff}}_{\mu\nu}V^\mu V^\nu = -\frac{A^2+B^2}{r^4\rho c_\mathrm{s}}$$

- This violates the WEC.
- This is not a surprise; the metric isn't GR.
- But neither can we say with certainty where physical metrics come from...

[13] SeSc 2023 arXiv:2305.08725

Physicality towards Space-Times

Caveats & Conventions, Part II

- **Warning!** Work in progress!

Caveats & Conventions, Part II

- **Warning!** Work in progress!
- There exist already nice calculations, but we're short a nifty result

Caveats & Conventions, Part II

- Warning! Work in progress!
- There exist already nice calculations, but we're short a nifty result
- $\Longrightarrow$ **The *talk's* goal:** Context, concepts, tools, minimal[14] maths

[14] Well . . . given the topic

Caveats & Conventions, Part II

- **Warning!** Work in progress!
- There exist already nice calculations, but we're short a nifty result
- $\Longrightarrow$ **The *talk's* goal:** Context, concepts, tools, minimal[14] maths
- **The goal:** Studying the physicality of metrics in a theory-agnostic way

[14] Well ... given the topic

Caveats & Conventions, Part II

- **Warning!** Work in progress!
- There exist already nice calculations, but we're short a nifty result
- $\Longrightarrow$ **The *talk's* goal:** Context, concepts, tools, minimal[14] maths
- **The goal:** Studying the physicality of metrics in a theory-agnostic way
- Uses simple **toy model**

[14]Well . . . given the topic

Physicality towards Space-Times:
The Context

The Problem of Time

- A generic feature of diffeomorphism-invariant theories: Tricky constraints.

$$\mathcal{H} \approx 0,$$
$$\mathcal{H}^i \approx 0.$$

- Classically, the problem of time ('frozen dynamics', 'gauge vs. evolution') is solved—carefully distinguish different roles of $\mathcal{H}$, carefully distinguish phase space and reduced phase space[15]

[15] Pons, Sundermeyer, Salisbury arXiv:1001.2726

The Problem of Time

- A generic feature of diffeomorphism-invariant theories: Tricky constraints.

$$\mathcal{H} \approx 0,$$
$$\mathcal{H}^i \approx 0.$$

- Classically, the problem of time ('frozen dynamics', 'gauge vs. evolution') is solved—carefully distinguish different roles of $\mathcal{H}$, carefully distinguish phase space and reduced phase space[15]

- After *quantization* of diffeomorphism-invariant theories, however, the problem of time remains (at least) much more hotly debated

[15] Pons, Sundermeyer, Salisbury arXiv:1001.2726

The Problem of Time

- A generic feature of diffeomorphism-invariant theories: Tricky constraints.

$$\mathcal{H} \approx 0,$$
$$\mathcal{H}^i \approx 0.$$

- Classically, the problem of time ('frozen dynamics', 'gauge vs. evolution') is solved—carefully distinguish different roles of $\mathcal{H}$, carefully distinguish phase space and reduced phase space[15]

- After *quantization* of diffeomorphism-invariant theories, however, the problem of time remains (at least) much more hotly debated

- Essentially: Extrinsic time (QM) versus intrinsic time (GR)

[15] Pons, Sundermeyer, Salisbury arXiv:1001.2726

Reminder: Geometric/Mathematical Context

In the first part, we saw:

- GR has strong theorems and no-go theorems
  - Positive mass
  - Singularities
  - Existence and uniqueness results
  - Censorship (various)
  - ...

Reminder: Geometric/Mathematical Context

In the first part, we saw:

- GR has strong theorems and no-go theorems
  - Positive mass
  - Singularities
  - Existence and uniqueness results
  - Censorship (various)
  - ...
- However, these rely not only on GR, but also on additional ‘physicality assumptions’

Reminder: Geometric/Mathematical Context

In the first part, we saw:

- GR has strong theorems and no-go theorems
  - Positive mass
  - Singularities
  - Existence and uniqueness results
  - Censorship (various)
  - ...
- However, these rely not only on GR, but also on additional 'physicality assumptions'
- Absence of these assumptions, or moving away from GR enlarges the space of solutions and 'solutions'
- Absence of these assumptions, or moving away from GR reduces available theorems and no-go theorems

The Problem

- In the absence of no-go theorems, or in the presence of quantum theory, potential problems occur
- Especially wormholes are often studied/found/claimed in- and outside of GR.

[16] Barceló, Visser arXiv:gr-qc/0205066, Santiago *et al.* arXiv:2105.03079, SeSc arXiv:2305.08725

The Problem

- In the absence of no-go theorems, or in the presence of quantum theory, potential problems occur
- Especially wormholes are often studied/found/claimed in- and outside of GR.
- Rather than fight over overly specific and often useless notions of physicality like energy conditions[16] focus on the effect

[16] Barceló, Visser arXiv:gr-qc/0205066, Santiago *et al.* arXiv:2105.03079, SeSc arXiv:2305.08725

The Problem

- In the absence of no-go theorems, or in the presence of quantum theory, potential problems occur
- Especially wormholes are often studied/found/claimed in- and outside of GR.
- Rather than fight over overly specific and often useless notions of physicality like energy conditions[16] focus on the effect
- Wormholes, Gödel universe, (superluminal) warp drives, Krasnikov tubes—their problem is time travel

[16] Barceló, Visser arXiv:gr-qc/0205066, Santiago *et al.* arXiv:2105.03079, SeSc arXiv:2305.08725

The Problem

- In the absence of no-go theorems, or in the presence of quantum theory, potential problems occur
- Especially wormholes are often studied/found/claimed in- and outside of GR.
- Rather than fight over overly specific and often useless notions of physicality like energy conditions[16] focus on the effect
- Wormholes, Gödel universe, (superluminal) warp drives, Krasnikov tubes—their problem is time travel
- Space-times may only be emergent

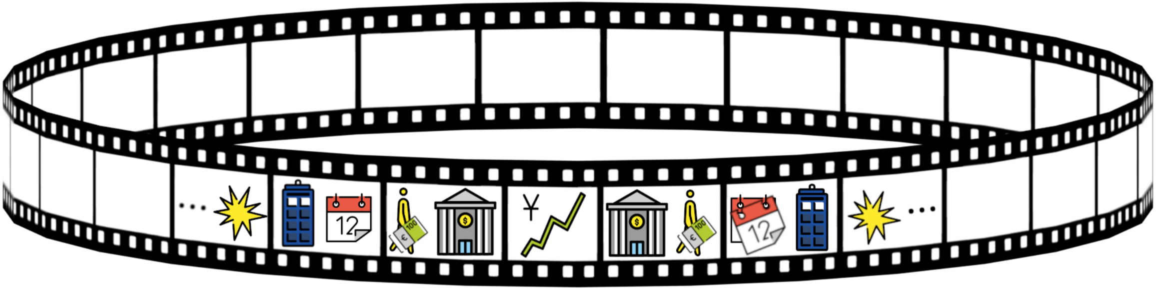
[16] Barceló, Visser arXiv:gr-qc/0205066, Santiago *et al.* arXiv:2105.03079, SeSc arXiv:2305.08725

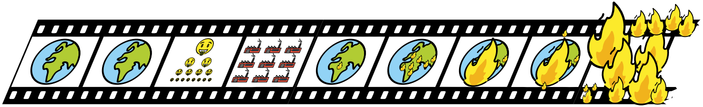
The Problem

- In the absence of no-go theorems, or in the presence of quantum theory, potential problems occur
- Especially wormholes are often studied/found/claimed in- and outside of GR.
- Rather than fight over overly specific and often useless notions of physicality like energy conditions[16] focus on the effect
- Wormholes, Gödel universe, (superluminal) warp drives, Krasnikov tubes—their problem is time travel
- Space-times may only be emergent
- Evaluate the physicality of time-travel not based on space-time/CTCs, but on time's origin

[16] Barceló, Visser arXiv:gr-qc/0205066, Santiago *et al.* arXiv:2105.03079, SeSc arXiv:2305.08725

The Picture





Ambient quantum system with *local* clocks for subsystems with different relational times

Physicality towards Space-Times:
The Tools

Positive Operator-Valued Measures

- It's a mouthful, so: **POVM**

Positive Operator-Valued Measures

- It's a mouthful, so: **POVM**
- A way to formalize imprecise measurements in quantum theory[17]

[17] Busch, Grabowski, Lahti — '*Operational Quantum Physics*', ISBN: 3-540-59358-6

Positive Operator-Valued Measures

- It's a mouthful, so: **POVM**
- A way to formalize imprecise measurements in quantum theory[17]
- Ingredients:
  - Hilbert space $\mathcal{H}$ and its states Ψ/density matrices $\hat{\rho}$
  - The totality of measurement outcomes Ω and its σ-algebra M of subsets

[17] Busch, Grabowski, Lahti — '*Operational Quantum Physics*', ISBN: 3-540-59358-6

Positive Operator-Valued Measures

- It's a mouthful, so: **POVM**
- A way to formalize imprecise measurements in quantum theory[17]
- Ingredients:
  - Hilbert space $\mathcal{H}$ and its states Ψ/density matrices $\hat{\rho}$
  - The totality of measurement outcomes Ω and its σ-algebra M of subsets
- A POVM F is a function such that:

(1) $\forall X \in M \ : \ F(X) \geq 0$. ('positive')

(2) $F(\Omega) = \mathbb{1}_{\mathcal{H}}$

(3) For disjoint $X_i \in M$, $E(\cup_i X_i) = \sum_i E(X_i)$

[17] Busch, Grabowski, Lahti — '*Operational Quantum Physics*', ISBN: 3-540-59358-6

Positive Operator-Valued Measures

- It's a mouthful, so: **POVM**
- A way to formalize imprecise measurements in quantum theory[17]
- Ingredients:
  - Hilbert space $\mathcal{H}$ and its states Ψ/density matrices $\hat{\rho}$
  - The totality of measurement outcomes Ω and its σ-algebra M of subsets
- A POVM F is a function such that:
  (1) $\forall X \in M \ : \ F(X) \geq 0$. ('positive') **Warning!** May have improper eigenstates!
  (2) $F(\Omega) = \mathbb{1}_{\mathcal{H}}$
  (3) For disjoint $X_i \in M$, $E(\cup_i X_i) = \sum_i E(X_i)$
- The Born rule reads

$$P(F|\rho) = \mathrm{Tr}(F\hat{\rho})$$

[17] Busch, Grabowski, Lahti — '*Operational Quantum Physics*', ISBN: 3-540-59358-6

Positive Operator-Valued Measures

- It's a mouthful, so: **POVM**
- A way to formalize imprecise measurements in quantum theory[17]
- Ingredients:
  - Hilbert space $\mathcal{H}$ and its states Ψ/density matrices $\hat{\rho}$
  - The totality of measurement outcomes Ω and its σ-algebra M of subsets
- A POVM F is a function such that:
  (1) $\forall X \in M \; : \; F(X) \geq 0$. ('positive') **Warning!** May have improper eigenstates!
  (2) $F(\Omega) = \mathbb{1}_{\mathcal{H}}$
  (3) For disjoint $X_i \in M$, $E(\cup_i X_i) = \sum_i E(X_i)$
- The Born rule reads

$$P(F|\rho) = \mathrm{Tr}(F\hat{\rho})$$

- Projection-valued measures exchange (1) for the stricter $E(X)^2 = E(X)$;
  $\iff$ standard QM operators

[17] Busch, Grabowski, Lahti — '*Operational Quantum Physics*', ISBN: 3-540-59358-6

The Page–Wootters Formalism: Steps towards Relational Quantum Time

- 'Times is what one reads off a clock.'[18]
- First attempt: A self-adjoint operator ('clock') canonically conjugate to a/the Hamiltonian

[18] Paraphrasing Einstein's 1905 article on special relativity.

The Page–Wootters Formalism: Steps towards Relational Quantum Time

- ‘Times is what one reads off a clock.’[18]
- First attempt: A self-adjoint operator (‘clock’) canonically conjugate to a/the Hamiltonian
- Problem: Pauli’s(?) no-go result (Pauli 1990, doi:10.1007/978-3-642-62187-9, p.84)

[18] Paraphrasing Einstein’s 1905 article on special relativity.

The Page–Wootters Formalism: Steps towards Relational Quantum Time

- ‘Times is what one reads off a clock.’[18]
- First attempt: A self-adjoint operator (‘clock’) canonically conjugate to a/the Hamiltonian
- Problem: Pauli’s(?) no-go result (Pauli 1990, doi:10.1007/978-3-642-62187-9, p.84)

Page–Wootters formalism (Page, Wootters 1983 doi:10.1103/PhysRevD.27.2885)

- Separate full Hilbert space: $\mathcal{H} = \mathcal{H}_\mathrm{C} \otimes \mathcal{H}_\mathrm{R}$

[18] Paraphrasing Einstein’s 1905 article on special relativity.

The Page–Wootters Formalism: Steps towards Relational Quantum Time

- 'Times is what one reads off a clock.'[18]
- First attempt: A self-adjoint operator ('clock') canonically conjugate to a/the Hamiltonian
- Problem: Pauli's(?) no-go result (Pauli 1990, doi:10.1007/978-3-642-62187-9, p.84)

Page–Wootters formalism (Page, Wootters 1983 doi:10.1103/PhysRevD.27.2885)

- Separate full Hilbert space: $\mathcal{H} = \mathcal{H}_{\mathrm{C}} \otimes \mathcal{H}_{\mathrm{R}}$
- Introduce a clock Hamiltonian $\hat{H}_{\mathrm{C}}$ of a subsystem

[18] Paraphrasing Einstein's 1905 article on special relativity.

The Page–Wootters Formalism: Steps towards Relational Quantum Time

- 'Times is what one reads off a clock.'[18]
- First attempt: A self-adjoint operator ('clock') canonically conjugate to a/the Hamiltonian
- Problem: Pauli's(?) no-go result (Pauli 1990, doi:10.1007/978-3-642-62187-9, p.84)

Page–Wootters formalism (Page, Wootters 1983 doi:10.1103/PhysRevD.27.2885)

- Separate full Hilbert space: $\mathcal{H} = \mathcal{H}_\mathrm{C} \otimes \mathcal{H}_\mathrm{R}$
- Introduce a clock Hamiltonian $\hat{H}_\mathrm{C}$ of a subsystem
- Specify some chosen, 'initial' clock state ψ_C

[18] Paraphrasing Einstein's 1905 article on special relativity.

The Page–Wootters Formalism: Steps towards Relational Quantum Time

- 'Times is what one reads off a clock.'[18]
- First attempt: A self-adjoint operator ('clock') canonically conjugate to a/the Hamiltonian
- Problem: Pauli's(?) no-go result (Pauli 1990, doi:10.1007/978-3-642-62187-9, p.84)

Page–Wootters formalism (Page, Wootters 1983 doi:10.1103/PhysRevD.27.2885)

- Separate full Hilbert space: $\mathcal{H} = \mathcal{H}_{\mathrm{C}} \otimes \mathcal{H}_{\mathrm{R}}$
- Introduce a clock Hamiltonian $\hat{H}_{\mathrm{C}}$ of a subsystem
- Specify some chosen, 'initial' clock state ψ_C
- Define time through evolution of this state with $\hat{H}_{\mathrm{C}}$

[18] Paraphrasing Einstein's 1905 article on special relativity.

The Page–Wootters Formalism: Steps towards Relational Quantum Time

- 'Times is what one reads off a clock.'[18]
- First attempt: A self-adjoint operator ('clock') canonically conjugate to a/the Hamiltonian
- Problem: Pauli's(?) no-go result (Pauli 1990, doi:10.1007/978-3-642-62187-9, p.84)

Page–Wootters formalism (Page, Wootters 1983 doi:10.1103/PhysRevD.27.2885)

- Separate full Hilbert space: $\mathcal{H} = \mathcal{H}_\mathrm{C} \otimes \mathcal{H}_\mathrm{R}$
- Introduce a clock Hamiltonian $\hat{H}_\mathrm{C}$ of a subsystem
- Specify some chosen, 'initial' clock state ψ_C
- Define time through evolution of this state with $\hat{H}_\mathrm{C}$
- Measure time evolution of an operator $\hat{A}$, stationary w.r.t. $\hat{H}_\mathrm{C}$, as

$$E(A|\tau) = \mathrm{tr}\left(\hat{A}\hat{P}_\tau\hat{\rho}\right) / \, \mathrm{tr}\left(\hat{P}_\tau\hat{\rho}\right),$$

where

$$\hat{P}_\tau = |\psi_C(\tau)\rangle \langle\psi_C(\tau)| \otimes \mathbb{1}_\mathrm{R}, \qquad \text{and} \qquad \hat{\rho} \in \mathcal{L}(\mathcal{H})$$

[18] Paraphrasing Einstein's 1905 article on special relativity.

Relational Time: Other Perspectives

- Meanwhile, POVM are another alternative (Busch *et al.* 1994 doi:10.1016/0375-9601(94)90785-4)
- Here, a *non-unique* time-operator appears as the first moment of a POVM

Relational Time: Other Perspectives

- Meanwhile, POVM are another alternative (Busch *et al.* 1994 doi:10.1016/0375-9601(94)90785-4)
- Here, a *non-unique* time-operator appears as the first moment of a POVM
- Höhn, Smith, Lock, Ahmadi (among others) placed the PW formalism on a gauge-theoretic footing,[19] and refined it using POVMs

[19] E.g., arXiv:1912.00033 and arXiv:2007.00580.

Relational Time: Other Perspectives

- Meanwhile, POVM are another alternative (Busch *et al.* 1994 doi:10.1016/0375-9601(94)90785-4)
- Here, a *non-unique* time-operator appears as the first moment of a POVM
- Höhn, Smith, Lock, Ahmadi (among others) placed the PW formalism on a gauge-theoretic footing,[19] and refined it using POVMs
- This also addressed various criticisms[20] aimed at the original formulation of PW

[19] E.g., arXiv:1912.00033 and arXiv:2007.00580.

[20] Unruh, Wald 1989 doi:10.1103/PhysRevD.40.2598, Kuchař 2011/1991, doi:10.1142/S0218271811019347

Relational Time: Other Perspectives

- Meanwhile, POVM are another alternative (Busch *et al.* 1994 doi:10.1016/0375-9601(94)90785-4)
- Here, a *non-unique* time-operator appears as the first moment of a POVM
- Höhn, Smith, Lock, Ahmadi (among others) placed the PW formalism on a gauge-theoretic footing,[19] and refined it using POVMs

- This also addressed various criticisms[20] aimed at the original formulation of PW

[19] E.g., arXiv:1912.00033 and arXiv:2007.00580.
[20] Unruh, Wald 1989 doi:10.1103/PhysRevD.40.2598, Kuchař 2011/1991, doi:10.1142/S0218271811019347

Relational Time: Other Perspectives

- Meanwhile, POVM are another alternative (Busch *et al.* 1994 doi:10.1016/0375-9601(94)90785-4)

- Here, a *non-unique* time-operator appears as the first moment of a POVM

- Höhn, Smith, Lock, Ahmadi (among others) placed the PW formalism on a gauge-theoretic footing,[19] and refined it using POVMs

- This also addressed various criticisms[20] aimed at the original formulation of PW

- A *time observable* of a clock, $\hat{H}_C$, is a POVM E_T covariant under $\hat{H}_C$'s time translation group, *i.e.*

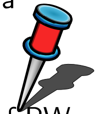
$$E_T(X + t) = U_C(t) E_T(X) U_C^\dagger(t)$$

[19] E.g., arXiv:1912.00033 and arXiv:2007.00580.

[20] Unruh, Wald 1989 doi:10.1103/PhysRevD.40.2598, Kuchař 2011/1991, doi:10.1142/S0218271811019347

Relational Time: Other Perspectives

- Meanwhile, POVM are another alternative (Busch *et al.* 1994 doi:10.1016/0375-9601(94)90785-4)
- Here, a *non-unique* time-operator appears as the first moment of a POVM
- Höhn, Smith, Lock, Ahmadi (among others) placed the PW formalism on a gauge-theoretic footing,[19] and refined it using POVMs



- This also addressed various criticisms[20] aimed at the original formulation of PW
- A *time observable* of a clock, $\hat{H}_C$, is a POVM E_T covariant under $\hat{H}_C$'s time translation group, *i.e.*

$$E_T(X+t) = U_C(t) E_T(X) U_C^\dagger(t)$$

- For us of particular relevance: The POVM bit of these developments.

[19] E.g., arXiv:1912.00033 and arXiv:2007.00580.

[20] Unruh, Wald 1989 doi:10.1103/PhysRevD.40.2598, Kuchař 2011/1991, doi:10.1142/S0218271811019347

Physicality towards Space-Times:
A First Toy Model

When a Physicist Gets Stuck: The Harmonic Oscillator

- Separate Hilbert space as:

$$\hat{H}_C = \hat{n}_C + \frac{1}{2}\mathbb{1}_C.$$

- Define *non*-unitary $\hat{W}$ through

$$\hat{a} = \hat{W}\widehat{|a|}, \qquad \text{with} \qquad \widehat{|a|} := \hat{n}^{1/2}$$

having improper eigenstates $|\theta\rangle$

$$\hat{W}\,|\theta\rangle = e^{i\theta}\,|\theta\rangle\,, \qquad \text{with} \qquad |\theta\rangle = \sum_{n\geq 0} e^{in\theta}\,|n\rangle\,.$$

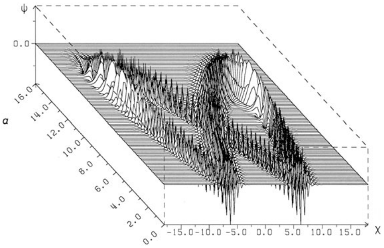
- The relevant POVM:

$$B_0(f) := \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\theta\; f(\theta)\;\; |\theta\rangle\,\langle\theta| = \sum_{n,m\geq 0} \frac{1}{2\pi}\int_0^{2\pi} e^{i(n-m)\theta} f(\theta)\,\mathrm{d}\theta\;\; |n\rangle\,\langle m|\,.$$

- Get one of many possible time operators for $f(\theta) = \theta$ as:

$$\hat{T}_0 = B_0(\theta) = \sum_{n\neq m\geq 0} \frac{1}{i(n-m)}\,|n\rangle\,\langle m| + \pi\mathbb{1}.$$

Modify Toy Model of Quantum Cosmology



Source: Kiefer 1990, doi:10.1016/0550-3213(90)90271-E

- Modify minisuperspace of **closed Friedmann universe** + **conformally coupled scalar**:

$$\hat{H}\Psi(\varphi,\chi) = \left(\frac{\partial^2}{\partial\varphi^2} - \omega_\varphi^2\varphi^2 - \frac{\partial^2}{\partial\chi^2} + \omega_\chi^2\chi^2\right)\Psi = 0$$

- Normalizability of Ψ gives two integers n_φ, n_χ fulfilling

$$\frac{\omega_\varphi}{\omega_\chi} = \frac{2n_\chi + 1}{2n_\varphi + 1}$$

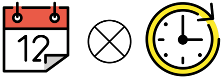
- Instead of φ, use phase as in harmonic oscillator as time; larger range for φ than a in QC

Outlook

Objectives

- Distinguish:
  - Periodic clock

  - Periodic clock with calendar

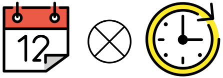


  - Time travel

Objectives

- Distinguish:

  - Periodic clock



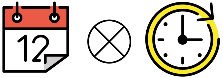
  - Periodic clock with calendar

  - Time travel
- Make self-consistency non-binary by getting a notion of ‘close to’ self-consistency

Objectives

- Distinguish:
  - Periodic clock

  - Periodic clock with calendar

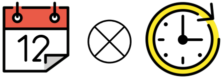


  - Time travel

- Make self-consistency non-binary by getting a notion of ‘close to’ self-consistency
- **Long term goal:** Using entropy for closed systems[21], rule out time travel thermodynamically with only a relative notion of time.

[21] Safranek *et al.*, arXiv:1803.00665

Objectives

- Distinguish:
  - Periodic clock 
  - Periodic clock with calendar 

  - Time travel 
- Make self-consistency non-binary by getting a notion of ‘close to’ self-consistency
- **Long term goal:** Using entropy for closed systems[21], rule out time travel thermodynamically with only a relative notion of time.
- Aim for arguments against space-times with CTCs, while staying agnostic about precise space-time notions of physicality

[21] Safranek *et al.*, arXiv:1803.00665

Summary

- Physicality needs context
- Please, don't evaluate physicality only based on energy conditions
- Please, use energy conditions *correctly*
- Let's explore
    - what 'unphysical' space-times can teach us,
    - what limits space-times in the first place.

Thank you!

References: Part I—Santiago, SeSc, Visser arXiv:2105.03079, arXiv:2106.05002, arXiv:2205.15950;
Part II—Höhn *et al.* arXiv:1912.00033, SeSc arXiv:2305.08725, [Alonso-Serrano, SeSc, Visser—To Appear]

References I

AFP09 G. Auletta, M. Fortunato & G. Parisi. *Quantum Mechanics.* ISBN: 978-0-521-86963-8 (Cambridge University Press, 2009).

Alc94 M. Alcubierre. The warp drive: hyper-fast travel within general relativity. *Classical and Quantum Gravity* **11,** L73. doi:10.1088/0264-9381/11/5/001. arXiv: gr-qc/0009013 [gr-qc] (1994).

BGL94 P. Busch, M. Grabowski & P. J. Lahti. Time observables in quantum theory. *Physics Letters A* **191,** 357–361. doi:10.1016/0375-9601(94)90785-4 (Aug. 1994).

BGL95a P. Busch, M. Grabowski & P. J. Lahti. *Operational Quantum Measurement.* ISBN: 3-540-59358-6 (Springer, 1995).

BGL95b P. Busch, M. Grabowski & P. J. Lahti. Who Is Afraid of POV Measures? Unified Approach to Quantum Phase Observables. *Annals of Physics* **237,** 1–11. doi:10.1006/aphy.1995.1001 (Jan. 1995).

BV02 C. Barcelo & M. Visser. Twilight for the energy conditions? *International Journal of Modern Physics D* **11,** 1553–1560. doi:10.1142/S0218271802002888. arXiv: gr-qc/0205066 [gr-qc] (Mar. 2002).

References II

Cur17 E. Curiel. in *Towards a Theory of Spacetime Theories* (eds D. Lehmkuhl, G. Schiemann & E. Scholz) 43–104 (Springer, 2017). ISBN: 978-1-4939-3209-2. doi:10.1007/978-1-4939-3210-8_3. arXiv: 1405.0403 [physics.hist-ph].

FPS18 C. J. Fewster, C. Pfeifer & D. Siemssen. Quantum energy inequalities in premetric electrodynamics. *Physical Review D* **97**, 025019. doi:10.1103/PhysRevD.97.025019. arXiv: 1709.01760 (Jan. 2018).

Göd49 K. Gödel. An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation. *Review of Modern Physics* **21**, 447–450. doi:10.1103/RevModPhys.21.447 (July 1949).

HE74 S. W. Hawking & G. F. R. Ellis. *The large scale structure of space-time.* ISBN: 978-0-521-09906-6. doi:10.1017/CBO9780511524646 (Cambridge University Press, 1974).

HSL21a P. A. Höhn, A. R. H. Smith & M. P. E. Lock. Equivalence of Approaches to Relational Quantum Dynamics in Relativistic Settings. *Frontiers in Physics* **9**, 181. doi:10.3389/fphy.2021.587083. arXiv: 2007.00580 [gr-qc] (Mar. 2021).

References III

HSL21b P. A. Höhn, A. R. H. Smith & M. P. E. Lock. The Trinity of Relational Quantum Dynamics. *Physical Review D* **104,** 066001. doi:10.1103/PhysRevD.104.066001. arXiv: 1912.00033 [quant-ph] (Sept. 2021).

Kie90 C. Kiefer. Wave packets in quantum cosmology and the cosmological constant. *Nuclear Physics B* **341,** 273–293. doi:10.1016/0550-3213(90)90271-E (Sept. 1990).

Kuc11 K. V. Kuchař. Time and Interpretations of Quantum Gravity. *International Journal of Modern Physics D* **20,** 3–86. doi:10.1142/S0218271811019347 (Supplement 1 2011). Contribution to: 4th Canadian Conference on General Relativity and Relativistic Astrophysics.

Man09 JB. Manchak. On the Existence of "Time Machine" in General Relativity. *Philosophy of Science* **76,** 1020–1026. doi:10.1086/605806. https://sites.socsci.uci.edu/~jmanchak/oteotmigr.pdf (2009).

References IV

Man21 JB. Manchak. in *Hajnal Andréka and István Németi on the Unity of Science: From Computing to Relativity Theory Through Algebraic Logic* (eds J. Madarász & G. Székely) 409–425 (Springer, 2021). ISBN: 978-3-030-64186-3. doi:10.1007/978-3-030-64187-0_17. http://philsci-archive.pitt.edu/14882/.

MT88 M. S. Morris & K. S. Thorne. Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity. *American Journal of Physics* **56,** 395. doi:10.1119/1.15620 (July 1988).

MV17 P. Martín-Moruno & M. Visser. in *Wormholes, Warp Drives and Energy Conditions* (ed F. S. N. Lobo) 193–213 (Springer, 2017). ISBN: 978-3-319-55181-4. doi:10.1007/978-3-319-55182-1_9. arXiv: 1702.05915 [gr-qc].

Nat02 J. Natário. Warp drive with zero expansion. *Classical and Quantum Gravity* **19,** 1157. doi:10.1088/0264-9381/19/6/308. arXiv: gr-qc/0110086 [gr-qc] (Mar. 2002).

References V

Pau90 W. Pauli. *Die allgemeinen Prinzipien der Wellenmechanik.* German (ed N. Straumann) ISBN: 978-3-540-51949-2. doi:10.1007/978-3-642-62187-9 (Springer, 1990). Neu herausgegeben und mit historischen Anmerkungen versehen von Norbert Straumann.

PS16 C. Pfeifer & D. Siemssen. Electromagnetic potential in pre-metric electrodynamics: Causal structure, propagators and quantization. *Physical Review D* **93,** 105046. doi:10.1103/PhysRevD.93.105046. arXiv: 1602.00946 (May 2016).

PSS10 J. M. Pons, K. A. Sundermeyer & D. C. Salisbury. Observables in classical canonical gravity: folklore demystified. *Journal of Physics: Conference Series* **222,** 012018. doi:10.1088/1742-6596/222/1/012018. arXiv: 1001.2726 [gr-qc] (1st Mediterranean Conference on Classical and Quantum Gravity Jan. 2010).

PW83 D. N. Page & W. K. Wootters. Evolution without evolution: Dynamics described by stationary observables. *Physical Review D* **27,** 2885–2892. doi:10.1103/PhysRevD.27.2885 (June 1983).

References VI

Sch21 S. Schuster. Sparsity of Hawking Radiation in $D+1$ Space-Time Dimensions for Massless and Massive Particles. *Classical and Quantum Gravity* **38,** 047002. doi:10.1088/1361-6382/abd144. arXiv: 1910.07256 [gr-qc] (Feb. 2021).

Sch23 S. Schuster. *Frenemies with Physicality: Manufacturing Manifold Metrics.* Essay written for the Gravity Research Foundation 2023 Awards for Essays on Gravitation. Mar. 2023. arXiv: 2305.08725 [gr-qc].

ŠDA19 D. Šafránek, J. M. Deutsch & A. Aguirre. Quantum coarse-grained entropy and thermalization in closed systems. *Physical Review A* **99,** 012103. doi:10.1103/PhysRevA.99.012103. arXiv: 1803.00665 [quant-ph] (Jan. 2019).

SSV21 J. Santiago, S. Schuster & M. Visser. Tractor beams, pressor beams, and stressor beams in general relativity. *Universe* **7,** 271. doi:10.3390/universe7080271. arXiv: 2106.05002 [gr-qc] (July 2021). Invited for submission by *Universe*.

SSV22 J. Santiago, S. Schuster & M. Visser. Generic warp drives violate the null energy condition. *Physical Review D* **105,** 064038. doi:10.1103/PhysRevD.105.064038. arXiv: 2105.03079 [gr-qc] (Mar. 2022).

References VII

UW89 W. G. Unruh & R. M. Wald. Time and the interpretation of canonical quantum gravity. *Physical Review D* **40,** 2598–2614. doi:10.1103/PhysRevD.40.2598 (Oct. 1989).

Zel62 Y. B. Zel'dovich. The equation of state at ultrahigh densities and its relativistic limitations. *Soviet Physics JETP* **14,** 1143–1147. http://jetp.ras.ru/cgi-bin/e/index/r/41/5/p1609?a=list (May 1962). Translation of Zhur. Eksptl'. i Teoret. Fiz. 41 (1961) 1609–1615 by W.H. Furry.

Modifications—And Recent Publicity

- Natário, a.k.a., zero expansion: Demand

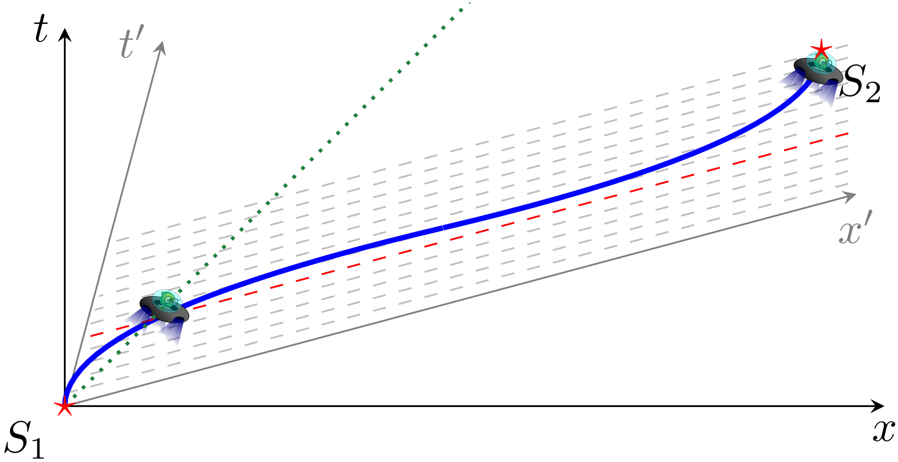
$$\nabla \cdot \mathbf{v} = 0$$

- Zero vorticity (arXiv:2006.07125):

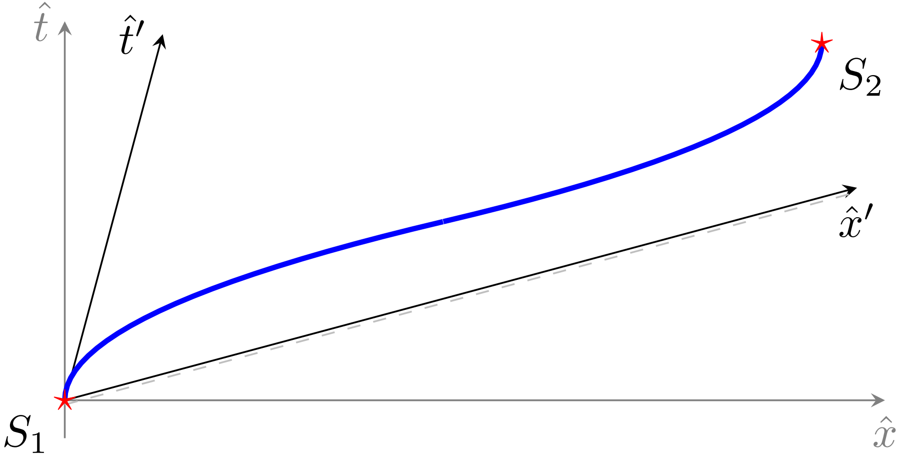
$$\nabla \times \mathbf{v} = 0 \qquad \Longrightarrow \qquad \mathbf{v} = \nabla \cdot \Phi$$

- **Warning!**
  - arXiv:2006.07125 does not provide an explicit example that can be checked; but zero-vorticity warp drives in general violate the NEC
  - arXiv:2104.06488 only uses metrics not fulfilling junction conditions
  - arXiv:2102.06824 only provides static, spherically symmetric metrics, no warp drives
  - arXiv:2102.05119, arXiv:2101.11467, arXiv:2008.06560 have issues of their own (require conflicting assumptions, giving empty space, wrong & important index placement, . . .)
  - All six (and others before them) claim fulfilment of the energy conditions by finding one(!) observer, usually the Eulerian, to fulfil the necessary inequalities.
  - The '$\forall$' in the EC is not, and cannot be shown.

Travelling with It—The 'Rest Frame'



Travelling with It—The ‘Boosted Frame’

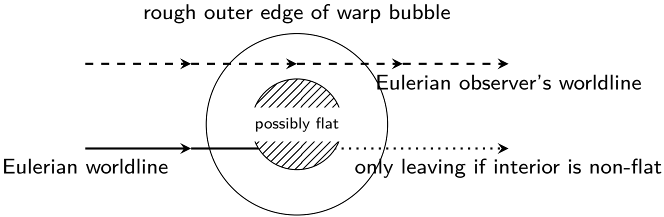


Sketch of Proof for NEC Violation in Warp Drives

- NEC for $\mathrm{tr}(K_{ij}) =: K = 0, \implies \rho + \bar{p} = -\frac{1}{8\pi}\,\mathrm{tr}\left(K_{ij}K^{jk}\right) \leq 0$
- NEC for $K = 0$ fulfilled $\implies K_{ij} = 0 \implies$ Minkowski

Sketch of Proof for NEC Violation in Warp Drives

- NEC for $\mathrm{tr}(K_{ij}) =: K = 0$, $\implies \rho + \bar{p} = -\frac{1}{8\pi}\,\mathrm{tr}\big(K_{ij}K^{jk}\big) \leq 0$
- NEC for $K = 0$ fulfilled $\implies K_{ij} = 0 \implies$ Minkowski



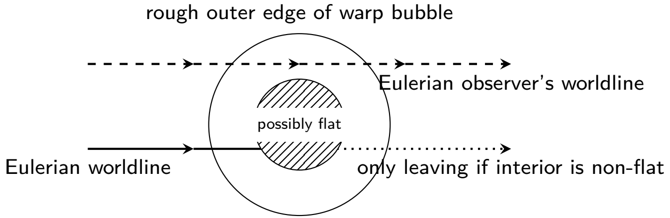
- If $K \neq 0$, Eulerian obs. see: $K \simeq 0 \to K \neq 0 \to K \simeq 0$ (due to asymptotics)
- In their proper time τ, however:

$$\text{NEC} \qquad \Rightarrow \qquad \frac{\mathrm{d}K}{\mathrm{d}\tau} \leq -\frac{3}{2}\,\mathrm{tr}\big([K_{ij}^{\mathrm{tf}}]^2\big)$$

- So, either:

Sketch of Proof for NEC Violation in Warp Drives

- NEC for $\mathrm{tr}(K_{ij}) =: K = 0, \implies \rho + \bar{p} = -\frac{1}{8\pi}\,\mathrm{tr}\big(K_{ij}K^{jk}\big) \leq 0$
- NEC for $K = 0$ fulfilled $\implies K_{ij} = 0 \implies$ Minkowski



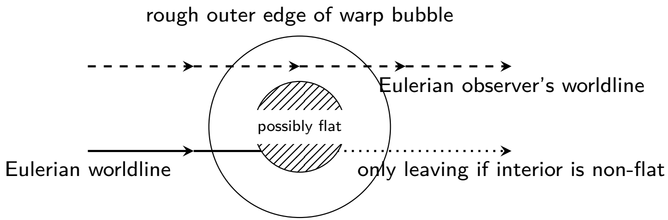
- If $K \neq 0$, Eulerian obs. see: $K \simeq 0 \to K \neq 0 \to K \simeq 0$ (due to asymptotics)
- In their proper time τ, however:

$$\text{NEC} \qquad \Rightarrow \qquad \frac{\mathrm{d}K}{\mathrm{d}\tau} \leq -\frac{3}{2}\,\mathrm{tr}\big([K_{ij}^{\mathrm{tf}}]^2\big)$$

- So, either:
    - K decreases monotonically if NEC fulfilled

Sketch of Proof for NEC Violation in Warp Drives

- NEC for $\operatorname{tr}(K_{ij}) =: K = 0, \implies \rho + \bar{p} = -\frac{1}{8\pi} \operatorname{tr}\left(K_{ij} K^{jk}\right) \leq 0$
- NEC for $K = 0$ fulfilled $\implies K_{ij} = 0 \implies$ Minkowski



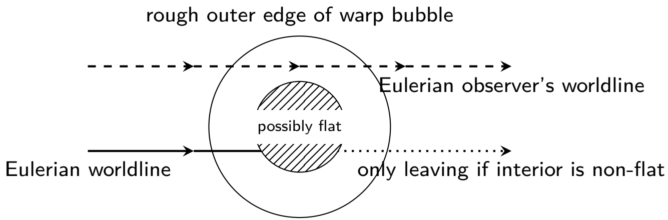
- If $K \neq 0$, Eulerian obs. see: $K \simeq 0 \to K \neq 0 \to K \simeq 0$ (due to asymptotics)
- In their proper time τ, however:

$$\text{NEC} \qquad \Rightarrow \qquad \frac{\mathrm{d}K}{\mathrm{d}\tau} \leq -\frac{3}{2} \operatorname{tr}\left([K_{ij}^{\mathrm{tf}}]^2\right)$$

- So, either:
  - K decreases monotonically if NEC fulfilled ✗, as $K \to 0$, eventually

Sketch of Proof for NEC Violation in Warp Drives

- NEC for $\mathrm{tr}(K_{ij}) =: K = 0, \implies \rho + \bar{p} = -\frac{1}{8\pi}\,\mathrm{tr}\left(K_{ij}K^{jk}\right) \leq 0$
- NEC for $K = 0$ fulfilled $\implies K_{ij} = 0 \implies$ Minkowski



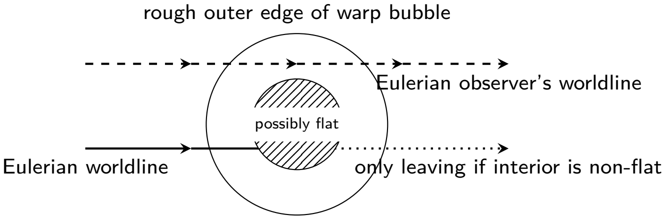
- If $K \neq 0$, Eulerian obs. see: $K \simeq 0 \to K \neq 0 \to K \simeq 0$ (due to asymptotics)
- In their proper time τ, however:

$$\text{NEC} \qquad \Rightarrow \qquad \frac{\mathrm{d}K}{\mathrm{d}\tau} \leq -\frac{3}{2}\,\mathrm{tr}\left([K_{ij}^{\mathrm{tf}}]^2\right)$$

- So, either:
  - K decreases monotonically if NEC fulfilled ✗, as $K \to 0$, eventually
  - K stays 0

Sketch of Proof for NEC Violation in Warp Drives

- NEC for $\mathrm{tr}(K_{ij}) =: K = 0$, $\implies \rho + \bar{p} = -\frac{1}{8\pi}\,\mathrm{tr}\big(K_{ij}K^{jk}\big) \leq 0$
- NEC for $K = 0$ fulfilled $\implies K_{ij} = 0 \implies$ Minkowski



- If $K \neq 0$, Eulerian obs. see: $K \simeq 0 \to K \neq 0 \to K \simeq 0$ (due to asymptotics)
- In their proper time τ, however:

$$\text{NEC} \qquad \Rightarrow \qquad \frac{\mathrm{d}K}{\mathrm{d}\tau} \leq -\frac{3}{2}\,\mathrm{tr}\big([K_{ij}^{\mathrm{tf}}]^2\big)$$

- So, either:
  - K decreases monotonically if NEC fulfilled ✗, as $K \to 0$, eventually
  - K stays 0 ✗, as now $K \neq 0$

Talking about T_{ab} without T_{ab}

- In a given orthonormal frame, the components have an easy interpretation:

$$(T_{\hat{a}\hat{b}})_{\hat{a},\hat{b}} = \begin{pmatrix} \rho & \mathbf{f}^t \\ \mathbf{f} & \begin{pmatrix} p_{\hat{1}} & T_{\hat{1}\hat{2}} & T_{\hat{1}\hat{3}} \\ T_{\hat{1}\hat{2}} & p_{\hat{2}} & T_{\hat{2}\hat{3}} \\ T_{\hat{1}\hat{3}} & T_{\hat{2}\hat{3}} & p_{\hat{3}} \end{pmatrix} \end{pmatrix}$$

where ρ energy density, $\mathbf{f}$ energy flux, $p_{\hat{\imath}}$ pressures, $T_{\hat{\imath}\hat{\jmath}}$ shear[22]

- In many contexts, one has relations between these components; 'equations of state'

[22] Assuming GR; hence $T_{ab} = T_{ba}$.

Talking about T_{ab} without T_{ab}

- In a given orthonormal frame, the components have an easy interpretation:

$$(T_{\hat{a}\hat{b}})_{\hat{a},\hat{b}} = \begin{pmatrix} \rho & \mathbf{f}^t \\ \mathbf{f} & \begin{pmatrix} p_{\hat{1}} & T_{\hat{1}\hat{2}} & T_{\hat{1}\hat{3}} \\ T_{\hat{1}\hat{2}} & p_{\hat{2}} & T_{\hat{2}\hat{3}} \\ T_{\hat{1}\hat{3}} & T_{\hat{2}\hat{3}} & p_{\hat{3}} \end{pmatrix} \end{pmatrix}$$

  where ρ energy density, $\mathbf{f}$ energy flux, $p_{\hat{\imath}}$ pressures, $T_{\hat{\imath}\hat{\jmath}}$ shear[22]

- In many contexts, one has relations between these components; 'equations of state'—but GR does not have a lot

[22] Assuming GR; hence $T_{ab} = T_{ba}$.

Talking about T_{ab} without T_{ab}

- In a given orthonormal frame, the components have an easy interpretation:

$$(T_{\hat{a}\hat{b}})_{\hat{a},\hat{b}} = \begin{pmatrix} \rho & \mathbf{f}^t \\ \mathbf{f} & \begin{pmatrix} p_{\hat{1}} & T_{\hat{1}\hat{2}} & T_{\hat{1}\hat{3}} \\ T_{\hat{1}\hat{2}} & p_{\hat{2}} & T_{\hat{2}\hat{3}} \\ T_{\hat{1}\hat{3}} & T_{\hat{2}\hat{3}} & p_{\hat{3}} \end{pmatrix} \end{pmatrix}$$

  where ρ energy density, $\mathbf{f}$ energy flux, $p_{\hat{\imath}}$ pressures, $T_{\hat{\imath}\hat{\jmath}}$ shear[22]

- In many contexts, one has relations between these components; 'equations of state'—but GR does not have a lot

- Instead of such equalities, find more general inequalities $\Rightarrow$ Energy Conditions (ECs)

[22] Assuming GR; hence $T_{ab} = T_{ba}$.

An Important Technicality

- There is some reliance on the 'Hawking–Ellis classification' of stress-energy tensors[23]
- This is based on classifying eigenvectors of $T^{\hat{a}}{}_{\hat{b}}$
- **Warning!**
  - $T^{\hat{a}}{}_{\hat{b}}$ is *not* necessarily symmetric, even in GR!
  - Equivalently, not every self-adjoint ('symmetric') endomorphism T is *real* diagonalizable if the scalar product g is Lorentzian
  - Equivalently, there is not necessarily a *real* tetrad diagonalizing T

[23] arXiv:1802.00865

An Important Technicality

- There is some reliance on the 'Hawking–Ellis classification' of stress-energy tensors[23]
- This is based on classifying eigenvectors of $T^{\hat{a}}{}_{\hat{b}}$
- **Warning!**
  - $T^{\hat{a}}{}_{\hat{b}}$ is *not* necessarily symmetric, even in GR!
  - Equivalently, not every self-adjoint ('symmetric') endomorphism T is *real* diagonalizable if the scalar product g is Lorentzian
  - Equivalently, there is not necessarily a *real* tetrad diagonalizing T
- Care is needed if diagonalizability of $T^{\hat{a}}{}_{\hat{b}}$ is assumed

[23] arXiv:1802.00865

An Important Technicality

- There is some reliance on the 'Hawking–Ellis classification' of stress-energy tensors[23]
- This is based on classifying eigenvectors of $T^{\hat{a}}{}_{\hat{b}}$
- **Warning!**
  - $T^{\hat{a}}{}_{\hat{b}}$ is *not* necessarily symmetric, even in GR!
  - Equivalently, not every self-adjoint ('symmetric') endomorphism T is *real* diagonalizable if the scalar product g is Lorentzian
  - Equivalently, there is not necessarily a *real* tetrad diagonalizing T
- Care is needed if diagonalizability of $T^{\hat{a}}{}_{\hat{b}}$ is assumed
- Much.

[23] arXiv:1802.00865

Extensions, Part I: Averaged Energy Conditions

Maybe, the issue is the ‘pointwise’. Instead average over various things:

Maybe, the issue is the ‘pointwise’. Instead average over various things:

- R_{ab} or T_{ab} over null curves $\longrightarrow$ **ANEC**

Extensions, Part I: Averaged Energy Conditions

Maybe, the issue is the 'pointwise'. Instead average over various things:

- R_{ab} or T_{ab} over null curves $\longrightarrow$ **ANEC**
- R_{ab} or T_{ab} over *achronal*, null curves $\longrightarrow$ **AANEC**

Extensions, Part I: Averaged Energy Conditions

Maybe, the issue is the 'pointwise'. Instead average over various things:

- R_{ab} or T_{ab} over null curves $\longrightarrow$ **ANEC**
- R_{ab} or T_{ab} over *achronal*, null curves $\longrightarrow$ **AANEC**
- G_{ab} or T_{ab} over timelike curves $\longrightarrow$ **AWEC**

Extensions, Part I: Averaged Energy Conditions

Maybe, the issue is the 'pointwise'. Instead average over various things:

- R_{ab} or T_{ab} over null curves $\longrightarrow$ **ANEC**
- R_{ab} or T_{ab} over *achronal*, null curves $\longrightarrow$ **AANEC**
- G_{ab} or T_{ab} over timelike curves $\longrightarrow$ **AWEC**
- R_{ab} or $T_{ab} - \frac{1}{2} T g_{ab}$ over null curves $\longrightarrow$ **ASEC**

Extensions, Part I: Averaged Energy Conditions

Maybe, the issue is the 'pointwise'. Instead average over various things:

- R_{ab} or T_{ab} over null curves $\longrightarrow$ **ANEC**
- R_{ab} or T_{ab} over *achronal*, null curves $\longrightarrow$ **AANEC**
- G_{ab} or T_{ab} over timelike curves $\longrightarrow$ **AWEC**
- R_{ab} or $T_{ab} - \frac{1}{2}Tg_{ab}$ over null curves $\longrightarrow$ **ASEC**

Still, especially (plausible) quantum matter can violate them.

Extensions, Part I: Averaged Energy Conditions

Maybe, the issue is the 'pointwise'. Instead average over various things:

- R_{ab} or T_{ab} over null curves $\longrightarrow$ **ANEC**
- R_{ab} or T_{ab} over *achronal*, null curves $\longrightarrow$ **AANEC**
- G_{ab} or T_{ab} over timelike curves $\longrightarrow$ **AWEC**
- R_{ab} or $T_{ab} - \frac{1}{2}Tg_{ab}$ over null curves $\longrightarrow$ **ASEC**

Still, especially (plausible) quantum matter can violate them.

Especially ANEC and AANEC found use, *e.g.*, in the topological censorship theorem, see arXiv:gr-qc/9305017

Extensions, Part II: Quantum Energy Inequalities[24]

- Instead of trying to guess the conditions, start from first principles.

[24] See arXiv:1208.5399, or arXiv:2108.12668

Extensions, Part II: Quantum Energy Inequalities[24]

- Instead of trying to guess the conditions, start from first principles.
- Choose a quantum field, compare possible (Hadamard) states with a reference state (*e.g.*, normal-ordered, . . .)

[24] See arXiv:1208.5399, or arXiv:2108.12668

Extensions, Part II: Quantum Energy Inequalities[24]

- Instead of trying to guess the conditions, start from first principles.
- Choose a quantum field, compare possible (Hadamard) states with a reference state (*e.g.*, normal-ordered, . . .)
- Get a lower (negative) bound that cannot be broken

[24] See arXiv:1208.5399, or arXiv:2108.12668

Extensions, Part II: Quantum Energy Inequalities[24]

- Instead of trying to guess the conditions, start from first principles.
- Choose a quantum field, compare possible (Hadamard) states with a reference state (*e.g.*, normal-ordered, . . .)
- Get a lower (negative) bound that cannot be broken
- Some averaged energy conditions can be regained sometimes

[24] See arXiv:1208.5399, or arXiv:2108.12668

Extensions, Part II: Quantum Energy Inequalities[24]

- Instead of trying to guess the conditions, start from first principles.
- Choose a quantum field, compare possible (Hadamard) states with a reference state (*e.g.*, normal-ordered, . . .)
- Get a lower (negative) bound that cannot be broken
- Some averaged energy conditions can be regained sometimes
- Finally a definitive application of algebraic QFT

[24] See arXiv:1208.5399, or arXiv:2108.12668

Kuchař's Criticisms

1. Wrong localization for relativistic particles
   $\rightarrow$ Covariant POVM allow approximate Newton–Wigner localization[2]

2. Constraint violation
   $\rightarrow$ PW's conditional probabilities as gauge-fixed expressions of a gauge-invariant ('clock-neutral') quantity[1]

3. Predict wrong propagators
   $\rightarrow$ Resolved by introducing a two-time conditional probability[1]

[2] Höhn *et al.* arXiv:2007.00580
[1] Höhn *et al.* arXiv:1912.00033

Unruh and Wald's Criticism

Lack of monotonicity (variant of Pauli/Schrödinger result)
$\rightarrow$ Covariance of POVM saves the day[1]

[1] Höhn *et al.* arXiv:1912.00033